

Processing-induced anisotropy

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ABSTRACT

Processing of seismic data is often performed under the assumption that the velocity distribution in the subsurface can be approximated by a macromodel composed of isotropic homogeneous layers or blocks. Despite being physically unrealistic, such models are believed to be sufficient for describing the kinematics of reflection arrivals.

In this paper, we examine the distortions in normal-moveout (NMO) velocities caused by the intralayer vertical heterogeneity unaccounted for in velocity analysis. To match P-wave moveout measurements from a horizontal or a dipping reflector overlaid by a vertically heterogeneous isotropic medium, the effective homogeneous overburden has to be anisotropic. This apparent anisotropy is caused not only by velocity monotonically increasing with depth, but also by random

velocity variations similar to those routinely observed in well logs.

Assuming that the effective homogeneous medium is transversely isotropic with a vertical symmetry axis (VTI), we express the VTI parameters through the actual depth-dependent isotropic velocity function. If the reflector is horizontal, combining the NMO and vertical velocities always results in nonnegative values of Thomsen's coefficient δ . For a dipping reflector, the inversion of the P-wave NMO ellipse yields a nonnegative Alkhalifah-Tsvankin coefficient η that increases with dip. The values of η obtained by two other methods (2-D dip-moveout inversion and nonhyperbolic moveout analysis) are also nonnegative but generally differ from that needed to fit the NMO ellipse. For truly anisotropic (VTI) media, the influence of vertical heterogeneity above the reflector can lead to a bias toward positive δ and η estimates in velocity analysis.

INTRODUCTION

Rapid progress in the development of anisotropic velocity-analysis methods has made transverse isotropy with a vertical symmetry axis (VTI) a common model in time and depth processing of P-wave data (e.g., Alkhalifah et al., 1996; Williamson et al., 1997; Toldi et al., 1999). P-wave depth imaging in VTI media requires estimates of the P-wave vertical velocity V_0 and Thomsen's (1986) anisotropic coefficients ϵ and δ , whereas time imaging for models with a laterally homogeneous overburden is controlled by the normal-moveout (NMO) velocity for horizontal reflectors V_{nmo} and the anellipticity coefficient η , defined as (Alkhalifah and Tsvankin, 1995)

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}. \quad (1)$$

Both time-imaging parameters can be obtained from P-wave reflection traveltimes using either dip-dependent P-wave NMO velocity or nonhyperbolic moveout from horizontal

interfaces (e.g., Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998a; Toldi et al., 1999; for a detailed overview, see Tsvankin, 2001).

In contrast, estimation of the parameters V_0 , ϵ , and δ from surface P-wave data is possible for only a certain class of models with dipping intermediate interfaces or some other types of lateral heterogeneity in the overburden (Grechka et al., 2002). Therefore, values of δ are often determined at borehole locations by combining the NMO (stacking) velocity V_{nmo} measured from surface seismic data and the vertical velocity V_0 derived from check shots or well logs (e.g., Tsvankin, 2001). For a single homogeneous VTI layer, the two velocities are related by (Thomsen, 1986)

$$V_{\text{nmo}} = V_0 \sqrt{1 + 2\delta}. \quad (2)$$

If the medium is horizontally layered, the interval NMO velocity can be found from the conventional Dix formula and used to obtain the interval parameter δ from equation (2). After

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estimating both δ and η , it is possible to find the remaining parameter ϵ using equation (1).

In addition to their key role in anisotropic processing, the parameters ϵ , δ , and η contain useful information about the petrophysical and lithologic properties of the subsurface formations. Hence, it becomes increasingly important to understand the physical origin of the anisotropic coefficients and their relationship to lithology. Transverse isotropy in sedimentary basins is believed to be caused by two main factors. One of them is the intrinsic anisotropy of shales resulting from preferential orientation of clay particles aligned by gravity (Sayers, 1994). The second factor is the interbedding of thin (compared to the predominant seismic wavelength) isotropic layers that creates an effective TI medium in the long-wavelength limit. The anisotropic parameters of this effective model can be found from either Backus (1962) averaging or the more general procedure of Schoenberg and Muir (1989). In both cases, the properties of seismic waves propagating through the medium are indistinguishable from those observed in a homogeneous VTI model.

In this paper, we discuss a different reason for effective transverse isotropy: the assumption of homogeneity commonly used in seismic processing. High-quality reflection events are recorded from only a limited number of interfaces, and it is often assumed that the medium between the interpreted reflectors is homogeneous. If vertical heterogeneity in isotropic media is not properly accounted for, description of P-wave reflection traveltimes requires making the velocity field anisotropic. We show that estimation of the parameters of this “apparent” (or effective) VTI model using the anisotropic velocity-analysis methods discussed above leads to always non-negative values of δ (for a horizontal reflector) and η (for a dipping or a horizontal reflector).

EFFECTIVE NMO VELOCITY FROM A HORIZONTAL REFLECTOR

Figure 1 shows the vertical (zero-offset) and nonzero-offset rays for the simple model of a horizontal reflector beneath a

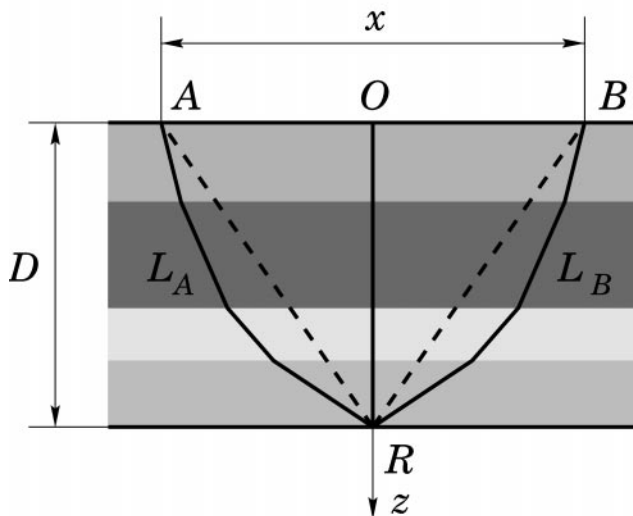


FIG. 1. Zero-offset ray ORO and ray AL_ARL_BB at offset $x = AB$ for the reflection from an interface beneath a vertically heterogeneous isotropic medium.

stack of homogeneous isotropic layers. If the reflector depth D is known, the zero-offset (vertical) traveltime T_0 can be used to compute the average vertical velocity V_0 :

$$V_0 = \frac{2D}{T_0}. \quad (3)$$

Since the model is vertically heterogeneous, V_0 becomes an effective quantity that averages the interval (or local) velocities $v(z)$. If the interfaces in the overburden are not strong enough to generate detectable reflection events, it is natural to treat the whole section above the reflector as homogeneous. Such an assumption, which clearly contradicts the actual model in Figure 1, is the consequence of our inability to reconstruct the vertically varying velocity $v(z)$ using the reflection traveltimes from the bottom of the layer. Note that any permutation of layers in Figure 1 produces exactly the same reflection traveltime $T(x)$ at any offset x ; this phenomenon is called the “O-equivalence” of velocity functions (Goldin, 1986). Thus, for processing purposes it is necessary to *assume* a certain velocity distribution $v(z)$ above the reflector.

If the composite layer in Figure 1 is treated as homogeneous and isotropic in accordance with the usual practice of velocity analysis, the reflection traveltime is supposed to be a hyperbola parameterized by the vertical velocity V_0 :

$$\hat{T}^2(x) = T_0^2 + \frac{x^2}{V_0^2}. \quad (4)$$

The traveltime \hat{T} from equation (4) corresponds to the straight ray ARB (Figure 1), which does not satisfy Fermat’s principle in the actual heterogeneous model. According to Fermat’s principle, the reflection time $T(x)$ along the actual ray AL_ARL_BB (e.g., Taner and Koehler, 1969) is smaller than that predicted by equation (4). As a result, the NMO velocity V_{nmo} for the model in Figure 1 is always greater than the vertical velocity V_0 , and the data cannot be explained in terms of a homogeneous isotropic model. Also, vertical heterogeneity makes the moveout nonhyperbolic, and equation (4) should be treated as the first two terms of the Taylor series expansion in x^2 of the squared reflection traveltime.

Next, we express the ratio V_{nmo}/V_0 through the interval-velocity function $v(z)$. The effective vertical velocity V_0 is given by

$$V_0 = \left[\frac{T_0}{2D} \right]^{-1} = \left[\frac{1}{D} \int_0^D \frac{dz}{v(z)} \right]^{-1}, \quad (5)$$

while the effective V_{nmo} is obtained from the Dix (1955) equation:

$$V_{\text{nmo}}^2 = \frac{1}{T_0} \int_0^{T_0} v^2(t) dt = \frac{2}{T_0} \int_0^D v(z) dz. \quad (6)$$

Using equations (5) and (6), we find

$$\frac{V_{\text{nmo}}^2}{V_0^2} = \frac{T_0}{2D^2} \int_0^D v(z) dz = \frac{\left[\frac{1}{D} \int_0^D v(z) dz \right]}{\left[\frac{1}{D} \int_0^D \frac{dz}{v(z)} \right]^{-1}}. \quad (7)$$

Equation (7) implies that V_{nmo}^2/V_0^2 can be interpreted as the ratio of the arithmetic and harmonic averages of the interval

velocities; it is well known that this ratio cannot be smaller than unity. The same result can be rigorously obtained from the integral version of the Cauchy-Schwartz inequality (e.g., Abramovitz and Stegun, 1965):

$$\int_0^D v(z) dz \int_0^D \frac{dz}{v(z)} \geq \left[\int_0^D dz \right]^2, \quad \text{or} \\ \left[\frac{1}{D} \int_0^D v(z) dz \right] \left[\frac{1}{D} \int_0^D \frac{dz}{v(z)} \right] \geq 1. \quad (8)$$

From equations (7) and (8), it follows that

$$\frac{V_{\text{nmo}}}{V_0} \geq 1. \quad (9)$$

The obtained relationship (9) between V_{nmo} and V_0 is typical for transversely isotropic media with a positive value of δ [see equation (2)]. In terms of the interval velocity $v(z)$, the parameter δ of the effective VTI model is given by

$$\delta = \frac{1}{2} \left[\frac{V_{\text{nmo}}^2}{V_0^2} - 1 \right] \\ = \frac{1}{2} \left\{ \left[\frac{1}{D} \int_0^D v(z) dz \right] \left[\frac{1}{D} \int_0^D \frac{dz}{v(z)} \right] - 1 \right\} \geq 0. \quad (10)$$

Equation (10) might also explain a certain bias toward positive δ values derived from stacking (NMO) and vertical velocities for purposes of anisotropic parameter estimation. Note that such a bias can also be caused by the influence of non-hyperbolic moveout on P-wave stacking velocity, particularly for spreadlength-to-depth ratios substantially higher than unity (Tsvankin and Thomsen, 1994).

EFFECTIVE NMO ELLIPSE FROM A DIPPING REFLECTOR

Next, consider P-wave reflection moveout from a plane dipping reflector beneath a vertically heterogeneous isotropic medium (Figure 2). The azimuthally dependent P-wave NMO velocity for this model is described by the NMO ellipse with axes in the dip and strike directions of the reflector (Grechka and Tsvankin, 1998b). Since the dip plane of the reflector represents a plane of symmetry for the whole model, the Dix-type averaging of the interval NMO ellipses (Grechka et al., 1999) reduces to the conventional Dix (1955) formula for the NMO velocities in the dip ($V_{\text{nmo,dip}}$) and strike ($V_{\text{nmo,str}}$) directions:

$$V_{\text{nmo,dip}}^2 = \frac{1}{T} \int_0^T v_{\text{nmo,dip}}^2(t) dt, \quad (11)$$

$$V_{\text{nmo,str}}^2 = \frac{1}{T} \int_0^T v_{\text{nmo,str}}^2(t) dt. \quad (12)$$

Here, T is the two-way zero-offset traveltime, and $v_{\text{nmo,dip}}$ and $v_{\text{nmo,str}}$ are the interval (local) dip-line and strike-line NMO velocities computed along the zero-offset ray (Figure 2), as described in Grechka et al. (1999). Since the medium is isotropic, the values of the interval NMO velocities can be adapted from Levin (1971):

$$v_{\text{nmo,dip}}(t) = \frac{1}{q(t)}, \quad (13)$$

$$v_{\text{nmo,str}}(t) = v(t), \quad (14)$$

where $v(t)$ is the interval (isotropic) velocity and

$$q(t) = \sqrt{\frac{1}{v^2(t)} - p^2} \quad (15)$$

is the vertical component of the slowness vector; $p = \sin \theta / v(t)$ is the horizontal slowness component (ray parameter).

Note that the ray parameter p is preserved along any ray propagating in vertically heterogeneous media in accordance with Snell's law. Since we are not interested in treating inhomogeneous (evanescent) waves, $q(t)$ is taken to be real and positive for any t , which means that p satisfies the inequality

$$p^2 v^2(t) < 1. \quad (16)$$

Substituting equations (13)–(15) into equations (11) and (12) yields

$$V_{\text{nmo,dip}}^2 = \frac{1}{T} \int_0^T \frac{v^2 dt}{1 - p^2 v^2}, \quad (17)$$

$$V_{\text{nmo,str}}^2 = \frac{1}{T} \int_0^T v^2 dt. \quad (18)$$

If the medium above the reflector were homogeneous, the dip and strike components of the NMO velocity would satisfy the well-known cosine-of-dip relationship that follows from equations (13) and (14):

$$V_{\text{nmo,dip}}^2 (1 - p^2 V_{\text{nmo,str}}^2) = V_{\text{nmo,str}}^2. \quad (19)$$

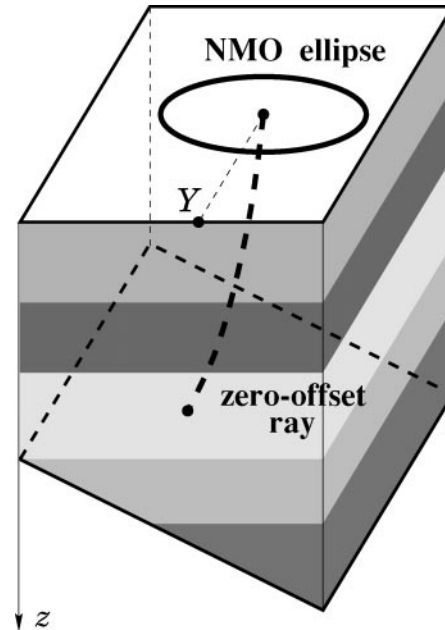


FIG. 2. P-wave NMO ellipse from a dipping reflector overlaid by a vertically heterogeneous isotropic medium.

Denoting the left-hand side of equation (19) by $\tilde{V}_{\text{nmo,dip}}^2$ and using equations (17) and (18), we prove in Appendix A that

$$\tilde{V}_{\text{nmo,dip}}^2 \equiv V_{\text{nmo,dip}}^2 (1 - p^2 V_{\text{nmo,str}}^2) \geq V_{\text{nmo,str}}^2 \quad (20)$$

for any interval-velocity function $v(z)$. The equality $\tilde{V}_{\text{nmo,dip}} = V_{\text{nmo,str}}$ is reached only in the special cases of a horizontal reflector ($p = 0$) or homogeneous media [$v(z) = \text{const}$].

Figure 3 shows the two-way zero-offset traveltime T and the velocities $\tilde{V}_{\text{nmo,dip}}$ and $V_{\text{nmo,str}}$ computed for a model that contains a plane dipping reflector $z = 1 + Y \tan \phi$ (ϕ is the dip) beneath an isotropic medium with a constant vertical-velocity gradient (Figure 3a). The common-midpoint (CMP) locations Y in Figure 3 record reflections from the segment of the interface within the depth range $1 \text{ km} \leq z \leq 2 \text{ km}$. Note that the zero-offset time T (Figure 3b) varies with Y because of the combined influence of the reflector dip and vertical heterogeneity. The gradual decrease of the reflection slope $p(Y) = (1/2)(dT/dY)$ in Figure 3b and the corresponding increase of the velocities in Figure 3c may lead to the conclusion that the subsurface is laterally heterogeneous.

Also, since $\tilde{V}_{\text{nmo,dip}} \neq V_{\text{nmo,str}}$ at any CMP location, the medium above the reflector may be mistakenly identified as anisotropic. It is natural to assume that such a model is transversely isotropic with a vertical symmetry axis, because the actual heterogeneous isotropic medium does not produce any effective azimuthal anisotropy. Below, we invert the dip- and strike-components of the NMO velocity for the effective anellipticity coefficient η under the assumption that the model is homogeneous and has VTI symmetry.

ESTIMATION OF THE EFFECTIVE V_{NMO} AND η

Clearly, the subsurface parameters obtained from the traveltimes and velocities in Figures 3b and 3c will depend on the selected model of the overburden. If the model is assumed (correctly) to be vertically heterogeneous and isotropic, the reflection data can be used to estimate the actual function $v(z)$ within the depth range $1 \text{ km} \leq z \leq 2 \text{ km}$ covered by the reflection points (Goldin, 1986). Still, the velocity $v(z)$ cannot be

found uniquely for depths $z < 1 \text{ km}$ because the data do not include reflections from the shallow segment of the interface.

Therefore, for practical purposes of velocity model-building, it may be more attractive to adopt a model that is vertically homogeneous but changes laterally. Then, it is possible to compute medium parameters uniquely for the whole overburden, although the inverted model cannot be isotropic (indeed, $\tilde{V}_{\text{nmo,dip}} > V_{\text{nmo,str}}$). If we ignore the contribution of lateral heterogeneity to the NMO-velocity measurements on the scale of a single CMP gather, as is usually done in practice, the velocities $V_{\text{nmo,dip}}(Y)$ and $V_{\text{nmo,str}}(Y)$ can be inverted for the VTI parameters at each CMP location. Then, the dependence of the obtained parameters on Y can be interpreted in terms of lateral heterogeneity.

Since the reflection traveltimes are symmetric with respect to the dip plane, it is natural to use the azimuthally isotropic VTI model for the inversion. We applied the algorithm of Grechka and Tsavkin (1998b) based on the exact equation of the NMO ellipse to estimate the zero-dip NMO velocity $V_{\text{nmo}}(Y)$ and the anellipticity coefficient $\eta(Y)$ of the effective VTI medium. Both V_{nmo} and η vary with the CMP coordinate Y (Figure 4), which creates an apparent lateral heterogeneity.

As illustrated by the example in Figure 4, the effective parameter η for our model is always non-negative. This is a general result that follows directly from inequality (20) in the limit of weak anisotropy (the proof is given in Appendix B); η vanishes only for the trivial special case of a homogeneous medium. Thus, vertical heterogeneity unaccounted for in velocity analysis yields an effective VTI medium with the same (positive) sign of the coefficients δ and η . It is interesting that the effective η estimated from the long-spread (nonhyperbolic) move-out of P-waves in vertically heterogeneous isotropic media is also nonnegative (Fomel and Grechka, 2001).

The value of η is controlled not only by the magnitude of vertical heterogeneity, but also by reflector dip. To study the dependence of the effective parameters η and V_{nmo} on dip, it is convenient to express the difference $\tilde{V}_{\text{nmo,dip}}^2 - V_{\text{nmo,str}}^2$ as a function of the horizontal slowness p of the zero-offset ray. Combining equations (20), (A-2), and (A-5), we find

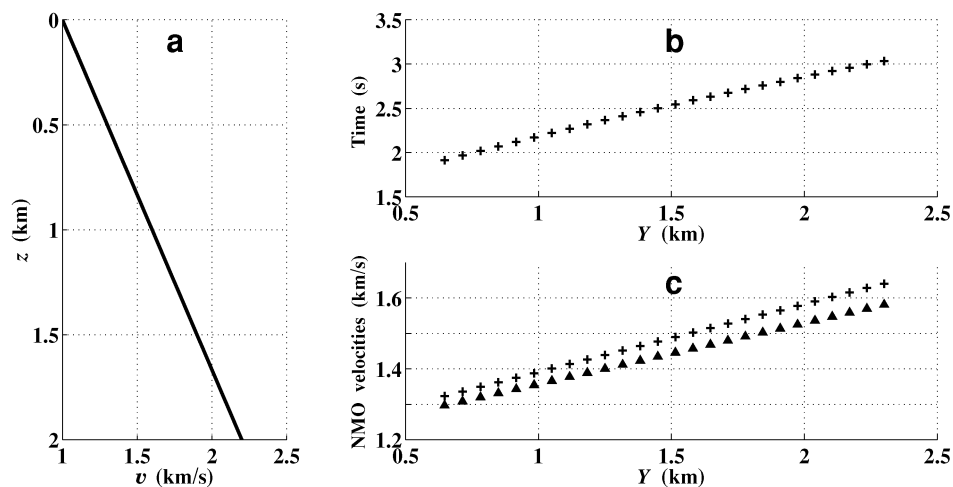


FIG. 3. Normal moveout for a dipping reflector beneath a vertically heterogeneous isotropic medium. (a) Linear velocity function $v(z) = 1 + 0.6z$; (b) two-way zero-offset traveltime T as a function of the CMP coordinate Y in the dip plane of the reflector (the dip $\phi = 40^\circ$); (c) quantity $\tilde{V}_{\text{nmo,dip}}$ (crosses) defined by equation (20) and the strike-line NMO velocity $V_{\text{nmo,str}}$ (triangles).

$$\begin{aligned} \tilde{V}_{\text{nmo,dip}}^2 - V_{\text{nmo,str}}^2 &= \frac{1}{T} \sum_{i=1}^{\infty} p^{2i} \left[\int_0^T v^{2(i+1)} dt \right. \\ &\quad \left. - \frac{1}{T} \int_0^T v^2 dt \int_0^T v^{2i} dt \right]. \end{aligned} \quad (21)$$

On the other hand, in the limit of weak anisotropy of the effective medium the same difference can be represented as [using equations (20) and (B-1)–(B-3)]

$$\begin{aligned} \tilde{V}_{\text{nmo,dip}}^2 - V_{\text{nmo,str}}^2 &= 8p^2\eta V_{\text{nmo}}^4 (1 - p^2 V_{\text{nmo}}^2) \\ &= 8p^2\eta V_{\text{nmo}}^4 - 8p^4\eta V_{\text{nmo}}^6. \end{aligned} \quad (22)$$

Clearly, for a horizontal reflector ($p=0$), the NMO ellipse degenerates into a circle, and the effective η is undefined. For $p \rightarrow 0$, however, η can be found by matching the coefficients of the p^2 -term in equations (21) and (22). According to the Chebyshev inequality (A-6), the term in the brackets in equation (21) is nonnegative for any value of i , and goes to zero only for a homogeneous medium ($v=\text{const}$). Therefore, for subhorizontal reflectors beneath a vertically heterogeneous isotropic medium, the effective η is always positive (see also Appendix B).

Furthermore, analysis of the expansion coefficients in equations (21) and (22) suggests that the effective η increases with p (i.e., with dip) for mild dips. In contrast, the effective NMO velocity V_{nmo} should become slightly smaller with increasing dip. Figure 5 demonstrates that the variation of the effective V_{nmo} and η with dip is qualitatively predicted by the weak-anisotropy approximation (22). Both V_{nmo} and η are plotted in Figure 5 as functions of the depth z of reflection points because the effective velocities $\tilde{V}_{\text{nmo,dip}}$ and $V_{\text{nmo,str}}$ in equations (21) and (22) refer to a fixed value of z .

It might be thought that the nonnegligible values of η for the model from Figure 3 are associated with the monotonic increase in velocity with depth. However, since the apparent

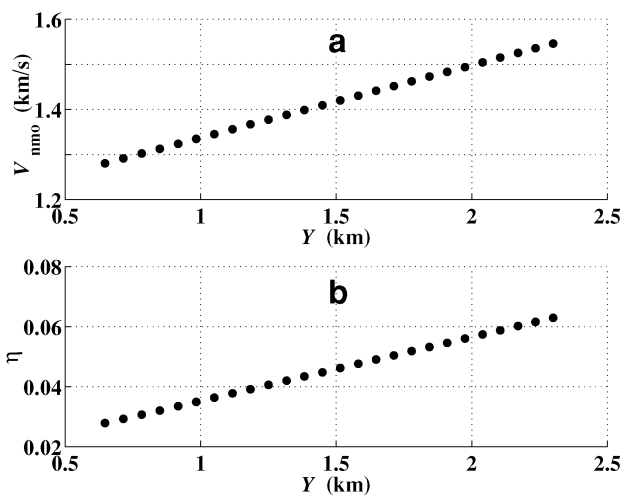


FIG. 4. Effective zero-dip NMO velocity V_{nmo} (a) and the anisotropy coefficient η (b) estimated from the NMO velocities shown in Figure 3c under the assumption of a vertically homogeneous VTI model.

anisotropy is caused only by the different types of averaging applied to the vertically varying velocity to obtain the measured (effective) quantities, the phenomena discussed above can be observed in any $v(z)$ media. For example, Figure 6 shows the inverted effective V_{nmo} and η for the isotropic velocity $v(z)$ specified as a random Gaussian function (the mean is 1 km/s, the standard deviation is 0.1 km/s). Similar to the model from Figure 3, the velocities V_{nmo} are smaller than the mean of $v(z)$ (Figure 6b), and the coefficients η are positive (Figure 6c). Note that to treat this model as layered and apply the equations for the effective NMO velocities given above, the wavelength should be sufficiently small (i.e., the frequency should reach about 100 Hz).

As illustrated by Figure 7, for more pronounced random velocity variations the effective η can reach 0.1–0.2, values that can cause serious distortions in isotropic imaging. Although it might seem that the model in Figure 7 exaggerates the magnitude of velocity variations observed in typical well logs, this conclusion is supported by the real-data example in Figure 8. The velocity function in Figure 8 was taken from a well log recorded in a borehole on the U.S. Gulf Coast (Dvorkin et al., 2002). For a wide range of CMP coordinates, the values of η estimated for a reflector dip of 60° are higher than 0.05 and exceed 0.1 for $Y \approx 0.2$. In agreement with the synthetic examples, the effective η becomes much smaller when the dip decreases to 30 – 40° . However, note that the log in Figure 8 is quite short, and we had to make the heterogeneous layer under the midpoint $Y=0$ only about 0.1 km thick. Increasing the layer thickness to 0.3–0.4 km would yield larger η values for the same reflector dip. Evidently, small-scale layering may have a substantial influence on the effective anisotropic coefficients.

DISCUSSION AND CONCLUSIONS

Complicated, spatially varying isotropic velocity fields are sometimes kinematically equivalent to simpler effective anisotropic models, which poses a serious challenge for anisotropic velocity analysis. Here, we examined one of

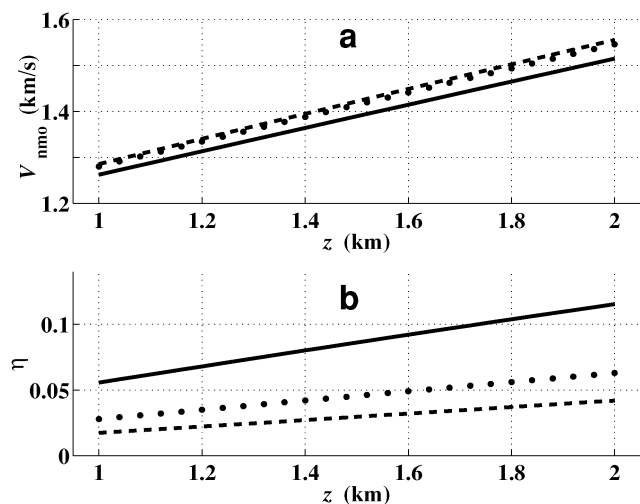


FIG. 5. Effective V_{nmo} (a) and η (b) estimated for reflectors with the dips $\phi = 30^\circ$ (dashes), $\phi = 40^\circ$ (dots), and $\phi = 50^\circ$ (solid) overlaid by a heterogeneous isotropic medium with the velocity function $v(z) = 1 + 0.6z$. The depths of the zero-offset reflection points (the horizontal axis) range from 1 to 2 km.

the consequences of approximate treatment of vertical heterogeneity in estimating the subsurface velocity field. If a heterogeneous medium between reflectors is treated as a homogeneous layer, the traveltimes cannot be fit without introducing apparent (nonexistent) anisotropy.

The apparent (or effective) VTI model, equivalent to a vertically heterogeneous isotropic medium above a horizontal reflector, has a nonnegative coefficient δ . If the reflector is dipping, the relationship between the semi-axes of the P-wave NMO ellipse yields a nonnegative effective anellipticity coefficient $\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$. Therefore, Thomsen (1986) parameters of the effective VTI model always satisfy the inequality $\epsilon \geq \delta \geq 0$.

It is interesting to note that although the inequality $\eta \geq 0$ was obtained from 3-D (azimuthal) moveout inversion using the dip- and strike-components of the P-wave NMO ellipse ($V_{\text{nmo,dip}}$ and $V_{\text{nmo,str}}$), it can be shown that the same result would follow from the 2-D dip-moveout (DMO) method of

Alkhalifah and Tsvankin (1995), which operates with $V_{\text{nmo,dip}}$ and the NMO velocity from a horizontal reflector V_{nmo} . The values of η obtained by the Alkhalifah–Tsvankin method, however, would not necessarily be the same as those determined from the NMO ellipse because the relationship between V_{nmo} , V_{dip} , and $V_{\text{nmo,str}}$ in our model differs from that in homogeneous VTI media. To fit all three velocities (V_{nmo} , V_{dip} , and $V_{\text{nmo,str}}$) using a homogeneous anisotropic medium, it is necessary to assume orthorhombic (Grechka and Tsvankin, 1999) or even lower symmetry. Note that the effective η needed to describe nonhyperbolic reflection moveout of P-waves over a heterogeneous isotropic medium is also nonnegative (Alkhalifah, 1997; Fomel and Grechka, 2001; Tsvankin, 2001).

Positive values of the effective anisotropic coefficients stem from the common physical origin—Fermat’s principle, which requires that reflected rays bend in such a way that the traveltimes for all source-receiver pairs reach their minimum values.

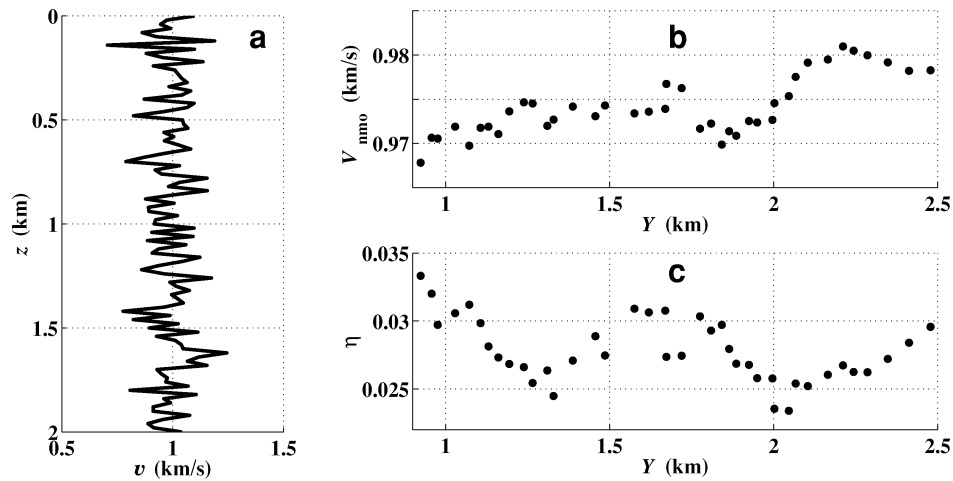


FIG. 6. Isotropic velocity model (a) and the inverted effective V_{nmo} (b) and η (c). The mean of the velocity function is 1 km/s, the standard deviation is 0.1 km/s, the reflector dip $\phi = 40^\circ$, and the depths of the reflection points range from 1 to 2 km (Y increases in the downdip direction).

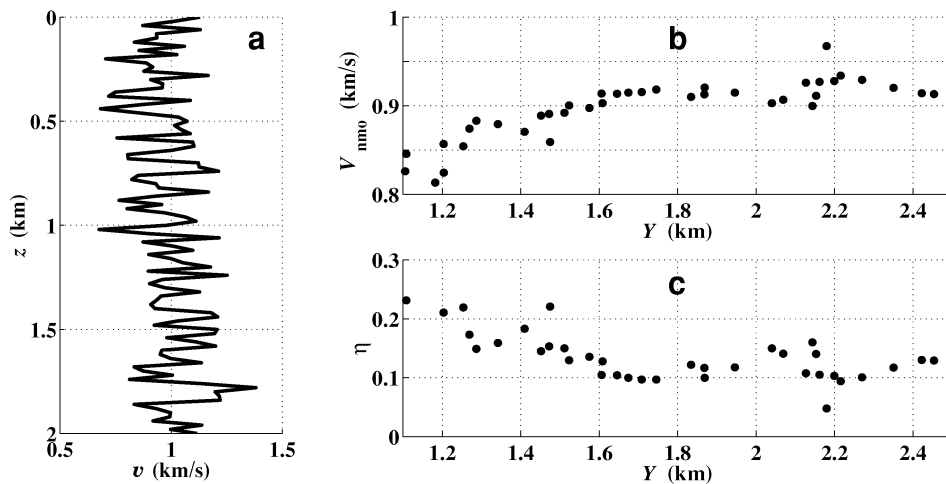


FIG. 7. Isotropic velocity model (a) and the inverted effective V_{nmo} (b) and η (c). The mean of the velocity function is 1 km/s, the standard deviation is 0.15 km/s, the reflector dip $\phi = 45^\circ$, and the depths of the reflection points range from 1 to 2 km.

As discussed above, Fermat's principle directly explains non-negative effective δ values for reflections from horizontal interfaces. When the reflector is dipping, we observe a similar phenomenon: because of ray bending, the reflection traveltime increases with offset slower than that in a reference homogeneous medium, and the corresponding NMO velocity is higher. Since the influence of ray bending is more pronounced in the dip plane, the vertical heterogeneity increases the difference between the dip-line and strike-line NMO velocities, which translates into positive values of η of the effective homogeneous VTI model.

The analysis of the effective anisotropic coefficients of the apparent VTI model also provides insight into potential biases in anisotropic moveout inversion. For example, ignoring the vertical velocity gradient between reflectors in VTI media should lead to overestimating the parameters η and δ . This may partially explain the discrepancy between predominantly positive δ values obtained from reflection data (e.g., Alkhalifah et al., 1996; Williamson et al., 1997) and sometimes negative values of δ derived from core measurements and VSP surveys (e.g., Thomsen, 1986; Vernik and Liu, 1997; Jakobsen and Johansen, 2000). Another possible reason for overstated values of δ inferred from reflection traveltimes is the increase in stacking (moveout) velocity with spreadlength caused by nonhyperbolic moveout. To remove or mitigate this distortion, it is necessary to apply a nonhyperbolic equation $t^2(x^2)$ in moveout velocity analysis (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995). Note that if vertical transverse isotropy is caused by fine isotropic layering on a scale small compared to the predominant wavelength (see Backus, 1962), the parameters η and ϵ are positive, while δ can be either positive or negative (Berryman, 1979; Berryman et al., 1999).

Seismic processors are certainly aware of the influence of vertical heterogeneity on reflection moveout and typically try to incorporate velocity gradients into their models. However, a fundamental problem in velocity analysis of reflection data is that one always has to rely on some kind of interpolation

when building velocity functions between reflecting interfaces. In the absence of direct information (e.g., from well logs) about intralayer vertical heterogeneity, the vertical velocity variation within the layer has to be *assumed* since there is an infinite number of different intralayer velocity functions that can produce the same moveout velocity from the underlying reflector. In our paper, we analyzed the consequences of making the simplest assumption that the layer is homogeneous. If the processor introduces, for example, an inaccurate value of the velocity gradient, the moveout still cannot be described by this (distorted) isotropic model, but the values of the corresponding effective anisotropic coefficients will be different. However, it is beyond the scope of this paper to discuss all possible erroneous assumptions about intralayer vertical heterogeneity.

Problems of the type considered here are typical for seismic velocity analysis and inversion. Since the available data usually cannot constrain all components of the parameter vector \mathbf{m} , the actual velocity distribution $v(\mathbf{m}, \mathbf{x})$ is often replaced by a certain model $\hat{v}(\hat{\mathbf{m}}, \mathbf{x})$ with fewer unknowns, so that all model parameters $\hat{\mathbf{m}}$ can be resolved uniquely. For that reason, some anisotropy-related components of the vector $\hat{\mathbf{m}}$ may be invoked in traveltime inversion for complex spatially varying isotropic velocity fields. Improved understanding of various types of interplay between anisotropy and heterogeneity remains one of the key problems in anisotropic velocity analysis.

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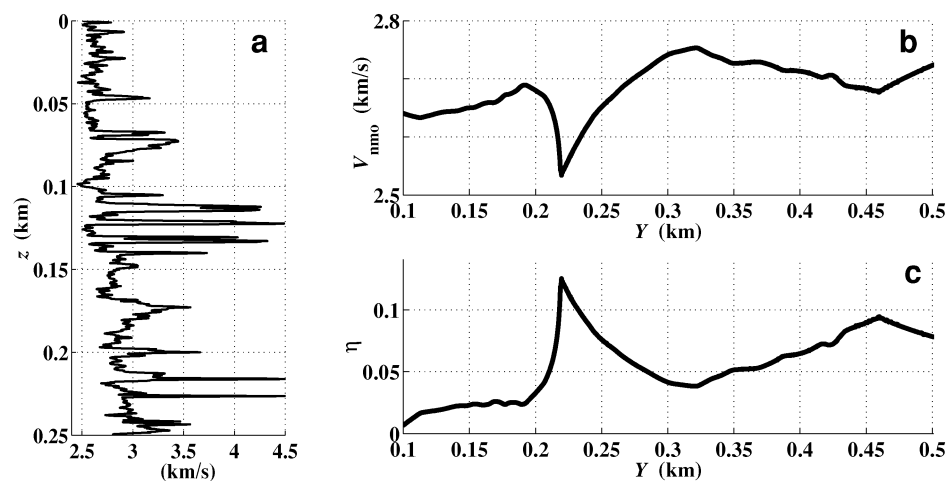


FIG. 8. Portion of a well log from the U.S. Gulf Coast (a) and the inverted effective V_{nmo} (b) and η (c). The effective V_{nmo} and η were estimated for a reflector with the dip $\phi = 60^\circ$ located at the depth $z = 0.1$ km under the CMP $Y = 0$. Prior to the inversion, the log was smoothed by applying a 40-m-wide running window (the width is based on an average velocity of 3 km/s taken from the log and a frequency of 75 Hz).

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APPENDIX A

RELATIONSHIP BETWEEN THE SEMI-AXES OF THE NMO ELLIPSE

Here, we prove inequality (20), which involves the semi-axes of the P-wave NMO ellipse ($V_{\text{nmo,dip}}$ and $V_{\text{nmo,str}}$) measured above a vertically heterogeneous isotropic medium:

$$\tilde{V}_{\text{nmo,dip}}^2 \equiv V_{\text{nmo,dip}}^2 (1 - p^2 V_{\text{nmo,str}}^2) \geq V_{\text{nmo,str}}^2, \quad (\text{A-1})$$

where p is the horizontal component of the slowness vector of the zero-offset ray. After substituting equations (17) and (18) and multiplying by T^2 , inequality (A-1) takes the form

$$\left(\int_0^T \frac{v^2 dt}{1 - p^2 v^2} \right) \left(T - p^2 \int_0^T v^2 dt \right) \geq T \int_0^T v^2 dt. \quad (\text{A-2})$$

Using inequality (16), the denominator of the first integral in equation (A-2) can be replaced by the converging series

$$\frac{1}{1 - p^2 v^2} = \sum_{i=0}^{\infty} p^{2i} v^{2i}. \quad (\text{A-3})$$

Since p is independent of t , the left-hand side of inequality (A-2) can be rewritten as

$$\begin{aligned} F &\equiv \left(\int_0^T \frac{v^2 dt}{1 - p^2 v^2} \right) \left(T - p^2 \int_0^T v^2 dt \right) \\ &= \left(\sum_{i=0}^{\infty} p^{2i} \int_0^T v^{2(i+1)} dt \right) \left(T - p^2 \int_0^T v^2 dt \right) \end{aligned}$$

$$\begin{aligned} &= T \sum_{i=0}^{\infty} p^{2i} \int_0^T v^{2(i+1)} dt \\ &\quad - \sum_{i=0}^{\infty} p^{2(i+1)} \int_0^T v^2 dt \int_0^T v^{2(i+1)} dt \\ &= T \int_0^T v^2 dt + T \sum_{i=1}^{\infty} p^{2i} \int_0^T v^{2(i+1)} dt \\ &\quad - \sum_{i=1}^{\infty} p^{2i} \int_0^T v^2 dt \int_0^T v^{2i} dt. \quad (\text{A-4}) \end{aligned}$$

Therefore,

$$\begin{aligned} F &= T \int_0^T v^2 dt + \sum_{i=1}^{\infty} p^{2i} \left[T \int_0^T v^{2(i+1)} dt \right. \\ &\quad \left. - \int_0^T v^2 dt \int_0^T v^{2i} dt \right]. \quad (\text{A-5}) \end{aligned}$$

Note that the first integral in equation (A-5) coincides with the right-hand side of inequality (A-2). The terms in the brackets of equation (A-5) are always nonnegative, which follows from the Chebyshev inequality [Abramovitz and Stegun, 1965,

equation (3.2.7)]:

$$T \int_0^T v^{2(i+1)} dt \geq \int_0^T v^2 dt \int_0^T v^{2i} dt. \quad (\text{A-6})$$

The equality is reached only if $v(t) = \text{const}$. Also, note that for $i = 1$, inequality (A-6) takes the well-known Cauchy-Schwartz form

$$T \int_0^T v^4 dt \geq \left(\int_0^T v^2 dt \right)^2. \quad (\text{A-7})$$

Thus, from equation (A-5), we conclude that

$$\begin{cases} F = T \int_0^T v^2 dt & \text{if } v(t) = \text{const} \text{ or } p = 0 \text{ and} \\ F > T \int_0^T v^2 dt & \text{otherwise,} \end{cases} \quad (\text{A-8})$$

which proves inequality (A-2).

APPENDIX B

INEQUALITY FOR THE EFFECTIVE PARAMETER η

In this appendix, we show that the parameter η determined from the NMO ellipse for the effective VTI model is always nonnegative. The P-wave NMO velocities in the dip ($V_{\text{nmo,dip}}$) and strike ($V_{\text{nmo,str}}$) directions satisfy inequality (A-1) proved in Appendix A. Assuming that the anisotropy is weak and $|\eta| \ll 1$, we can use the following linearized approximations for $V_{\text{nmo,dip}}$ and $V_{\text{nmo,str}}$ (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998b):

$$V_{\text{nmo,dip}}^2(p) = \frac{V_{\text{nmo}}^2}{1-\xi} \left[1 + \frac{2\eta\xi}{1-\xi} (6-9\xi+4\xi^2) \right], \quad (\text{B-1})$$

$$V_{\text{nmo,str}}^2(p) = V_{\text{nmo}}^2 [1 + 2\eta\xi(2-\xi)]; \quad (\text{B-2})$$

$$\xi \equiv p^2 V_{\text{nmo}}^2. \quad (\text{B-3})$$

Equations (B-1) and (B-2) are derived under the assumption that

$$\xi < 1, \quad (\text{B-4})$$

which effectively removes from consideration large dips close to 90° .

Substituting equations (B-1) and (B-2) into inequality (A-1) leads to

$$\frac{1}{1-\xi} \left[1 + \frac{2\eta\xi}{1-\xi} (6-9\xi+4\xi^2) \right] [1-\xi-2\eta\xi^2(2-\xi)] \geq 1 + 2\eta\xi(2-\xi). \quad (\text{B-5})$$

Further linearization in η yields

$$\frac{1}{1-\xi} [1-\xi-2\eta\xi^2(2-\xi)+2\eta\xi(6-9\xi+4\xi^2)] \geq 1 + 2\eta\xi(2-\xi), \quad (\text{B-6})$$

or

$$8\eta\xi(1-\xi) \geq 0. \quad (\text{B-7})$$

Since $0 \leq \xi < 1$ [see inequality (B-4)],

$$\eta \geq 0. \quad (\text{B-8})$$

As discussed in Appendix A, except for the special cases $p = \xi = 0$ (horizontal reflector) and $v(z) = \text{const}$ (homogeneous isotropic medium), inequality (A-1) becomes

$$V_{\text{nmo,dip}}^2(1-p^2V_{\text{nmo,str}}^2) > V_{\text{nmo,str}}^2, \quad (\text{B-9})$$

and

$$8\eta\xi(1-\xi) > 0. \quad (\text{B-10})$$

Therefore, for dipping reflectors ($\xi \neq 0$) beneath a heterogeneous isotropic medium, the effective η computed from the P-wave NMO ellipse is strictly positive. Clearly, for $v(z) = \text{const}$ the effective η vanishes because the medium is isotropic and homogeneous.