

Front Range Aggregates Optimizes Feeder Movements at Its Quarry

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Front-end loaders extract sand and gravel (aggregate) from a pit and haul it to a feeder, which releases the aggregate onto a conveyor belt that is connected to a stockpile; the material is subsequently distributed to a processing plant. As mining progresses, the mining frontier moves farther away from the feeder, increasing loader cycle time. In turn, plant managers add loaders to maintain production rates. Eventually, the feeder must be moved closer to the mining frontier. Such a move requires shutting down production so that a crew can move the feeder. Historically, because a feeder movement did not occur until all loaders were in operation, such feeder movements overtaxed the loaders and lacked advance warning. We present a model to determine how often the feeder should be moved to the mining frontier. A shortest-path algorithm can quickly solve our model to minimize feeder movement and loader cycle-time costs. This model revolutionizes how aggregate companies, specifically Front Range Aggregates, plan feeder movements.

Key words: optimization; network models; shortest-path models; applications; quarry-mining operations; production planning.

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Aggregate consists of (i) crushed stone, and (ii) sand and gravel, including cobble. Both categories include boulders. In 2007, US crushed stone output was 1.59 billion tons with a value of \$14 billion, distributed over 1,370 companies principally operating 3,360 quarries (Willett 2008). US sand and gravel output was an estimated 1.17 billion tons in 2007. Its value was about \$8 billion, distributed over 4,000 companies (Bolen 2008).

In 2004, Front Range Aggregates, with estimated annual sales of between \$1 million and \$2.5 million, acquired the Parkdale property, a 100-acre glacial granite deposit at the southern end of the Colorado Front Range. The deposit contains sand, gravel, cobble, and boulders. The sand and gravel resulted from a load of sediment that meltwater deposited in front of a glacier many thousands of years ago; the cobble arrived by glacial transport; the boulders were the result of a flood. Previous investigations suggest

that a few million tons of sand, gravel, and granite lie within the deposit. Another aggregate company had owned and operated the deposit from 1999–2003 before declaring bankruptcy in March 2003.

At its Parkdale Property, Front Range Aggregates employs a plant manager and 13 hourly workers in five 10-hour production shifts and five 8-hour maintenance shifts per week. Typically, three people perform maintenance and the remainder work on the production shift. The company uses five major pieces of machinery. Three wheel loaders, each with a capacity of approximately 8–10 tons and a capability of traveling at about 30 feet per second (20 mph), extract aggregate from the deposit and transport it to a feeder (a large funnel). A skid-steer loader clears large rock and other debris from the area around the loader paths and near the feeder. A track excavator is used for dewatering ditches. For the purposes of our study, we assume that all equipment in the mine has been

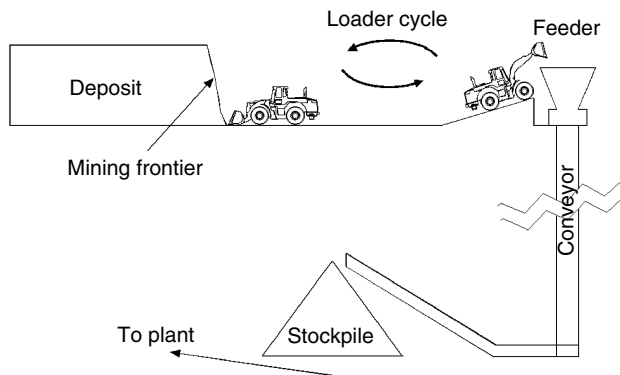


Figure 1: Loaders excavate material from a mining frontier that separates the excavated from the unexcavated material. They haul the material to a feeder, where the material is funneled onto a conveyor belt, transported to a (surge) stockpile, and then fed to a processing plant.

determined; i.e., we make no decisions regarding the type of equipment to purchase or use. We also assume that the loaders are placed into operation in decreasing order of their efficiency.

The feeder regulates the flow of suitably sized rock onto a conveyor belt, which runs to a stockpile near a processing plant. The stockpile acts as a buffer so that the plant receives material at a constant rate. Figure 1 depicts the flow of material from the mining frontier, i.e., the area from which material is currently being extracted, to the feeder, then to the stockpile, and finally to the plant (Figure 2).

At the processing plant, the rock is separated by size using gravity and agitation. Excavators first remove boulders from the material, which is then passed through a set of steel bars (the grizzly) that separates cobble from the rest of the material. A jaw crusher breaks the remaining material finely enough to pass through two rectangular 8 by 20-foot vibrating deck screens that categorize the aggregate as: (1) not fine enough to pass through the first screen, (2) fine enough to pass through the first, but not the second screen, or (3) fine enough to pass through both screens. The resulting fine-granularity rock bypasses the primary cone crusher, while the coarser rock is ground more finely than in the jaw crusher. Three 8 by 20-foot stacked, vibrating deck screens divide the aggregate into four different granularities. From there, the rock may be categorized into pea gravel or concrete stone, as sand and washed with a sand screw before being used in concrete, or the rock may be sent



Figure 2: A loader dumps material into a feeder (top); the feeder is connected to a series of conveyor belts (bottom) that transports the aggregate to a stockpile prior to transporting it to the processing plant. Sources: loader photo retrieved from Web August 31, 2005; conveyor photo retrieved from <http://www.rocksystems.com/images/inventory/506-P-4.jpg>.

through either another three deck screens (7 by 20 feet in dimension) and/or a secondary cone crusher and categorized as 3/4-inch rock, 1/2-inch rock, or crusher fines. Splitters, the granularity of the deck screens, and the settings on the crushers regulate the size of rock that flows through various parts of the plant and ultimately into piles of finished product. These settings are important for meeting demand specifications, and the combination of settings allows the plant to run smoothly. For example, a splitter might ensure that fine material avoids a crusher, and a crusher produces smaller, more-jagged rocks to meet product specifications. Figure 3 depicts a schematic of the processing plant.

The company's end-products are sand, concrete stone, 3/4-inch rock, crusher fines, 1/2-inch rock, pea

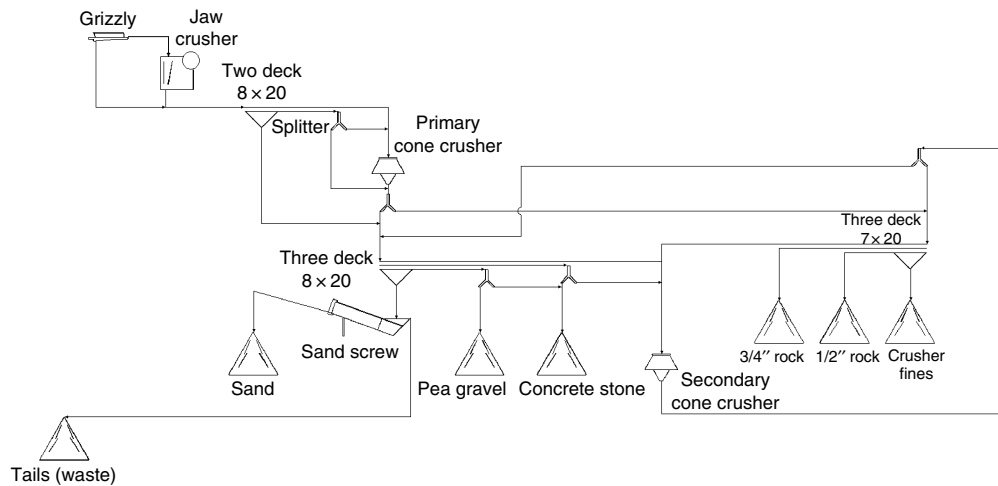


Figure 3: The processing plant uses gravity and agitation to recover sand, pea gravel, concrete stone, two sizes of rock, and crusher fines from the aggregate. Crushers grind the rock while splitters and screens appropriately categorize it.

gravel, and cobble, with percentages of total output as 29, 18, 18, 14, 11, 7, and 3 percent, respectively. Boulders contribute a negligible amount to overall output. Sand, pea gravel, concrete stone, 3/4-inch rock, and 1/2-inch rock are used in the production of concrete and asphalt. Crusher fines and sand are used primarily for structural fills and beddings (e.g., as a reliable material on which a road can be built). Cobble, rock fragment between 2.5 inches and 10 inches in diameter, is used for erosion control in construction such as housing developments. Boulders are used as landscaping materials. A loader transports end products from piles of output at the plant directly to a rail car on a track that circumscribes the plant. This spur connects to a main rail line, which runs to Front Range transloading sites, where customers (ready-mixed concrete and asphalt manufacturers, construction contractors, and landscape designers) use trucks to transport the end product to their sites.

Economic Analysis

Economic analysts use geographic sampling, expected operational costs, and market information to determine if a mine is likely to be profitable. Geological investigators sample the field by drilling holes to find the approximate composition of the proposed mining area. They consider fixed costs associated with depreciation, exploration, development, permitting,

insurance (e.g., for floods), equipment, employee salaries, and county, state, and federal income taxes. Analysts must also consider the cost of building and maintaining a rail line on the property. In addition to fixed costs, expenses include wages of hourly employees, and costs associated with plant and equipment maintenance and repair, loader fuel, utilities (primarily electricity), and stripping (i.e., removing vegetation, topsoil, and overlying waste material from the deposit).

Economic analysts compare these costs to the current average selling prices for aggregate products. If a mine could yield a profit, the mining company must ensure that its operations are efficient and its extraction costs are low. Some gravel pits and aggregate quarries operate on such a thin margin that the company realizes a profit only when, at the end of the mine's life, the company seals the resulting hole and sells it to a city or county for use as a water reservoir.

Related Work

A large body of optimization research in surface mining addresses the *ultimate pit limit problem*, i.e., determining the boundaries of the mine such that the extracted material is, on average, profitable. Ahuja et al. (1993) show that the ultimate pit limit problem is a maximum-flow model. In this context, research often focuses on making efficiency improvements to

Lerchs and Grossmann's (1965) seminal exact algorithm for determining ultimate pit limits, e.g., Underwood and Tolwinski (1998) and Hochbaum (2001).

Other authors provide heuristic approaches to the problem without including assumptions, e.g., Sevim and Lei (1998). Wilke and Reimer (1977) and Johnson (1969) formulate linear programming production-scheduling models for use in the short and long terms, respectively. Generally speaking, their models determine the amount of material to extract and process either over a single period in the short term or over multiple periods in the long term to maximize profits subject to operational (e.g., block sequencing) or quality constraints, and production-capacity constraints. Johnson (1969) suggests a decomposition procedure to solve problem instances. Fytas et al. (1993) combine simulation (to model long-term decisions) and linear programming (to model short-term decisions) to maximize cash flow over the life of a mine. Akaike and Dagdelen (1999), Erarslan and Celebi (2001), Johnson et al. (2002), and Caccetta and Hill (2003) propose integer programming generalizations of the above. Onur and Dowd (1993) and Ramazan and Dimitrakopoulos (2004) take the ultimate pit limits as given; however, they consider the inclusion of roadways in a mine and variability in the grade of a production block, respectively. For a more complete review of the literature on open pit mine scheduling, we refer the reader to the references contained in the above papers, especially in Erarslan and Celebi (2001), who provide a thorough review of optimizing open pit mine scheduling operations.

Aggregate quarries are homogeneous and usually relatively shallow because they contain solid rock or shale. Ultimate pit limits are relevant in a 2,000 foot-deep deposit that might have metal of varying qualities at various depths; however, low-quality aggregate is simply left unmined because the quality of the material deeper in the shallow pit is not likely to improve. Aggregate mining operations are fundamentally different, and, in some sense, less complicated than those described in the generalizations of the work of Lerchs and Grossmann (1965). However, aggregate mines do use some quantitative models. For example, Norton (1991) describes mine- and quarry-design software to locate haulage roads and dewatering pipelines, and to quantify the economic

impacts of certain mine-planning decisions. Gove and Morgan (1994) describe software designed to balance the number and type of trucks and the number and type of loaders in a simple truck-and-shovel operation. The goal is to meet production levels while minimizing operating costs. Our operation is slightly different in that it uses a conveyor belt, rather than trucks, to "haul" the excavated material. Additionally, neither of these software applications describes a formal optimization model.

Optimization applications in underground mining also exist, as Carlyle and Eaves (2001), Kuchta et al. (2004), and Sarin and West-Hansen (2005) discuss, although there are fundamental differences in mining methods and their corresponding models.

Previous Method of Operation

Prior to using an optimization model, mine managers usually placed the feeder in a centralized, "intuitive" location and ran loaders between the feeder and the mining frontier, adding loaders as the distance between the feeder and the mining frontier increased. However, mine managers would become reluctant to remove loaders from operation to perform maintenance; the delayed maintenance had adverse long-term effects on equipment functionality and resulted in excessively high operational costs. When all loaders were in use and the mine managers realized that the loader cycle times were so long that planned production levels could not be met, mine managers stopped operations and moved the feeder. However, there was such a short time between the realization that the managers had to move the feeder and its actual movement that stockpiles could not meet demand during stopped production. Furthermore, the lack of advance planning precluded other types of maintenance, e.g., plant modifications, from being scheduled simultaneously with the feeder movement; this increased total downtime at the facility.

Hence, the lack of a robust model to systematically determine feeder-movement policies had a twofold adverse effect: (1) movement policies were suboptimal, and (2) lack of advance warning of a feeder movement resulted in a crisis when the feeder was moved. Our model mitigates both of these problems.

Optimization Model

Our mathematical model determines feeder movements in multiples of 20 feet in a forward direction down the midpoint of a straight-line trajectory of length L and width $2a$ from an initial (predetermined) location. When the area along the trajectory is completely mined out, the feeder is moved laterally a distance of $2a$; the same moves can be followed along the trajectory back in the opposite direction. In total, the feeder is moved laterally $p - 1$ times, where $2ap$ represents the total width of the pit. We assume that the aggregate is fairly homogeneous in its composition, which precludes a need to move the feeder to another area of the mine to meet production requirements for different product types. We also assume that the area in which we make feeder movements is devoid of irregularities, i.e., areas that must be avoided, such as a lake or an already mined-out area. Figure 4 shows a typical mining area with irregularities at each end, and a regular, rectangular area divided into widths of $2a$ along which our model determines feeder movements. Mine planners can use our model to enumerate alternatives and evaluate them to determine feeder movements in the irregular areas of the pit.

We model this problem as a dynamic program on a network consisting of collections of nodes and arcs,

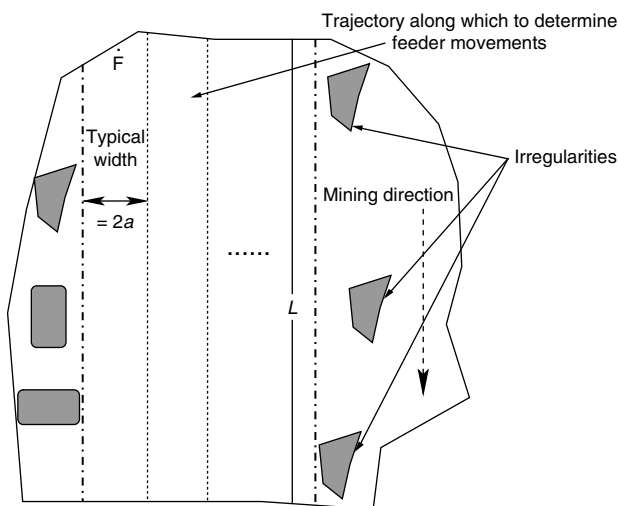


Figure 4: The rectangular area (bounded horizontally by the dotted and dashed lines) is suitable for dividing into trajectories of length L and width $2a$ along which our model determines how far to move the feeder (F) at one time.

where each node in the network is spaced 20 feet from the previous node and represents a possible feeder location. Forward arcs exist between a node and all nodes of greater distance than the current node. An arc (i, j) is used in the network if the feeder has been at location i and is moved to location j . Each arc, if used, incurs a cost, which is the sum of a fixed and a variable cost.

Our optimization model requires input data on the fixed cost of a feeder movement and the variable cost of traveling from the mining frontier to the feeder, dumping a load of excavated material, and returning to the mining frontier. The feeder is moved using one or more 20-foot conveyor-belt extensions. These extensions, and the labor and equipment required to move the feeder, are sunk costs. The mine owns the conveyor belt extensions and equipment; mine workers, who would otherwise be performing other jobs (e.g., running the loaders) move the feeder. Therefore, the fixed cost of moving the feeder does not consist of labor or equipment costs, but rather includes only the opportunity cost of deferring production for the time required to move the feeder. We can compute this opportunity cost as the product of the profit margin per ton of aggregate, the required production rate per day (based on initial economic analysis, including anticipated selling prices), and the number of days required to move the feeder. This fixed cost, which is incurred each time the feeder is moved, is invariant with the distance that the feeder is moved.

The variable cost is more complicated to compute. Given that the feeder is at location i and is not moved again until the mining frontier reaches location j (a specified number of feet away from i), we can compute the average haul distance for a loader moving between the feeder and the mining frontier by approximating the number of feet a loader moves, on average, between i and j within a rectangle with a length of $j - i$ and a width of a (because the feeder position is symmetric about the width). We compute the distance based on rectangular shapes, which, when divided into small squares (appendix), closely approximate arc-like frontiers. In the appendix, we also show the high dependency of average costs on the chosen geometry, and therefore the criticality of an appropriately chosen geometry. Using this average haul distance, we can compute the average cycle

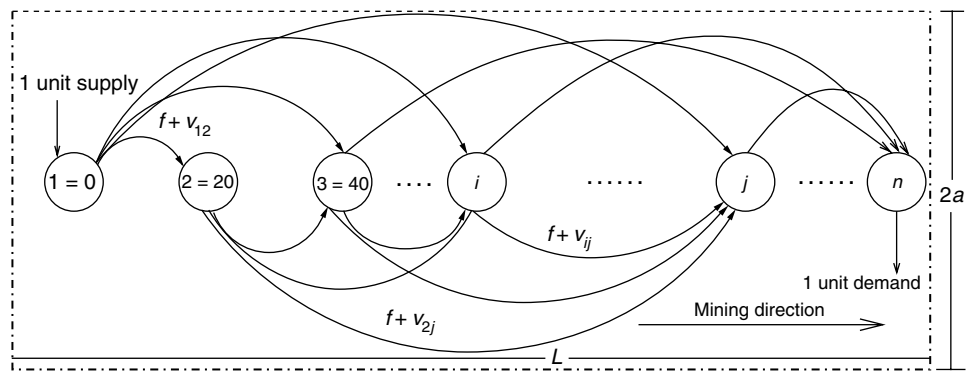


Figure 5: The depiction of the dynamic program (rotated 90 degrees with respect to Figure 4) has nodes that represent possible feeder locations and arcs that represent possible feeder movements. The parameter f represents the fixed cost of moving the feeder; v_{ij} represents the variable cost of mining from location i to location j for a width of $2a$ feet and at a depth of 30 feet. Because conveyor-belt extensions are added in fixed increments of 20 feet, there is a finite number of (i, j) pairs, where the difference between j and i is a multiple of a 20-foot increment with a minimum positive distance of 20 feet and a maximum positive distance of approximately $20 * \lfloor L/20 \rfloor$ feet.

time for a loader, i.e., the time required to lower the bucket, pick up a full load, raise the bucket, return to the feeder with the full load, lower the bucket to the feeder and dump the full load in, and return to the mining frontier. The cycle time for each loader is based on its speed (factoring in loader efficiency and the loader-path topology) and on the distance the loader travels between the mining frontier and the feeder. From these cycle times, we can compute the average number of loads carried over a 10-hour shift. Using the number of tons each full load contains, we can compute the number of tons that a single loader can transport to the feeder within a 10-hour workshift and we can determine the number of loaders necessary to meet production requirements; from that, we can determine the total variable cost per day of running the loaders based on the cost of the labor, fuel, and maintenance necessary to run each type of loader. We can then compute the variable cost per ton as the quotient of the variable cost per day and the production level per day. The volume of material moved between i and j is the amount of aggregate in a three-dimensional area that is $j - i$ feet long, $2a$ feet wide, and 30 feet deep. The product of this quantity with its density yields the number of tons moved. The total variable cost, v_{ij} , of running the loaders between i and j is then the product of the variable cost per ton and the total number of tons

moved in a rectangular volume $j - i$ feet long, $2a$ feet wide, and 30 feet deep. The sum of the fixed and variable costs between locations i and j is the cost on arc (i, j) . From the fixed and variable costs, the model determines where to next place the feeder given its current location. Figure 5 depicts the topology of our dynamic programming model; we give the algebraic formulation in the appendix.

Recall that we assume that the area in which we make feeder movements is devoid of irregularities, and we know the trajectory width. If the former assumption is violated, we might determine feeder placements to excavate these areas that contain irregularities by enumerating all operationally reasonable alternatives, and evaluating the associated costs. Similarly, if we violate the latter assumption, we can enumerate a reasonable set of widths (e.g., integer multiples of the total width), determine the least-cost solution for each trajectory width using our model, compute the total cost of mining p trajectories of width $2a$ in the deposit, and select the width that yields the minimum total cost. Practical implementation of feeder movements would require that the feeder be placed at approximately, not exactly, the location at which mining ceases prior to the feeder move; this precludes unsensible behavior, such as undermining the earth on which the feeder is sitting, or tramping up a steep mound of dirt.

Dynamic programming models with a structure similar to ours have been successfully applied in industries other than in mining, e.g., to minimize travel expenses for federal employees (Huising et al. 2001), to determine the locations at which larvicide should be sprayed (Solomon et al. 1992), and to schedule projects (Darrah 1984).

Results and Comparisons

The Parkdale property consists of a deposit divided into two parts, each 1,200 feet wide. One deposit part is 1,800 feet long; the second is 1,000 feet long. These deposit parts are free from irregularities. Our goal is to determine for each deposit part: (1) an optimal trajectory width from a set of candidate widths, and (2) optimal feeder moves along the trajectory of that given width. Mine planners are interested in considering evenly spaced trajectory widths, i.e., trajectory widths that are multiples of 1,200 feet. Therefore, for each of the two deposit parts, we compute the optimal trajectory moves and the associated total cost of mining the deposit for widths of 100, 200, 300, 400, 600, and 1,200 feet. Because each network model solves quickly, the computational burden of solving six model instances for each deposit part is minimal.

We implement the network as a shortest-path model and process the results in a series of Excel worksheets. In the first worksheet, we accept cost information and the desired trajectory width and compute arc costs based on this information (appendix). We set up the second worksheet to handle the shortest-path network topology of a deposit part of a given length; as inputs, it takes the costs in the first worksheet and computes the optimal series of feeder movements along a trajectory of that given length for the width specified in the first worksheet. The underlying solver, invoked with a click of the “solve” button, is the Jensen Network Solver (coded by Paul Jensen at the University of Texas at Austin); it operates in a spreadsheet and can solve network models as large as ours (i.e., models that contain 50–100 nodes and a few thousand arcs) in a matter of seconds. The solver displays the solution and objective function value in that second spreadsheet, in which we have created a macro, “capture flow,” that translates the arcs with a corresponding value of 1 in the optimal solution to feeder locations and places these feeder moves in another worksheet.

Figure 6 shows the worksheet in which the network is solved as a shortest-path model. The right side depicts representative cost information that is collected on the first of the three worksheets. In the upper right corner, the spreadsheet shows the results of the “capture flow” macro, which translates the shortest-path solution into feeder locations and places the information in a third worksheet.

For the 1,800-foot-length part of the deposit, optimal feeder moves are spaced either 440 or 460 feet apart, and occur along a trajectory width of 1,200 feet. Specifically, the optimal mining plan is to place the feeder at an initial location in the deposit that is 600 feet away from its sides, excavate until the mining frontier reaches 440 feet from that initial location, move the feeder approximately 440 feet, excavate until the mining frontier reaches 900 feet from the initial location, move the feeder 460 feet, excavate until the mining frontier reaches 1,360 feet from the initial location, move the feeder to that point, and then excavate until the end of that part of the deposit. Optimal feeder moves for the 1,000-foot part of the deposit are similar: for a 1,200-foot-deposit width, move the feeder once to 500 feet. In each case, no more than three loaders are needed.

Without our model, mine planners would ignore feeder movements until production requirements could no longer be met using the existing fleet of loaders. If we compute the costs associated with this suboptimal policy using the trajectory widths that are multiples of 1,200 feet, the lowest-cost widths for the 1,800 and 1,000-foot-deposit parts are 1,200 and 600 feet, respectively. For the 1,800-foot part of the deposit, the suboptimal policy would result in the feeder being moved once, at 1,260 feet (after which three loaders could not meet production requirements). For the 1,000-foot part of the deposit, no feeder move would be necessary. For the 1,800-foot part of the deposit, the cost of the suboptimal policy is 14 percent higher than the optimal solution; results for the 1,000-foot part of the deposit are similar; the cost of the suboptimal policy is 13 percent higher. The savings gained from the use of our model apply to costs that constitute about half of the operating expenses at the quarry. (The other half is associated with plant operation and train loading.) Note that these percentages are actually lower bounds on the cost savings for two

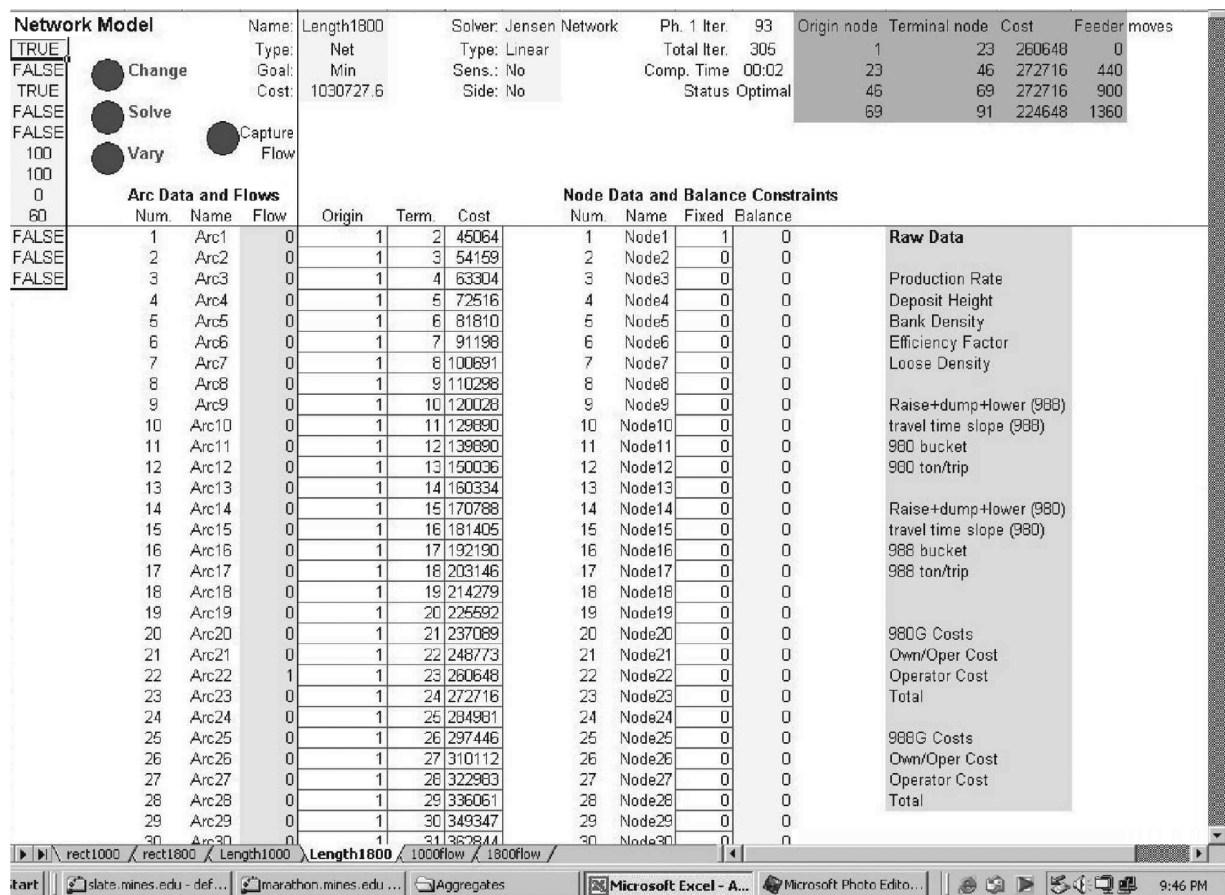


Figure 6: We implement the network model and process the results using three Excel worksheets, which gather cost information (see the right side in this example), use this information and an underlying network solver (i.e., the Jensen solver) to compute an optimal solution, and translate the optimal solution into a series of feeder movements (see the upper right corner).

reasons: (1) we assume that the mine would choose the optimal trajectory width while determining feeder moves without our model, and (2) we do not consider the detrimental effects of poor loader-maintenance schedules, lack of coordination between feeder movements and scheduled downtime at the plant, and low customer satisfaction because of late delivery resulting from suboptimal feeder movements.

We began to develop the model at the start of 2004. After working on it for a few months, we realized that a dynamic program solvable as a shortest-path model would accurately capture the relevant decisions. In early 2005, we embedded our application in a spreadsheet; we transitioned the model to the mine that fall. While developing the model, two coauthors

met with the mine manager to verify the model's usefulness and validate the assumptions and calculations therein. Upon its completion, one coauthor (and her research assistant) visited the mine site for several hours to install the software on the mine manager's computer, explain the software to him, and give him an instruction sheet. That manager, who has been using the software since its installation in October 2005, now runs the model himself; he can make changes to the number and type of operating equipment and to the characteristics of the operating equipment, such as costs and efficiency factors.

Front Range Aggregates began to implement the model recommendations mentioned above at its Parkdale property in January of 2006. Prior to the

implementation, the company used the authors' recommendations to mine its irregular areas. Although these recommendations were not the result of the shortest-path solutions, they are relevant to the discussion because: (1) they validated the cost structures we use in our network model, and (2) they were done using our advice regarding the (small) enumeration procedure we mention above. Subsequent to mining the irregularities, the mine manager has been mining the 1,000-foot part of the deposit, having placed the feeder according to our recommendations. The model is closely tracking the actual loader time required to feed the plant as the frontier moves away from the feeder. The manager plans to continue to move the feeder according to our recommendations. More than one year later (as of this writing), the company has verified the cost savings we estimate, and has realized pit-haulage cost improvements approaching 33 percent compared with the previous year's levels. These savings are better than predicted because of both the conservative nature of our estimates and external factors. For example, the plant was able to run continuously because of, *inter alia*, favorable market conditions; additionally, the loaders experienced no major mechanical problems. The company plans to continue to use the model where mine reserve characteristics allow its application.

The usefulness of our dynamic program is evident from this cost comparison. Our model uses only basic data and is relatively straightforward to implement; the model does not require understanding of sophisticated mathematical modeling and yields easily executable and simple-to-understand policies. Finally, it produces solutions that improve the way in which aggregate quarries in general can plan feeder movements.

Appendix

Formulation

Indices

i, j = location of feeder along a straight-line trajectory.
 n = last node = $\lfloor L/20 \rfloor$.

Sets

N = nodes, i.e., feasible feeder locations (multiples of 20 feet from an initial location).

A = feasible adjacent feeder-location pairs (multiples of 20 feet apart, terminal node > incident node).

Parameters

f = fixed cost of moving feeder (\$).
 v_{ij} = variable cost of extracting and hauling all mined material in the pit between location j and the feeder located at i given a width of $2a$ feet and a depth of 30 feet (\$).

Decision Variables

$$x_{ij} = \begin{cases} 1 & \text{if the feeder is moved along its straight-line trajectory to location } j, \text{ having last been moved to location } i, \\ 0 & \text{otherwise.} \end{cases}$$

Objective

$$\min \sum_{(i,j) \in A} (f + v_{ij})x_{ij}.$$

Constraints

$$\sum_j x_{1j} = 1, \quad (1)$$

$$\sum_i x_{ij} = \sum_k x_{jk} \quad \forall j \in N, j \neq 1, n, \quad (2)$$

$$\sum_i x_{in} = 1, \quad (3)$$

$$0 \leq x_{ij} \leq 1 \quad \forall (i, j) \in A. \quad (4)$$

We minimize the total fixed costs of moving the feeder and the variable extraction costs between feeder movements. The constraint set forms simple flow-balance requirements for a shortest-path problem, with a single unit supplied at the origin (node 1) and a single unit demanded at the destination (node n).

Deriving the Average Haul Distance for a Loader

Suppose that we assume, without loss of generality, that the feeder is at the corner of an $a \times b$ rectangle (Figure A.1). We wish to compute the average haul distance between the feeder and the mining frontier, where the mining frontier lies within the rectangle. We divide the rectangle into small squares, each of length Δ , and approximate the distance between the feeder and the mining frontier within one of the small

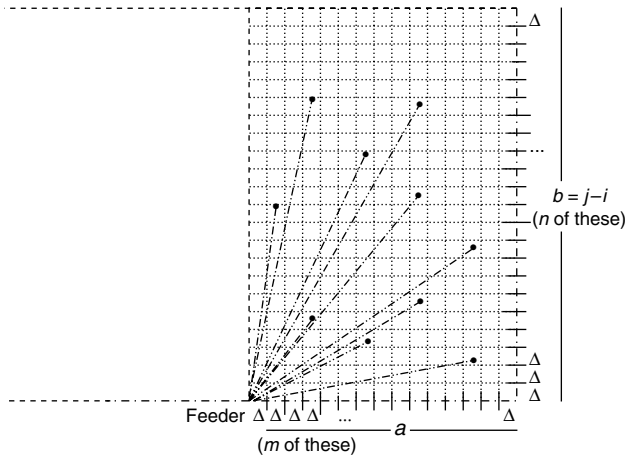


Figure A.1: To derive the average haul distance for a loader between the feeder and the mining frontier in an $a \times b$ rectangle, we divide the rectangle into squares, and take the average distance from the feeder (at the bottom left corner) to the center of all squares inside the rectangle.

squares as the distance between the feeder and the center of said square. The average haul distance is then the average (two-way) distance between the feeder and the center of all such squares within the $a \times b$ rectangle. The lines in Figure A.1 comprised of both dashes and dots show some representative distances.

If we assume that the horizontal side of the rectangle consists of m intervals of length Δ , and that the vertical side of the rectangle consists of n such intervals, then $(i\Delta + \Delta/2, j\Delta + \Delta/2)$ for $i = 0, \dots, m-1$ and $j = 0, \dots, n-1$ are the x and y coordinates, (\hat{x}_i, \hat{y}_j) , respectively, of the center of any small square within the $a \times b$ rectangle. The distance between the feeder and the center of the (i, j) th square is

$$d_{ij} = \sqrt{\Delta^2(0.5 + i)^2 + \Delta^2(0.5 + j)^2},$$

and the haul distance D between the feeder and the mining frontier in the $a \times b$ rectangle is the average distance across all such squares as the size of the squares approaches zero:

$$\begin{aligned} D &= \lim_{m, n \rightarrow \infty} \frac{1}{nm} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} d_{ij} = \frac{1}{ab} \int_0^a \int_0^b \sqrt{x^2 + y^2} dx dy \\ &= \frac{\sqrt{a^2 + b^2}}{3} + \frac{a^2}{6b} \ln \left(\frac{a + b + \sqrt{a^2 + b^2}}{a - b + \sqrt{a^2 + b^2}} \right) \\ &\quad - \frac{b^2}{6a} \ln \left(\frac{b}{a + \sqrt{a^2 + b^2}} \right). \end{aligned} \quad (5)$$

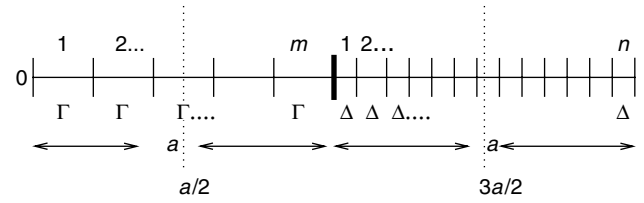


Figure A.2: We show using a one-dimensional example how the approximation for the distance between the feeder and the mining frontier depends on the geometry of the section.

Equation (5) follows from the definition of integral as a limit of Riemann sums. We can then calculate the integral by dividing the rectangular domain into two equal triangles and doing a polar coordinate transformation in each triangle.

Geometry of the Mine

Note that the approximation for the distance between the feeder and the mining frontier depends on the geometry of the section, i.e., in our case, that we have elected to use a rectangle for the shape of the mined area, and squares for the subdivided areas, rather than other shapes. To see the importance of carefully selecting a geometry that represents or closely approximates the actual loader movements, consider the one-dimensional example in Figure A.2. We separate a line segment of length $2a$ into two equal segments, each of length a , and then subdivide one such segment into m subsegments and the other segment into n subsegments, where each of the first set of subsegments has width $\Gamma = a/m$ and each of the second set has width $\Delta = a/n$.

Then, the average distance $D(m, n)$ from the left endpoint to the $n + m$ segments along the line is

$$\begin{aligned} D(m, n) &= \frac{1}{m+n} \left(\sum_{i=1}^m i\Gamma + \sum_{i=1}^n (a + i\Delta) \right) \\ &= \frac{1}{m+n} [\Gamma m(m+1)/2 + an + \Delta n(n+1)/2] \\ &= \frac{a}{m+n} [(m+1)/2 + n + (n+1)/2] \\ &= a \left(\frac{1}{2} + \frac{1 + 1/n}{m/n + 1} \right). \end{aligned} \quad (6)$$

If we let $m = kn$, where k is a positive integer greater than 1, and substitute this value for m into (6),

as $n \rightarrow \infty$, we obtain

$$D(kn, n) = a \left(\frac{1}{2} + \frac{1 + 1/n}{k + 1} \right) \rightarrow \frac{a}{2} \left(\frac{k + 3}{k + 1} \right) \in [a/2, a].$$

Hence, depending on the value for k , the average distance from the left endpoint to any point on the line lies somewhere between halfway between the origin and the midpoint of the line segment, and the midpoint of the line segment.

If, however, we reverse the relationship between m and n such that $n = km$, by the same logic, it follows that the average distance from the left endpoint to anywhere on the line is

$$D(m, km) \rightarrow \frac{a}{2} \left(\frac{3k + 1}{k + 1} \right) \in [a, 3a/2]$$

as $m \rightarrow \infty$; thus, the average distance from the left endpoint to any point on the line lies somewhere between the midpoint of the line segment, and halfway between the midpoint of the line segment and the end of the segment.

Hence, we conclude that we cannot arbitrarily assign dimensions to subsections of the mining frontier to approximate average loader travel distance. Instead, we must carefully choose a pattern that closely approximates actual loader movement.

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Thomas M. Maul, General Manager, Front Range Aggregates, LLC, 3655 Outwest Drive, Colorado Springs, CO 80910, writes: “Planning of pit feeder movements in the aggregates industry is rarely performed using sophisticated techniques. Traditionally, mine managers have tended to defer the cost and effort of this task until absolutely necessary, i.e., until production requirements can no longer be met with full utilization of the existing mobile equipment fleet. Contributing to the problem is the fact that many aggregate mines exist on a scale that does not justify

a management “team,” many having a single person managing the entire mine. Such a person does not have a great deal of time to perform lengthy optimization analyses, and so the planning of unit operations is driven by the desire to meet short-term objectives; not long term performance.

“The spreadsheet model described in this paper allows mine planners (managers) to quickly enter relevant operating cost data, and describe pit dimensions. The optimization model then uses this data to compute a sequence of pit feeder movements which minimizes pit operating costs over the life of the pit. Changing the data inputs is relatively simple and does not take much time, making the model practically applicable to most equipment and pit configurations found in typical alluvial aggregate mining operations.

“Front Range Aggregates (FRA) implemented the model recommendations for pit feeder placement at our Parkdale aggregates mine in January of 2006, and to date (approximately 12 months of operation) have realized pit haulage cost improvements approaching 33% compared to 2005 levels. We have not yet completed one full cycle of pit feeder movements, but operational characteristics of the pit are closely conforming to those predicted in the model at this point, and we expect full cycle results to be consistent with those predicted by the model over the life of the mine, barring unforeseen external influences.

“FRA will continue to utilize the model where mine reserve characteristics allow its application, and we expect it to be a useful tool for our operation staff in our efforts to optimize operational performance.”