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To cite this article:

Barry King, Alexandra Newman (2018) Optimizing the Cutoff Grade for an Operational Underground Mine. *Interfaces*

Published online in Articles in Advance 01 Aug 2018

. <https://doi.org/10.1287/inte.2017.0934>

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Optimizing the Cutoff Grade for an Operational Underground Mine

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Received: May 4, 2016

Revised: January 3, 2017; June 17, 2017

Accepted: August 30, 2017

Published Online in Articles in Advance:
August 1, 2018

<https://doi.org/10.1287/inte.2017.0934>

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Abstract. An important strategic decision for any operational mine is the differentiation between ore and waste material; this differentiation is referred to as the cutoff grade. In underground mining, material classified as ore is extracted, while waste is left in situ. Our mixed-integer programming optimization framework determines the cutoff grades in three different, predetermined zones for a soon-to-be-operational underground mine. We fix all cutoff grades *a priori* to optimize the periods in which to complete each mining activity to maximize the net present value for this restricted problem. We then use an enumerative optimization framework that relaxes the fixed cutoff-grade assumption and constructs a schedule for each cutoff-grade combination for all three zones. This framework both exploits an underlying mathematical structure and identifies an optimum set of grades that unconditionally maximizes net present value under the existing zone configuration, thereby providing objective, repeatable, and superior solutions, verified by our industry partner, a major gold producer, for large-scale problems in a matter of days; current industry practice would produce solutions with lower net present value and based only on detailed analysis for a single zone, would require six to eight weeks and would preclude scenario analysis.

History: This paper was refereed.

Funding: Alexandra Newman and Barry King received funding from the Center for Innovation in Earth Resources Science and Engineering at the Colorado School of Mines.

Keywords: integer programming • enumeration • underground mine scheduling • cutoff grade

Introduction

Gold has always been a valuable mineral, not only for its use in luxury and commemorative items such as jewelry and Olympic medals, but also as an international monetary standard and as an industrial material. For example, computers rely on gold's corrosion resistance and electrical conductivity to function efficiently. In the harsh environment of space, gold is used to reflect infrared waves that continuously bombard satellites and astronauts. It is estimated that every American born in 2015 will require, on average, 1.59 troy ounces of gold in his or her lifetime (Mineral Education Coalition 2015). All this gold is mined or recycled; not surprisingly, mining produces billions of dollars worth of gold annually.

Mining is a major part of our global economy and, in general, is classified as surface or underground (Hustrulid 2001, Hustrulid et al. 2013). Open-pit mining, which can be used for gold extraction, starts from

the surface and proceeds downward while maintaining a safe angle of the pit walls, creating a cone shape. When ore is located sufficiently deep below the earth's surface, underground mining methods, depending on the geometry, size, and host-rock characteristics of the *orebody*, are used. A common underground mining method for gold extracts large rectangular boxes of material, called *stopes*, from the earth. There exist different variants of stoping, but we narrow our discussion to open stoping (i.e., when a stope is removed, the void is left open), because in situ rock pillars remain to maintain ground stability. See Hustrulid (2001) for a complete discussion of stoping methods, in particular, and underground mining methods, in general.

Our industry partner, a major gold producer, is planning to open another underground mine in one of two regions, in which multiple operating mines feed a single processing plant; mining is critical to the local economy. An orebody exists underneath an operating open-pit mine in which an underground mine is to be

constructed and operated for at least a decade and a half. The construction can be very costly, and the decisions regarding the underground mine design and production schedule have large monetary implications.

The *cutoff grade* at a mine is the minimum ratio of metal to host rock (i.e., gold ounces per tonne that will be processed into a salable good). For example, a one-troy-ounce-per-tonne cutoff grade indicates that any material containing less than one troy ounce of gold per tonne of material is either not extracted or, if it is extracted, it is sent to a waste dump. The selectivity of underground mining implies that all material extracted from stopes is at or above a predetermined cutoff grade unless it is removed to access high-value stopes. The cutoff grade affects the total extracted tonnes, sometimes by a factor of two or more. A low cutoff grade yields more ore tonnage, a longer mine life, and more overall metal production, but at the cost of additional *development meters*, that is, expensive and time-consuming underground construction that provides a route for material haulage to the surface. However, a high cutoff grade implies a shorter mine life, which may not justify the large capital cost of starting a mine. Too high a cutoff grade leaves valuable ore in the ground, which cannot be extracted at a later date. An optimal cutoff grade, that is, the one that maximizes net present value (NPV), must balance the revenue from salable gold, the cost of extraction, and the time value of money.

The purpose of our research is to determine a set of cutoff grades for three distinct zones, predetermined by mine planners based on geotechnical and safety considerations, which maximizes the NPV of the underground mine for our industry partner. In conjunction with the determination of cutoff grades, we wish to establish an extraction schedule in such a way as to maximize the NPV of the project while adhering to precedence and resource constraints for a specific cutoff grade. Precedence constraints define rules regarding the order in which activities can occur. Resource constraints preclude over-utilization of existing production and processing capacities in any given period. We define three specific attributes of production: (1) that just mentioned, namely *production capacity*, which signifies the maximum tonnage that can be extracted in a given period (because of labor or other considerations); (2) *production rate*, which expresses the

number of stopes that can be extracted per period based on the mining method and host rock properties; and (3) *production levels*, which represent the total tonnage that is scheduled to be extracted in a given period. Attributes (1) and (2) serve as inputs to the model; attribute (1) can be changed in the short term, while attribute (2) is a more permanent condition and is fixed in our application; attribute (3) represents output from the extraction schedule.

After identifying these extraction schedules, we relax the fixed cutoff-grade assumption by (1) constructing a schedule for each cutoff-grade combination for all three zones and (2) using this enumerative optimization framework to identify an optimum set of feasible grades that unconditionally maximizes NPV. As a byproduct, our framework determines a strategic production schedule for the underground mine described in the *Cutoff-Grade Determination and Sequencing* section. To expedite solutions, we leverage algorithms mentioned in the *Literature Review* section that exploit the mathematical structure of the formulation (see the *Integer Programming Solution Strategy* section). Our methodology represents a contribution mathematically because of our fast solution procedure, and a contribution in practice because said solution procedure allows for a much more detailed analysis than currently exists in practice.

This paper is organized as follows: in the next section, we present a literature review and follow it with a section in which we discuss cutoff-grade determination and sequencing; that is, a description of the mine relevant to our industry partner. We then provide sections on mine scheduling, which includes a description of the mine's current practice, and our modeling and solution procedure, and on computational results. The last section emphasizes the impact of our work on the operations of our industry partner and concludes this study. The appendix provides the integer programming formulation and details regarding solution times.

Literature Review

Integer programs help to schedule both open-pit and underground mines (Newman et al. 2010). Whereas seminal work on open-pit mine planning posed and solved as an integer program has existed since the 1960s (Johnson 1968), underground mine scheduling has lagged behind its open-pit counterpart; most

research has occurred within the last two decades. For example, Trout (1995) represents one of the first researchers to use integer programming to schedule underground mines, in this case, to maximize NPV associated with the extraction of base metals via stoping methods. Carlyle and Eaves (2001) examine a similar setting and demonstrate provable savings to the mine through their implemented schedules. Kuchta et al. (2004) and Martinez and Newman (2011) present a model also used for production scheduling of a base metal; these articles describe work for the same mine associated with an underground method known as sublevel caving; while the former addresses long-term schedules, the latter makes both long- and short-term decisions. O'Sullivan and Newman (2014) produce an end-of-mine-life extraction schedule for a complex operation that uses three different mining methods. Like the work in Carlyle and Eaves (2001) and Kuchta et al. (2004), the work was implemented. However, all this research uses a fixed cutoff grade, and most examine methods to efficiently solve the monolith for realistically sized instances.

Lane's (1988) seminal work determines the cutoff grade for an open-pit mine by maximizing NPV using a series of equations. Other work implicitly selects the cutoff grade while determining an extraction schedule for an open-pit mine; in this case, the variables represent not only whether to extract a notional, three-dimensional piece of ore (i.e., a block) at a given time, but also the destination to which said block is sent; see Osanloo et al. (2008), Cullenbine et al. (2011), and Asad and Dimitrakopoulos (2013) for examples of these formulations. Some research considers cutoff grade as an explicit variable while optimizing mining operations. For example, Asad (2002) suggests a methodology for determining the cutoff grade in an open-pit mine while maximizing its NPV subject to extraction, processing, and refining constraints. Rashidinejad et al. (2008) also determine cutoff grades in an open-pit mine, paying special attention to environmental effects caused by acid runoff and postponement of reclamation at the end of a mine's life. Hall (2014) provides a methodology for determining the cutoff grade based on evaluating an underground mine's NPV under a variety of production scenarios using rule-based scheduling. Roberts and Bloss (2014) adapt an open-pit optimizer to aid in determining the cutoff grade for an underground

mine by differentiating stopes at a finer level of detail and identifying whether these "sub-stopes" are to be extracted or left in the earth. Ataei and Osanloo (2004) and Gu et al. (2010) similarly determine cutoff grades for underground mines, the former via a heuristic and the latter via dynamic programming. However, this work fails to address the problem using the level of detail (in terms of the number of mining activities and periods within a production scheduling framework) that we do.

King et al. (2016) note that certain open-pit and underground mine planning problems can be formulated as resource-constrained project scheduling problems (RCSPPs) whose mathematical structure (Artigues et al. 2013) generally leads to long solution times if said structure is not exploited. Bienstock and Zuckerberg (2010) develop a novel algorithm for solving the linear programming (LP) relaxation of an RCSPP, which greatly expedites solutions. The OMP solver leverages this algorithm, with improved computational efficiency and methods for creating integer solutions from the LP relaxation (Chicoisne et al. 2012, Muñoz et al. 2017). Using reformulations of a model designed to establish a horizontal demarcation (i.e., a transition) between the open-pit and the underground mine for a deposit being extracted via these two methods, King et al. (2016) employ these algorithms to determine near-optimal solutions. Although the problems addressed by their and our models are different, their model possesses a mathematical structure similar to that in our paper; in both cases, the instances are large, obviating the ability to solve the monolith outright.

Software allows mine operators to rapidly change the mine design. Alford and Hall (2009) identify tools to create thousands of stopes in a matter of seconds for any cutoff grade, eliminating the tedious task of hand drawing. Therefore, the mining industry is examining more cutoff grades than ever. However, mine planners are limited in their ability to analyze the NPV of a mine schedule for each cutoff grade for the following reasons: (1) underground mine planning software is not very sophisticated, with many mines still relying on heuristics (O'Sullivan et al. 2015); (2) because production schedules are difficult to generate, even for a fixed cutoff grade, studies are often performed in isolation and (or) by examining a subset of cutoff grades; and (3) the sheer number of cutoff grades that can exist in an

underground mine precludes a thorough manual analysis. We leverage the work of Alford et al. (2007) to create the stopes, and of King et al. (2016) to expedite solutions. Our work combines the simultaneous decisions associated with underground production scheduling and cutoff-grade determination, which comprise a set of decisions that are usually made in isolation.

Cutoff-Grade Determination and Sequencing

In the following sections, we provide background on cutoff-grade determination and sequencing for an open-stopping mine that belongs to our industry partner and utilizes a top-down mining method. We also highlight how changes in the cutoff grade affect each aspect of the mine design, or layout.

Stope Layout

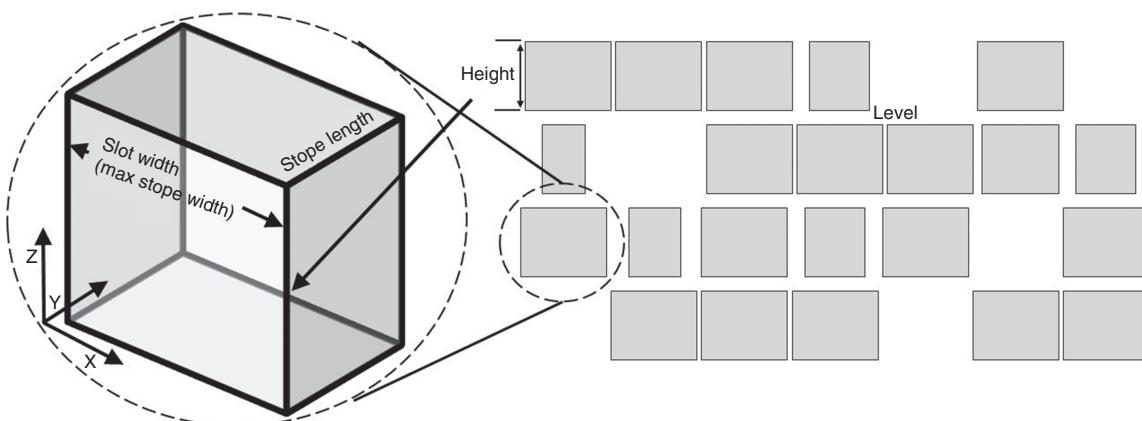
Let us define the x -direction as parallel to the earth's surface oriented along the longest section of the orebody, the y -direction as perpendicular to the x -axis in the same plane, and the z -direction as perpendicular to the earth's surface. Stopes are spaced along a regular grid of levels in the z -direction, defined by the elevation relative to sea level (Figure 1). A *level* is an elevation at which the top of a stope meets the bottom of the stope above it, and all stopes that have their base at the same level belong to that level. The vertical distance between horizontal levels is equal to the maximum height of the stopes between those levels. The width and length of any stope may vary within a

minimum and maximum size based on the geotechnical properties of the host rock, such as its strength, and thus its ability to remain an open void. A stope exists on a given level if the bottom of the stope coincides with the elevation of a level. In the x -direction, the grid consists of slots whose width is determined by the maximum stope width. If the length in the y -direction is greater than the maximum stope depth at a specific level-slot location, multiple stopes may exist at that location. As the cutoff grade increases, a stope's volume must remain the same or decrease because, as we raise the cutoff grade (i.e., become more selective), the amount of material that meets that level of selectivity must naturally decrease (or stay the same); in some cases, the stope is left in situ. However, the level height and minimum and maximum stope dimensions remain constant, regardless of the cutoff grade. That is, cutoff grade does not influence the constraints placed on the dimensions of a stope, but it changes the extractable volume within these bounds, because one must balance the ability to draw material out of a stope with the dilution that increases as more material is drawn.

Mining Zones

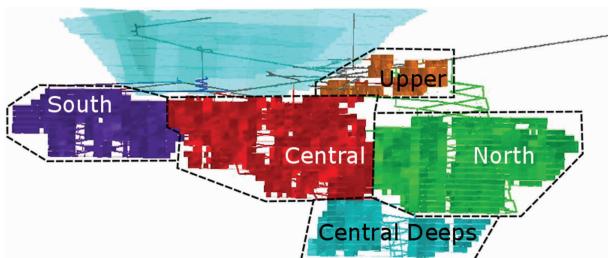
An underground mine consists of many stopes and is separated into distinct mining zones whose size and shape are dictated by the orebody configuration. More mining zones allow for direct access to an increased amount of ore but at a significant cost. Conversely, safety and geotechnical considerations preclude mining

Figure 1. In This Example of an Underground Stope Grid, Stopes Are Shown in Their Grid Locations at Various Widths



Note. We zoom in on one stope to display the different stope dimensions.

Figure 2. (Color online) Zones Within the Underground Data Set



Notes. The open-pit mine (transparent light blue, on top) is located above most of the underground mine. Each mining zone may have a different cutoff grade. Zones are delineated by dashed lines and are also color-coded as follows: North (green), South (purple), Central (red), Central Deep (light blue), and Upper (orange).

large orebodies as a single zone. Our orebody covers over two kilometers horizontally and extends over 750 meters in the vertical direction. Based on government mandates for safety, existing infrastructure, and geotechnical considerations regarding the strength of the host rock, prior to the start of our work, mine planners separated the orebody into four mining zones: Upper, North, Central (which includes the Central Deep), and South (Figure 2), each of which may have a distinct cutoff grade, although the zone boundaries remain constant.

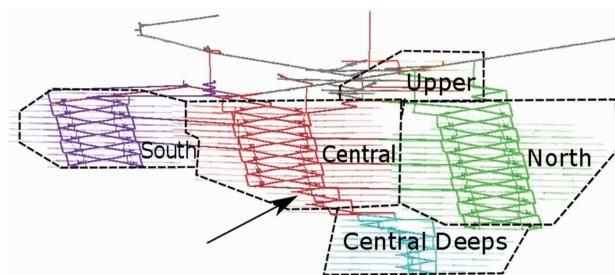
Decline

A *decline* is a downward-sloping ramp constructed primarily through waste rock and is used by rubber-tired equipment to haul ore to the surface. Once a decline reaches the ore body, twin declines (i.e., two corkscrew-shaped ramps placed side-by-side that are connected at their closest points) are used to allow for more extraction in the South, Central, and North Zones (Figure 3). Precedence dictates that the decline must be completed two levels below a given stope's level before the stope can be extracted. The main decline is part of the Upper and Central Zones, with branches that connect the South and North Zones. A portion of the Central Zone, known as Central Deep, uses a single decline.

Horizontal Development

Horizontal development is constructed on each underground level, designed to pass through each stope along its x -axis (Figure 3). Construction begins at the decline and proceeds toward the edge of the mining zone. The cutoff grade influences the amount of horizontal development that is required on each level,

Figure 3. (Color online) Decline and Horizontal Development Design



Notes. Zones are delineated by dashed lines and are also color-coded. The North (green) and South (purple) use a twin decline exclusively. The Central (red) uses a twin decline in areas with a high concentration of ore, and when the deposit narrows, it switches to a single decline as it reaches the Central Deep (light blue). The transition from twin to single decline is highlighted with an arrow. Horizontal development is depicted as a horizontal line for each level.

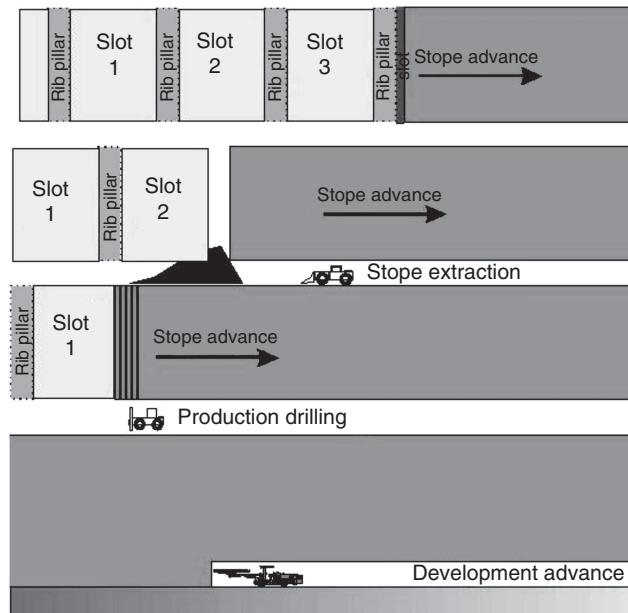
because it is correlated with the extractable volume contained in the stopes on the level. The total amount of horizontal development required differs by multiple kilometers depending on the cutoff grade, and an estimation of distance is required for each cutoff grade.

Extraction Sequencing

Stopes on each level are separated into a left and a right mining corridor, typically divided by a decline. Before a stope is extracted, drilling and blasting must have occurred. Drilling is done from the bottom of the stope upward, creating large columnar holes, which are then filled with explosives and blasted. This fragments the rock so that it can be extracted with equipment from the bottom of the stope. The mining sequence forces all horizontal development to be completed on the levels above and below the stopes in a corridor before any extraction in the corridor can occur.

Stope numbers, increasing from left to right, are assigned as a bookkeeping feature; for example, "stope 540-3" indicates that the stope is located on the 540 level at location 3, where no particular significance is associated with location 3. Stopes within a mining zone have three sequencing rules (Figure 4), invariant of cutoff grade: (1) stopes in the left corridor are extracted in increasing order of slot number, and stopes in the right corridor are extracted in decreasing order of slot number; (2) if multiple stopes exist at the same level-slot location, they are extracted in decreasing order of economic value; and (3) a stope may not be extracted unless the stope(s) directly above have been completely

Figure 4. The Progression of Extraction Sequencing in the Left Corridor of an Open Stoping Underground Mine



Notes. First, development must be constructed on the levels above and below the stope, which are labeled with a slot number. Then, drilling and blasting fragment the rock. Finally, stopes are extracted, leaving open voids in the ground. Rib pillars maintain stability of the host rock. In the left corridor, stopes are extracted in increasing order of slot number on the level. In addition, the stope with a slot number on the level above must be extracted before extraction of the stope with the same slot number on the level below.

extracted. If no stope(s) exist(s) in the same slot location on the level above, the stope below can be extracted after rules (1) and (2) are satisfied. A *rib pillar* exists between each pair of stopes to ensure stability. A fixed production rate determines the time required to extract each stope.

Mine Scheduling

In our framework, mine scheduling is categorized as *strategic* or *tactical*. A strategic schedule provides guidance as to which areas of the mine should be extracted in a given year, when access to a new mining zone should be constructed, and the remaining life expectancy of the mine. Tactical schedules provide specified starting dates for mining activities (e.g., decline construction, horizontal development, and stoping), and this schedule is updated frequently based on activity completion. The purpose of our work is to assist a mining company in making strategic decisions that it can then implement in its tactical schedule.

Determining the cutoff grade for a mine is typically done before a strategic schedule is created, which may result in a suboptimal NPV for the mine. Therefore, combining cutoff-grade selection into the strategic mine schedule, while difficult, is valuable to mining companies.

Current Practice at Our Industry Partner

Our industry partner uses the following analysis to determine the cutoff grade for each zone: (1) for a single zone (e.g., the Central Zone), mine planners begin by constructing a full mine design at a single cutoff grade; (2) for each relevant cutoff grade, stope shapes are altered, and the horizontal development is adjusted to match; (3) a genetic algorithm is used to create a schedule for each cutoff-grade option and production capacity; (4) the schedule is postprocessed via spreadsheet analysis to obtain a more accurate NPV based on specific costs not considered in the scheduling model; and (5) the procedure iterates for each operationally feasible cutoff-grade option and production capacity. After steps (1)–(5) are completed, the result is tailored to the other zones.

The mine planner requires approximately one day to complete a schedule for each cutoff-grade option and production capacity, resulting in a six- to eight-week exercise associated with evaluating a single zone's NPV with respect to all cutoff-grade options and production capacities. This excessive planning time results in only approximate solutions for the other zones based on simplified analysis and precludes the evaluation of alternate scenarios because of time limitations. Therefore, we introduce an integer programming model, which we use within an enumerative framework, to derive optimal solutions for all zones.

Data and Integer Programming Model Description

For our integer programming model, which we describe in the appendix, we use the same computer-generated stope shapes for each cutoff grade as our industry partner uses in current practice. The tonnage, average grade, and location of the stopes populate the model parameters. The quantity of extracted tonnes is given by the stope shape, and the development meters are estimated based on the number and size of the stopes in each zone. The cutoff grade drastically changes many attributes of the data; for example, at seven-units-per-tonne cutoff grade, the total

Figure 5. Number of Stopes per Grid Location for All Zones Except the Upper Zone; Lighter Areas Represent Fewer Stopes



Notes. The top image shows the quantity of stopes available for extraction at a one-unit-per-tonne cutoff grade, and the bottom image shows the quantity of stopes available for extraction at a seven-units-per-tonne cutoff grade. Underground mine design and cutoff grades change significantly based on the number of stopes that are to be extracted.

ounces are 83 percent less than at the one-unit-per-tonne cutoff grade (Figure 5). (Note that when referring to actual numerical values, we report the units generically for confidentiality.) The total number of stopes also decreases by 63 percent between the lowest and highest cutoff grades. There are 1,500 stoping activities, each associated with a profit and a quantity of extraction tonnes; there are 700 development activities, each associated with a cost and a longitudinal distance of development. We schedule with yearly periods over a 20-year mine life; the 9 percent discount rate we use is nonnegotiable in our computations as far as our industry partner is concerned, having been determined at the corporate level based on, for example, the perceived risk associated with operating within a particular geographical region. Variable costs consist of extraction, development, processing, and haulage on a per-tonne or per-meter basis. There exists an initial capital and annual fixed cost for each zone.

The integer programming model determines which underground mining activities to complete in each period to maximize the present value of an instance

subject to precedence relationships and resource constraints. The NPV is then calculated by postprocessing the fixed and capital costs, which we justify as follows: (1) we incur the fixed cost regardless of the cutoff grade or production capacity, the major decisions our model seeks to determine; and (2) we know the approximate time horizon during which the mine is operational (and therefore during which a bulk of the fixed costs are incurred) *a priori*. Excluding the fixed costs allows us to exploit the problem structure to use our specialized algorithms and solution techniques. Activities generally require one period to complete, and for an activity to be scheduled in a given period, all of its predecessors must be scheduled in the same or in a previous period. Stope extraction tonnage and development meters are limited by the resource capacity constraints.

Integer Programming Solution Strategy

The integer programming model (U) determines only the optimal schedule for a single cutoff grade in each zone; because we consider scheduling in three zones—North, Central, and South, we refer to each composite combination of cutoff grades as a *cutoff-grade triple*. The zones cannot be scheduled independently because they compete for overarching production capacity. To obtain a globally optimal solution across all zones considering all cutoff-grade triples, we exploit an enumeration strategy whose effectiveness relies on two key features: (1) fast solution times for each cutoff-grade triple and (2) the ability to bound the objective function value for each solve in feature (1).

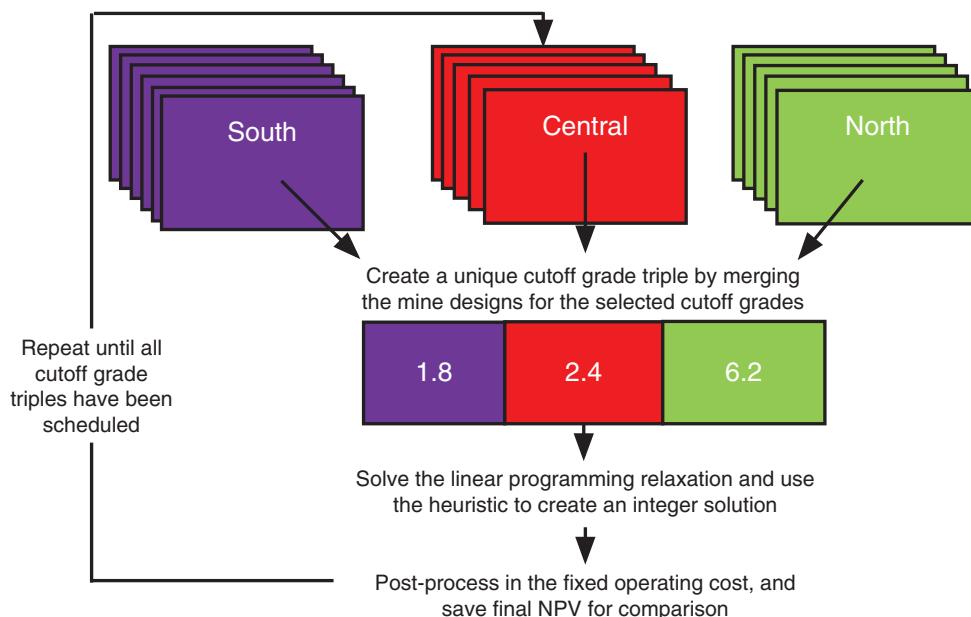
To address the first key feature, we employ the OMP solver (Muñoz et al. 2017) to quickly solve the LP relaxation of each cutoff-grade triple and to create an integer-feasible solution from the corresponding LP relaxation. The OMP solver is written in C, and tailored to solve RCPSP problems for which a vast majority of the constraints are precedence relationships, which is the case for our formulation. The algorithms used by this solver have been shown empirically to be two to three orders of magnitude faster than simplex-based methods for solving the LP relaxation of mine scheduling problems. A list-ordering heuristic, TopoSort, used to create near-optimal integer solutions from those corresponding to the LP relaxations

of problems with our mathematical structure, solves in seconds (Chicoisne et al. 2012, King et al. 2016).

To address the second key feature, we use the objective function value of the LP relaxation of our integer program to provide an upper bound on the NPV across all zones for a specific cutoff-grade triple. If the TopoSort heuristic yields an integer solution with an objective function value close to that of the LP relaxation, that integer solution is (near-) optimal. If the integer solution for a given cutoff-grade triple has an objective function value (i.e., a lower bound) that is greater than the LP relaxation's objective function value (i.e., the upper bound) of another cutoff-grade triple, we can guarantee through dominance arguments that the latter cutoff-grade triple cannot produce an optimal schedule. Refer to King et al. (2016) for a more detailed discussion of the use of dominance to eliminate feasible, yet nonoptimal, solutions from consideration.

With a reasonable number of cutoff-grade triples, fast solution times, and strong bounds, enumeration is a viable strategy for determining the cutoff grade for an underground mine. Figure 6 illustrates our iterative solution procedure.

Figure 6. (Color online) We Fix a Cutoff Grade for Each of the North, Central, and South Zones Individually, in This Case 6.2, 2.4, and 1.8, Respectively



Notes. We then solve the LP relaxation associated with this specific cutoff-grade triple, and employ the TopoSort heuristic to obtain a near-optimal integer solution for the triple. We postprocess fixed costs into the objective function, store the result, fix a different cutoff-grade triple, and repeat. The optimal solution is that with the highest NPV after all feasible cutoff-grade triples have been examined.

Computational Results

In this section, we provide computational results associated with determining the optimal cutoff grade, first for the Central Zone and then for the entire mine, with the exception of the Upper Zone, because its cutoff grade is already fixed by our industry partner. The Central Zone-only schedule (1) calibrates our model parameters, (2) shows the advantages of our optimization framework, and (3) provides a solution for the most profitable part of the orebody *a priori*. We use a deterministic approach because our industry partner bases its decisions on a standardized geological model of the deposit, which is updated frequently as new information becomes available. The speed with which we are able to solve our instances makes these resolves possible. A Dell PowerEdge R410 with 16 processors (2.72 GHz each) and 28 GB RAM with OMP Solver version 1854 produces all results.

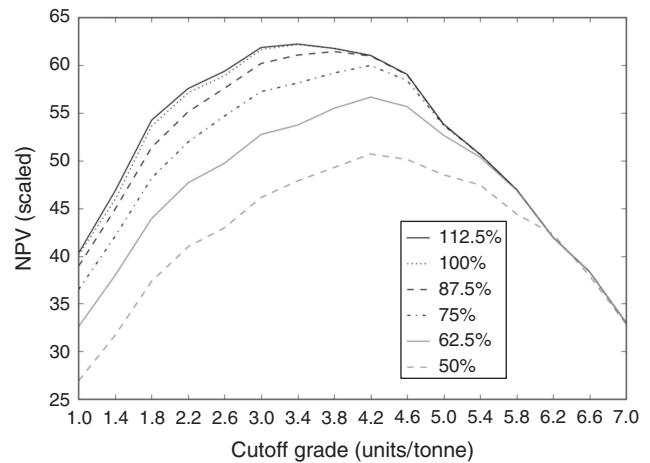
Single-Zone Scheduling

Our industry partner determines a “favorable” production capacity for each zone based on operations in

the Central Zone; this measure is subjective, and the mining engineers ultimately choose a capacity based on its ability to follow a “trapezoidal trend” in production during the time horizon over which the schedules extend to better manage capital equipment and personnel. That is, our industry partner prefers a profile that exhibits three phases: an initial phase of increasing production, followed by a period of consistent production, and finally a rapid decrease in production. The graphical depiction results in a trapezoidal shape. To this end, we create a schedule for a set of cutoff grades using a series of production capacities, each instance of which is referred to as a cutoff-grade production-capacity option, in the Central Zone to determine the production capacity above which the NPV does not significantly increase; that is, the mine becomes limited by precedence and production rates and not by production capacities. Although the lowest cutoff grade maximizes the amount of metal extracted, the cost of extracting the additional metal eventually outpaces the revenue gained. Additionally, this potential increase in ounces is spread over multiple years because of production capacity and may not yield the highest discounted profit. Specifically, we vary the cutoff grade used to create the stope shapes from 1 unit per tonne to 7 units per tonne, inclusive, by 0.4 units-per-tonne increments. From a practical standpoint, 0.4 units per tonne is very detailed for a strategic mine schedule. Any finer fidelity would result in unnecessary computation and add insignificant value to the mine plan. The detail in our cutoff-grade increments allows for the construction of an NPV curve to identify favorable cutoff grades. Because these curves have been observed to be unimodal in practice, finer refinement need only occur near the peak of the curve.

The cutoff grade and NPV change significantly when we alter the production capacity until we reach 100 percent; that is, the maximum capacity the mine can sustain for multiple consecutive years. The optimal cutoff grade ranges from 3.0 to 4.2 units per tonne depending on the production capacity. As the cutoff grade increases, the NPV curves for different production capacities become virtually identical, providing an indication of the maximum production capacity for each cutoff grade. For example, if the mine operates with a 6.2-unit-per-tonne cutoff grade, an annual production capacity greater than 50 percent of full capacity does not add value (Figure 7). There is no significant

Figure 7. The Central Zone’s NPV Changes as a Function of Cutoff Grade; for Low Cutoff Grades, Marginal Returns with Respect to NPV Increase, While at High Cutoff Grades, the NPV Starts to Diminish



Notes. Higher production capacities (given in the legend in the figure) yield higher NPVs. The optimal cutoff grade falls between 3.0 and 4.2 units per tonne, depending on the production capacity.

increase in NPV at any cutoff grade beyond 100 percent of production capacity. These solutions provide a good bound on minimum and maximum cutoff grades that are likely to be optimal; in practice, if computational time is a major concern when scheduling the entire mine, cutoff grades below 3 units per tonne or higher than 4.2 units per tonne may be omitted.

Our single-zone instances contain 12,014 variables and 73,946 constraints, on average; corresponding solution times for each cutoff-grade production capacity option are fewer than 10 seconds, and the time to enumerate all of the Central Zone LP relaxations and create an integer solution is under 25 seconds for an annual production capacity of 100 percent. For a given production capacity curve in Figure 7, the TopoSort heuristic is able to produce an integer solution that dominates all others; that is, its NPV is greater than the NPV associated with any LP relaxation of a different cutoff grade; this allows us to mathematically guarantee that there exists a single optimal cutoff grade for a given production capacity for these numerical experiments. The TopoSort heuristic finds solutions within 1 percent of optimality for our Central Zone-only instances, demonstrating empirically that the bounds provided by the linear programming relaxation are tight. The appendix provides detailed computational results.

Entire Mine Scheduling

Mine planners provide an overarching production capacity for all zones throughout the expected mine life. Within a zone, we establish an optimal, maximum capacity based on the same type of analysis as was conducted in the *Single-Zone Scheduling* section. The Central Zone may begin production in the first year, but the North and South Zones cannot begin until the fifth year. (Development may start as soon as necessary.) With the production capacities fixed for the entire mine and each zone, we identify the highest-value NPV from 16 cutoff-grade options, any of which might occur in the three mining zones: Central, South, and North. The enumeration of this complete set results in 16^3 (4,096) cutoff-grade triples.

We use the TopoSort heuristic to create an integer-feasible schedule associated with the LP solution corresponding to the highest NPV among all cutoff-grade triples, and we refer to this as the *original schedule* with a corresponding scaled NPV of 85.53. (We use the same scale as the Central Zone-only schedule.) The resulting relative difference between the LP bound and the integer-solution NPV is 0.11 percent. The cutoff grades corresponding to the highest NPV schedule are 3.8, 3.4, and 3.0 units per tonne for the South, Central, and North Zones, respectively. Unfortunately, our initial solution fails to consider some operational details. Specifically, the production levels from the individual zones fluctuate unacceptably (Figure 8).

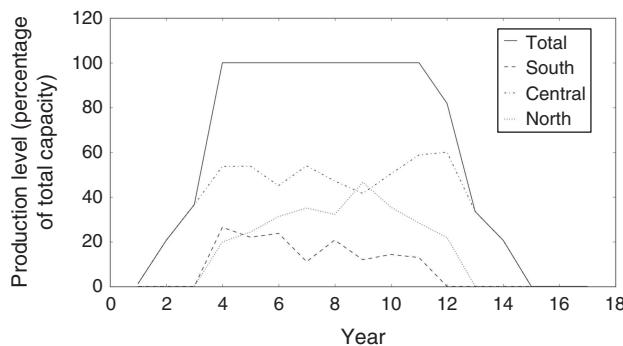
The South Zone production level peaks and then trends downward, while the North and Central Zone

production levels appear either in the shape of a single- or triple-hump, respectively; in no case does the production level smooth out at some consistent value. These features preclude the mine planner from coordinating production levels of each zone because, under this schedule, employee and production equipment would be oscillating between zones. Because we do not know *a priori* how these operational violations will appear and because we want to enforce as few constraints in the model as possible to sacrifice as little NPV as possible, we postprocess our integer programming-generated schedules.

Our industry partner desires that only two of the three zones operate at one time, unless one zone is ending and another zone is beginning. Both the North and the South Zones start stope extraction as soon as possible in the original schedule, because that stope value decreases with depth, and mining at the top of both the North and South Zones provides the most value. Mining in the South Zone first is preferred because of ventilation considerations; therefore, we adjust the production capacity constraints for the South Zone to start in the fifth year and delay extraction in the North Zone until the South Zone extraction is nearly complete. In this way, we alter the extraction and development capacities for the North Zone by preventing any stope extraction until year 9.

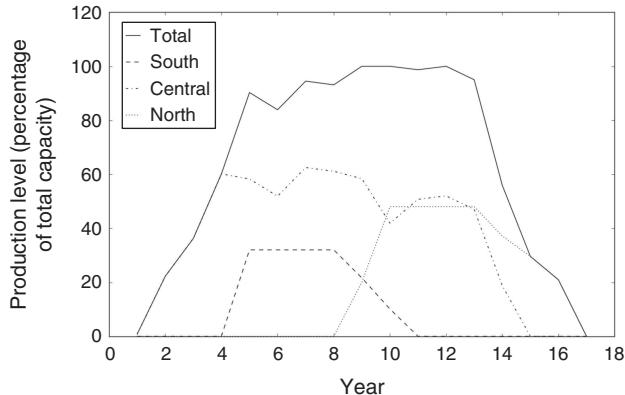
We rerun the entire enumeration procedure with this restriction, resulting in the *restricted schedule* integer-feasible solution constructed from the LP relaxation with the highest objective function value of 83.9, only 1.8 percent lower than that from the original schedule. Cutoff grades of 3.8, 3.4, and 3.0 units per tonne for the South, Central, and North Zones, respectively, remain the same. The South Zone has its development constructed in time for full stope extraction to occur in years 5–8 and end by year 11. The North Zone begins stope extraction in year 9 and operates at a constant production level in years 10–13 before decreasing for the remaining zone life. The Central Zone slowly increases stope extraction in years 2–4 and smooths out in years 5–9. Once the North Zone reaches full production, the Central Zone has a lower, but consistent, production level in years 10–14, before quickly dropping off. (During the Central Zone's ramp-up phase, the Upper Zone augments total ore tonnage.) These

Figure 8. Production Levels As a Percentage of the Total Production Capacity Used for the Entire Mine and for Each of the Three Zones for Each Year in the Planning Horizon



Note. The production levels in every zone vary too greatly to be operationally feasible.

Figure 9. Restricted Production Schedules for the Entire Mine and for Each of the Three Zones as a Percentage of the Total Maximum Production Capacity



Note. Although the Central Zone's stope extraction fluctuates slightly, that of the North and South Zones, as well as that overall, are smooth, which is desirable.

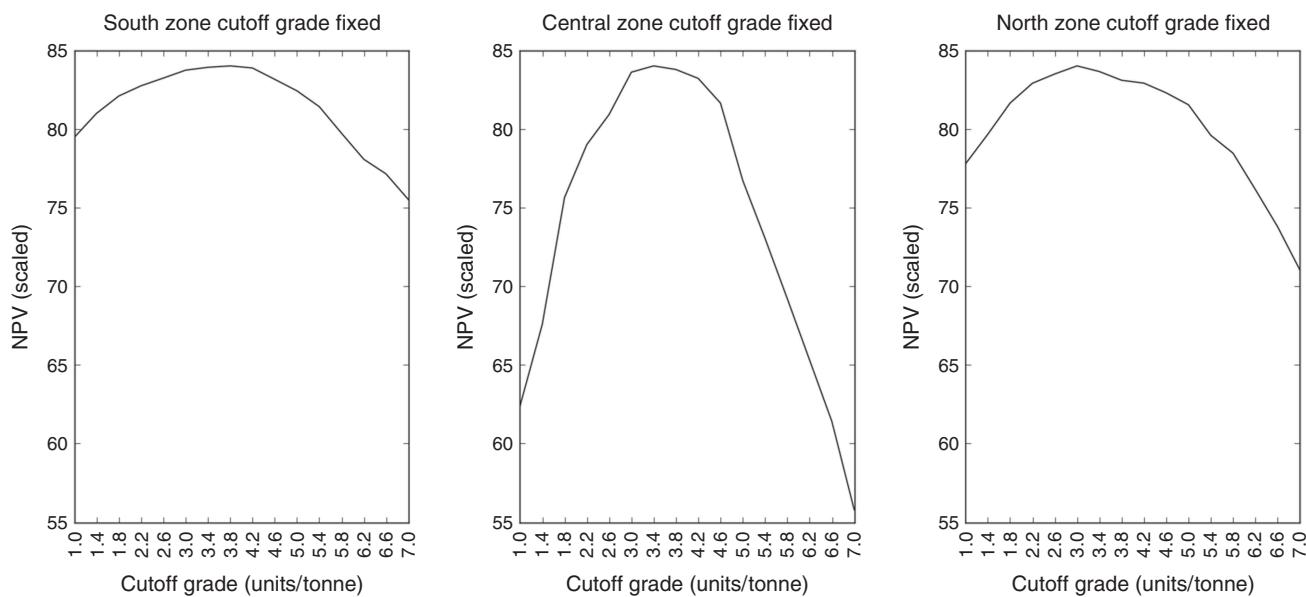
profiles yield a desirable production level for the entire mine. Figure 9 shows the restricted schedule.

On average, the cutoff-grade triple instances contain 31,575 variables and 230,117 constraints with corresponding solution times for the LP relaxation averaging 8.10 seconds. Similar to simplex-based methods, the OMP solver only utilizes one core when solving the LP relaxation. So, we can expedite solutions by

enumerating all 256 of the Central and North Zone cutoff-grade options for a given cutoff grade in the South Zone and running the 16 different South Zone cutoff-grade options on 16 different cores. With this type of parallel computation, we are able to solve all of the LP relaxations in 2,510 seconds; see the appendix. The minimum and maximum LP relaxation solution times across all 16 cores are 1.59 and 19.02 seconds, respectively. We obtain an integer solution for the top-10 highest cutoff-grade triples (with respect to the LP relaxation value) to provide our industry partner with multiple scheduling options.

We examine the effect of fixing the cutoff grade in each zone on the overall NPV of the mine by setting the cutoff grade of the selected zone to an arbitrarily specified value and allowing those in the other two zones to vary (Figure 10). The cutoff grade of the Central Zone has the largest effect on NPV; setting the cutoff grade in this zone to 7.0 units per tonne reduces the maximum attainable NPV by 33 percent. An NPV within 0.48 percent of optimality is attainable if the Central Zone's cutoff grade is between 3.0 and 3.8 units per tonne. Although the South Zone begins stope extraction earlier than the North Zone, the choice of a sub-optimal cutoff grade is buffered by the choice of the

Figure 10. Maximum Attainable NPV for the Mine When Optimizing Cutoff Grade for Two Zones, Given a Fixed Cutoff Grade for a Single Zone

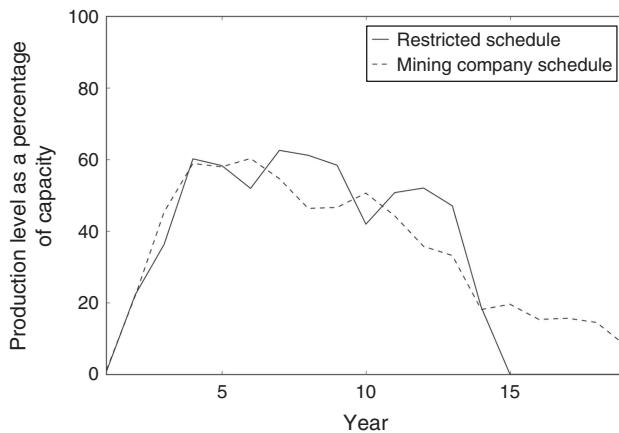


Note. The value curve indicates the degree to which the NPV may change with respect to the specified zone cutoff grade.

correct cutoff grade in the other zones. The North Zone has a small peak in NPV at 3.0 units-per-tonne cutoff grade, an attribute found in almost all 20 LP relaxation solutions with the highest NPV.

This work has been very influential on the decisions regarding cutoff grade. Specifically, our industry partner uses the analysis of different production capacities for the Central Zone to bound the cutoff grade between 3.0 and 4.2 units per tonne, after which it can use our recommended cutoff grades in the North and South Zones because of the successful calibration of our analysis to the Central Zone. Another result of our work is improvement of the production schedules over those the mine constructed manually. The mine expends the majority of its efforts in creating a schedule for the Central Zone. Yet, the Central Zone's restricted schedule exhibits significant improvements over current industry practice: (1) the production level is higher and more consistent, and (2) the mine life is four years shorter while producing the same amount of gold; because annual fixed costs are postprocessed into the NPV and variable costs are a function of production capacity, a shorter life implies less annual fixed and variable expenditures for extracting the same quantity of metal (Figure 11). By constructing the NPV curves in Figure 10, our industry partner is able to make more informed decisions, because it has a monetary value for every cutoff-grade triple.

Figure 11. Comparison of the Central Zone Production Schedule for Our Restricted Schedule and the Company's Schedule



Notes. Both follow a very similar trend in the first five years of production. However, the restricted schedule has a higher and more consistent level of production, which is desirable.

Impact and Conclusions

Determining the cutoff grade has a significant impact on the NPV and mine schedule. Current practice at our industry partner selects a cutoff grade for the main mining zone based on production capacity considerations, that is, the number of activities that can feasibly be completed during a given period because of equipment and personnel availability. This determination, which has traditionally been done manually, required six to eight weeks of a mine planner's time, even considering a small number of cutoff grades and production capacities, and a single zone. Our analysis shortens the planning time to a matter of days (including setup, computational runs, and scenario analysis) and results in an optimal cutoff grade for each zone individually. For our industry partner, these benefits result in savings of millions of dollars, as a result of both strategic planning changes and reductions in mine planners' time, over a 10-year life-of-mine period (and more, if the manual analysis fails to produce an optimal cutoff grade for the main zone). Furthermore, optimal cutoff grades by zone provide a mine planner with information that leads to second-order benefits; examples include better knowledge of end-of-mine life and decisions associated with asset planning. As a by-product, we create strategic extraction schedules with much smoother production levels than those obtained via manual methods. To the authors' knowledge, a cutoff-grade analysis such as the one we present here is not widespread in industry because it is so computationally burdensome, and our work represents the first published research that exploits the mathematical structure of the problem to reduce this computational burden.

Future work might include the following: (1) optimizing cutoff grade based on a reconfiguration of the zones to assess whether a better separation of the ore-body exists, and (or) (2) incorporating fixed and capital costs within the integer programming model to eliminate the postprocessing step and to more accurately reflect a mine's cost structure. Although we examine an open stoping operation, this optimization framework can be used in most stoping, drift-and-fill, and room-and-pillar mines or for a combination of mining methods, which may provide value to a mine operator, because the cutoff grade for each mining method is likely to be different.

Acknowledgments

This research benefited significantly from software created by Marcos Goycoola (Universidad Adolfo Ibañez), Daniel Espinoza (Gurobi), Eduardo Moreno (Universidad Adolfo Ibañez), and Orlando Rivera (Universidad Adolfo Ibañez). The authors are also grateful for the financial support provided by Alford Mining Systems.

Appendix

Underground Mine Scheduling Formulation

Our underground mine scheduling problem, adapted from King et al. (2016), is formulated to possess a resource-constrained project-scheduling-problem mathematical structure, with a majority of the constraints representing precedence relationships. This structure is well suited for the OMP solver and the TopoSort heuristic (Goycoolea et al. 2015, Brickey 2015). If all the coefficients in the resource constraints are nonnegative and the lower bounds are zero, TopoSort quickly provides not only feasible solutions, but solutions that are near optimal for resource-constrained project-scheduling problems. It is a list-ordering heuristic based on the premise that the earlier the expected completion time of a block or activity in the LP solution, the earlier the block or activity is scheduled in the integer programming solution. King et al. (2016) show the effectiveness of TopoSort for a model containing an underground mine scheduling formulation.

Indices and sets:

- $a \in \mathcal{A}$ set of all activities;
- $\check{a} \in \check{\mathcal{A}}_a$ set of predecessors for activity a ;
- $\bar{a} \in \bar{\mathcal{A}}_a$ set of predecessor activities \bar{a} that must be completed one period in advance of activity a ;
- $r \in \mathcal{R}$ set of resources, such as production and development capacity;
- $t \in \mathcal{T}$ periods.

Table A.1. In This Central Zone Scheduling Computational Summary, We Show That Depending on the Production Capacity, the Optimal Cutoff Grade Ranges from 3.4 to 4.2 Units Per Tonne, and for Each Production Capacity, the Solution Time Is Less Than 70 Seconds and the Optimality Gap Is Under 0.7 Percent

Production capacity (%)	Highest NPV cutoff	Total solution time (sec)	Optimality gap (%)
112.50	3.4	20.42	0.000
100.00	3.4	24.11	0.000
87.50	3.8	32.2	0.000
75.00	4.2	43.89	0.001
62.50	4.2	50.99	0.684
50.00	4.2	67.38	0.348

Notes. Optimality gaps are calculated using $100\% \cdot (1 - (\text{Integer Programming Obj. Func. Value}/\text{Linear Programming Relaxation Value}))$.

Parameters:

- c_a monetary value associated with completing activity a [\\$];
- q_{ra} quantity of resource r consumed when completing activity a [tonnes, meters];
- \bar{r}_{rt} maximum amount of resource r available in time t [tonnes, meters];
- δ_t discount factor for period t [fraction].

Decision variables:

- X_{at} 1 if activity a is completed by the end of time t ; 0 otherwise.

$$(\mathbf{U}) \quad \max \left\{ \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \delta_t c_a (X_{at} - X_{a,t-1}) \right\} \quad (\text{A.1a})$$

$$\text{s.t. } X_{a,t-1} \leq X_{at} \quad \forall a \in \mathcal{A}, t \in \mathcal{T}; \quad (\text{A.1b})$$

$$X_{at} \leq X_{\check{a}t} \quad \forall a \in \mathcal{A}, \check{a} \in \check{\mathcal{A}}_a, t \in \mathcal{T}; \quad (\text{A.1c})$$

$$X_{at} \leq X_{\bar{a},t-1} \quad \forall a \in \mathcal{A}, \bar{a} \in \bar{\mathcal{A}}_a, t \in \mathcal{T}; \quad (\text{A.1d})$$

$$\sum_{a \in \mathcal{A}} q_{ra} (X_{at} - X_{a,t-1}) \leq \bar{r}_{rt} \quad \forall r \in \mathcal{R}, t \in \mathcal{T}; \quad (\text{A.1e})$$

$$X_{at} \text{ binary, } \forall a \in \mathcal{A}, t \in \mathcal{T}. \quad (\text{A.1f})$$

The objective function (A.1a) maximizes NPV. Constraints (A.1b) ensure that once an activity is completed, it remains completed. Constraints (A.1c) enforce precedence. Constraints (A.1d) ensure that at least one period elapses between the completion of the specified pair of activities. Constraints (A.1e) bound stope extraction and development construction. All variables are required to be binary by constraints (A.1f).

Table A.2. Entire Mine Scheduling Parallel Computational Times

South Zone cutoff grade	LP solve time (sec.)
1	2,489.16
1.4	2,510.23
1.8	2,447.53
2.2	2,386.63
2.6	2,347.86
3	2,174.90
3.4	2,068.31
3.8	1,973.00
4.2	2,165.04
4.6	1,852.51
5	1,869.71
5.4	1,829.00
5.8	1,835.19
6.2	1,777.48
6.6	1,707.09
7	1,752.66

Notes. The South Zone's cutoff grade is fixed, and all 256 permutations of the Central and North Zone cutoff grades are run. With parallelization, the solution time is 2,510 seconds (i.e., the maximum value in column 2).

Central Zone Computational Results

Table A.1 provides a summary of the Central Zone solution attributes at different production capacities. The first column indicates the production capacity, and the second contains the cutoff grade that produces the highest net present value. The third column represents the total time required to solve the LP relaxations associated with all economically feasible cutoff grades for the production capacity specified in the first column and to obtain an integer solution. The final column represents the gap between the LP objective function value and the objective function value corresponding to the integer solution.

Parallel Computing Results

We highlight the effectiveness of parallelization by separating the enumerations across multiple cores (Table A.2). By dividing the work across 16 cores based on the South Zone's cutoff grade, we are able to solve all the LP relaxations in 2,510.23 seconds, the maximum value in column 2 of Table A.2, as opposed to 33,186.30 seconds for a serial execution. A significant correlation between cutoff grade and solution times exists, because lower cutoff grades contain more stopes and horizontal development, thus generally increasing the number of decisions that need to be made.

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Verification Letter

The *Interfaces* Editor-in-Chief has received a letter of verification from the company attesting to the impact of this work; however, the company wishes to remain anonymous.

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