GEOPHYSICS, VOL. 62, NO. 2 (MARCH-APRIL 1997); P. 614–629, 7 FIGS.

Reflection moveout and parameter estimation for horizontal transverse isotropy

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ABSTRACT

Transverse isotropy with a horizontal axis of symmetry (HTI) is the simplest azimuthally anisotropic model used to describe fractured reservoirs that contain parallel vertical cracks. Here, I present an exact equation for normal-moveout (NMO) velocities from horizontal reflectors valid for pure modes in HTI media with any strength of anisotropy. The azimuthally dependent P-wave NMO velocity, which can be obtained from 3-D surveys, is controlled by the principal direction of the anisotropy (crack orientation), the P-wave vertical velocity, and an effective anisotropic parameter equivalent to Thomsen's coefficient δ.

An important parameter of fracture systems that can be constrained by seismic data is the crack density, which is usually estimated through the shear-wave splitting coefficient γ. The formalism developed here makes it possible to obtain the shear-wave splitting parameter using the NMO velocities of P and shear waves from horizontal reflectors. Furthermore, γ can be estimated just from the P-wave NMO velocity in the special case of the vanishing parameter ϵ, corresponding to thin cracks and negligible equant porosity. Also, P-wave moveout alone is sufficient to constrain γ if either dipping events are available or the velocity in the symmetry direction is known. Determination of the splitting parameter from P-wave data requires, however, an estimate of the ratio of the P–to–S vertical velocities (either of the split shear waves can be used).

Velocities and polarizations in the vertical symmetry plane of HTI media, that contains the symmetry axis, are described by the known equations for vertical transverse isotropy (VTI). Time-related 2-D P-wave processing (NMO, DMO, time migration) in this plane is governed by the same two parameters (the NMO velocity from a horizontal reflector and coefficient η) as in media with a vertical symmetry axis. The analogy between vertical and horizontal transverse isotropy makes it possible to introduce Thomsen parameters of the "equivalent" VTI model, which not only control the azimuthally dependent NMO velocity, but also can be used to reconstruct phase velocity and carry out seismic processing in off-symmetry planes.

INTRODUCTION

In horizontally layered, isotropic media, normal-moveout (NMO) velocity of reflected waves (defined in the zero-spread limit) is equal to the rms of the velocities in each layer. Conventional velocity analysis takes advantage of this simple relation by obtaining interval velocities from the stacking (moveout) velocity via the Dix (1955) formula. If the medium is anisotropic, NMO velocity in a single layer is no longer equal to the vertical velocity, nor is V_{nmo} in multilayered media given by the rms average of vertical velocities (Hake et al., 1984). The difference between the vertical and moveout velocities in anisotropic formations, such as shales, causes errors in time-to-depth conversion (Banik,1984). On the other hand, inversion of moveout velocities can provide estimates of the anisotropic coefficients that can be used in seismic processing, amplitude variation with offset (AVO) analysis, and lithology discrimination.

Analytic expressions for NMO velocities from horizontal reflectors are well known for transversely isotropic models with a vertical symmetry axis (VTI media) (e.g., Lyakhovitsky and Nevsky, 1971; Hake et al., 1984; Thomsen, 1986). Using Thomsen's (1986) notation, the NMO velocities of the P-, SV-, and SH-waves[1] in a single VTI layer can be represented as

$$V_{\text{nmo}}[P\text{-wave}] = V_{P\text{vert}}\sqrt{1+2\delta}, \qquad (1)$$

$$V_{\text{nmo}}[SV\text{-wave}] = V_{S\text{vert}}\sqrt{1+2\sigma}, \qquad (2)$$

[1] I will omit the qualifiers in "quasi-P-wave" and "quasi-SV-wave."

Manuscript received by the Editor August 7, 1995; revised manuscript received June 28, 1996.
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$$V_{\text{nmo}}[SH\text{-wave}] = V_{S\text{vert}}\sqrt{1+2\gamma}, \tag{3}$$

with

$$\sigma \equiv \left(\frac{V_{P\text{vert}}}{V_{S\text{vert}}}\right)^2 (\epsilon - \delta), \tag{4}$$

where $V_{P\text{vert}}$ and $V_{S\text{vert}}$ are the vertical velocities of the P- and S-waves respectively, ϵ, δ, and γ are Thomsen's anisotropy parameters for vertical transverse isotropy, and σ is the effective parameter introduced in Tsvankin and Thomsen (1994) to describe SV-wave propagation. Equations (1)–(3) are valid for VTI models with arbitrary strength of the anisotropy. In horizontally layered media, normal-moveout velocity of each mode is given by the rms average of the interval NMO velocities (1)–(3).

As discussed in Tsvankin and Thomsen (1995), the NMO velocities from horizontal reflectors in VTI media are not sufficient to recover the vertical velocities and anisotropic parameters, even if all three waves are recorded. However, if some additional information is available (such as the reflector depth or one of the vertical velocities), equations (1)–(3) make it possible to obtain all anisotropic coefficients. The accuracy of this estimation procedure strongly depends on the quality of the tie between seismic and borehole data (the latter is usually needed to obtain the vertical velocity).

Another anisotropic model of practical importance is transversely isotropic with a horizontal symmetry axis (HTI). The most common physical reason for the HTI symmetry is a system of parallel vertical cracks (fractures), with quasi-circular shapes (like pennies), embedded in an isotropic matrix (Hudson, 1981; Crampin, 1985; Thomsen, 1988). It should be emphasized that while modeling and processing of reflection data are more complicated for horizontal transverse isotropy than for VTI media, the azimuthal dependence of moveout velocities and amplitudes in HTI models (if properly accounted for) provides additional information for seismic inversion. Obviously, it is impossible to carry out such an inversion procedure without relating the attributes of the reflected waves to the anisotropic parameters.

Thomsen (1988) presented the weak-anisotropy approximation for NMO velocities of P- and S-waves from a horizontal reflector in the symmetry plane of HTI media that contains the symmetry axis. More general nonhyperbolic ("skewed") moveout equations for pure modes both in the symmetry and off-symmetry planes were given in Sena (1991). His results, however, are valid only for weak anisotropy and horizontal reflectors. A weak-anisotropy formalism similar to that in Sena (1991) was employed in Li and Crampin (1993) to study the moveout from horizontal reflectors in a layer with transversely isotropic or orthorhombic symmetry. Dellinger and Muir (1993) suggested finding reflection traveltimes in horizontally layered anisotropic media as a power series in terms of ray parameter using the equivalent-medium formulation.

Here, I present an exact equation for normal-moveout velocities of pure modes valid for any orientation of the survey line over an HTI layer. If the anisotropy is caused by vertical cracks, P-wave moveout data can be used to find the crack orientation and, in some cases, estimate the shear-wave splitting parameter γ, which is close to the crack density in HTI models. Another practically important conclusion of this work is that 2-D time processing of P-wave data in the vertical plane that contains the symmetry axis is governed by the same two effective parameters that Alkhalifah and Tsvankin (1995) introduced for vertical transverse isotropy.

DESCRIPTION OF THE HTI MODEL

The transversely isotropic model with a horizontal symmetry axis has two mutually orthogonal vertical planes of symmetry shown in Figure 1. They will be referred to as the "isotropy plane" (the one normal to the symmetry axis) and the "symmetry-axis plane" (the one that contains the symmetry axis). Note that the kinematic signatures and polarizations of all three waves in the isotropy plane are described by just the isotropic equations. As shown below, the velocities and polarizations in the symmetry-axis plane can be found by analogy with VTI media.

The split shear waves in HTI media will be denoted as "$S^{\parallel}$" and "$S^{\perp}$," with the $S^{\parallel}$-wave polarized in the isotropy plane and the $S^{\perp}$-wave polarization vector being in the plane formed by the symmetry axis and the slowness vector. The form of the superscripts is explained by the fact that in HTI media caused by the parallel vertical cracks, the polarization vector of $S^{\parallel}$ is parallel to the crack planes, while the wave $S^{\perp}$ at vertical incidence is polarized normal to the cracks. In the symmetry-axis plane, the $S^{\perp}$-wave represents and in-plane (SV) motion, while the $S^{\parallel}$-wave is polarized in the direction orthogonal to the plane and may be called the SH-wave. Therefore, for this plane the $S^{\perp}$ and $S^{\parallel}$-waves can be denoted as the SV- and SH-waves, respectively. However, the polarizations of the shear waves recorded in any other plane do not conform to this simple rule. For instance, in the isotropy plane the particle motion of the wave that I refer to as $S^{\parallel}$ will be confined to the incidence plane, while the $S^{\perp}$-wave is polarized parallel to the symmetry axis and, therefore, orthogonally to the incidence plane. The mode $S^{\parallel}$ is often called the "fast" shear wave since at vertical incidence it propagates faster than $S^{\perp}$ (e.g., Crampin, 1985).

Conventionally, the HTI model is characterized by the stiffness tensor $c_{ijk\ell}$ that corresponds to the coordinate frame in which x_1 represents the symmetry axis (Figure 1). Taking advantage of the symmetries in the stiffness tensor and using the

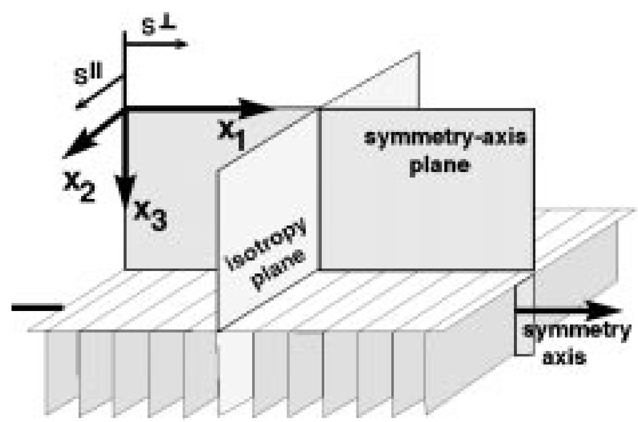

FIG. 1. Two vertical symmetry planes in HTI media. In the plane normal to the symmetry axis ("isotropy plane"), velocity is independent of propagation angle.

Voigt recipe, $c_{ijk\ell}$ can be written as a symmetric 6×6 matrix of the following form (only nonzero elements are shown):

$$\mathbf{c}_{\rm HTI} = \begin{pmatrix} c_{11} & c_{13} & c_{13} & & & \\ c_{13} & c_{33} & (c_{33}-2c_{44}) & & & \\ c_{13} & (c_{33}-2c_{44}) & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{55} \end{pmatrix}. \tag{5}$$

Alternatively, it is possible to use to stiffness tensor $c^{(R)}_{ijk\ell}$ (the R stands for "rotated") or Thomsen's (1986) parameters defined in a rotated coordinate system with the x_3-axis pointing in the symmetry direction. These "generic" Thomsen parameters include the P- and S-wave velocities ($V^{(R)}_{P0}$ and $V^{(R)}_{S0}$) in the symmetry (horizontal) direction and the anisotropic coefficients $\epsilon^{(R)}$, $\delta^{(R)}$ and $\gamma^{(R)}$. They are expressed through the components of the tensor $c^{(R)}_{ijk\ell}$ exactly in the same fashion as in VTI media (Appendix A). This set of Thomsen parameters makes it possible to apply the phase-velocity equations developed for VTI media (including the concise weak-anisotropy approximations) and express P-wave velocities and traveltimes as functions of just three coefficients—$V^{(R)}_{P0}$, $\epsilon^{(R)}$, and $\delta^{(R)}$ (plus the azimuth of the symmetry axis).

However, the analysis below shows that the generic Thomsen notation is not well-suited to describe normal-moveout velocities in HTI media that depend on near-vertical velocity variations. In the next section, I introduce another set of dimensionless anisotropic coefficients that is more convenient in characterizing moveout velocities and other reflection seismic signatures for horizontal transverse isotropy.

LIMITED EQUIVALENCE BETWEEN VERTICAL AND HORIZONTAL TRANSVERSE ISOTROPY

The results of moveout analysis for VTI models can be extended to the vertical plane that contains the symmetry axis in HTI media (the "symmetry-axis plane") by using an "equivalence" between vertical and horizontal transverse isotropy. By the "equivalent" VTI model, I will mean the VTI medium that can be used to describe velocities, traveltimes, and polarizations of body waves in the symmetry-axis plane of the original HTI model. To prove the equivalence between VTI media and the symmetry-axis plane of HTI media, it is sufficient to examine the Christoffel equation that determines the phase velocity V and polarization vector $\mathbf{U}$ of plane waves (e.g., Musgrave, 1970):

$$\left[G_{ik} - \rho V^2 \delta_{ik}\right] U_k = 0, \tag{6}$$

where ρ is the density, and G_{ik} is the Christoffel matrix given by

$$G_{ik} = c_{ijk\ell} n_j n_\ell, \tag{7}$$

$\mathbf{n}$ is the unit vector in the slowness direction. Let us find G_{ik} for wave propagation in the $[x_1, x_3]$ plane ($n_2 = 0$) of a transversely isotropic medium with the axis of symmetry pointing in the x_1 direction (Figure 2). Using equations (5) and (7), I find the nonzero components of the Christoffel matrix in the $[x_1, x_3]$ plane as

$$G_{11} = c_{11} n_1^2 + c_{55} n_3^2,$$

$$G_{33} = c_{55} n_1^2 + c_{33} n_3^2,$$

$$G_{13} = (c_{13} + c_{55}) n_1 n_3,$$

$$G_{22} = c_{66} n_1^2 + c_{44} n_3^2.$$

Although $c_{66} = c_{55}$, I will keep c_{66} in the expression for G_{22} to facilitate the comparison with VTI media.

The matrix $G_{ik} - \rho V^2 \delta_{ik}$ in the $[x_1, x_3]$ plane then becomes

$$G_{ik} - \rho V^2 \delta_{ik} = \begin{bmatrix} c_{11} n_1^2 + c_{55} n_3^2 - \rho V^2 & 0 & (c_{13} + c_{55}) n_1 n_3 \\ 0 & c_{66} n_1^2 + c_{44} n_3^2 - \rho V^2 & 0 \\ (c_{13} + c_{55}) n_1 n_3 & 0 & c_{55} n_1^2 + c_{33} n_3^2 - \rho V^2 \end{bmatrix}. \tag{8}$$

It is easy to verify that $G_{ik} - \rho V^2 \delta_{ik}$ from equation (8) and the Christoffel equation (6) as a whole are *identical* to the corresponding equations in the $[x_1, x_3]$ plane of VTI media. As for vertical transverse isotropy, equation (6) splits into two independent equations for the $SH(S^{\parallel})$ motion ($U_1 = U_3 = 0$) and $P - SV$ motion ($U_2 = 0$). I conclude that velocities and polarizations in the symmetry plane of TI media that contains the *horizontal* symmetry axis represent exactly the same functions of the stiffness coefficients and the slowness direction as for *vertical* transverse isotropy. For instance, we can use the well-known phase-velocity equations for $P - SV$-waves in VTI media expressed through the stiffness coefficients and the phase angle with vertical to describe phase velocities of $P - S^{\perp}$-waves in the symmetry-axis plane of HTI media. Note that the phase angle in the VTI equations applied to HTI media should be measured from *vertical* (as in VTI models), not from the actual symmetry axis of the HTI model.

Although the Christoffel equation has exactly the same form in VTI and HTI media, the relations between the stiffnesses c_{44}, c_{55}, and c_{66} do depend on whether the medium is VTI or HTI. If the symmetry axis is vertical, then $c_{44} = c_{55}$, while for a horizontal symmetry axis $c_{66} = c_{55}$. Thus, the coefficients c_{ij} in equation (8) do not specify the *same* equivalent VTI model (since $c_{44} \neq c_{55}$, the shear-wave vertical velocities are different). However, since $P - S^{\perp}$ and $S^{\parallel}$-waves in the symmetry-axis plane are decoupled, we can just use two different VTI models to describe $P - S^{\perp}$ and $S^{\parallel}$ propagation in the symmetry-axis plane. One VTI model, designed for $P - S^{\perp}$-waves, will be characterized by c_{11}, c_{33}, c_{55}, and c_{13}, while the other ("$S^{\parallel}$") model will include c_{44} and c_{66}.

Since phase velocity controls group (ray) velocity and group angle, all *kinematic* signatures and body-wave polarizations in the symmetry-axis plane of HTI media are given by the known VTI equations expressed through the stiffness constants c_{ij}. Seismic signatures for vertical transverse isotropy are particularly convenient to describe in Thomsen notation (Tsvankin, 1996), so it is natural to introduce the Thomsen parameters for this "equivalent" VTI medium. These parameters (I will denote them with the superscript V) are represented through the stiffness coefficients c_{ij} using the same equations as those used by Thomsen (1986) in VTI media:

$$\epsilon^{(V)} \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \tag{9}$$

$$\delta^{(V)} \equiv \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \tag{10}$$

$$\gamma^{(V)} \equiv \frac{c_{66} - c_{44}}{2c_{44}}. \tag{11}$$

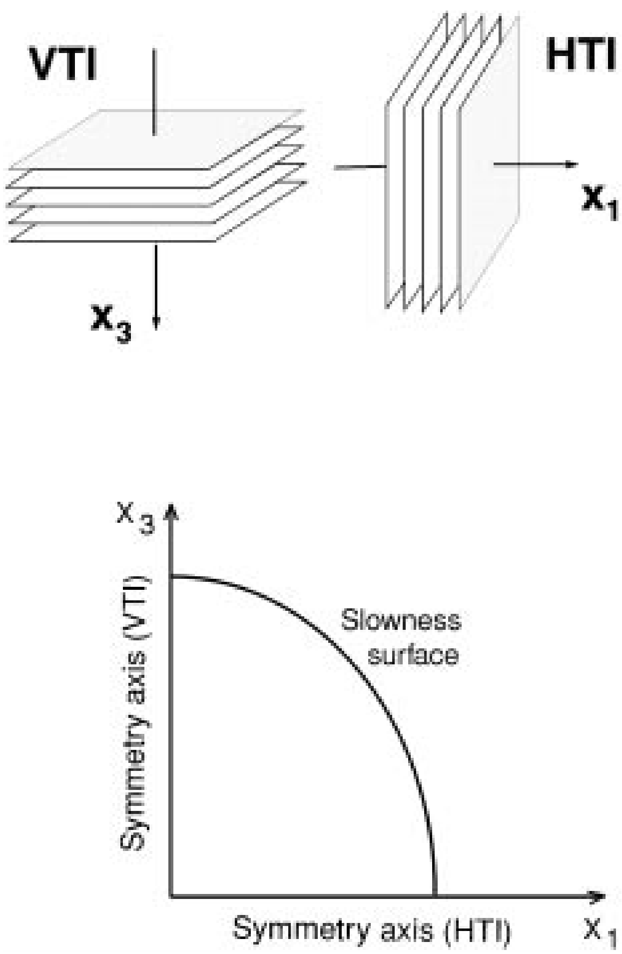

FIG. 2. Symmetry plane $[x_1, x_3]$ of a transversely isotropic medium with the symmetry axis pointing either in the x_3 (VTI) or in the x_1 (HTI) direction. The slowness surface in the symmetry-axis plane remains the same if the HTI medium is replaced with the equivalent VTI model.

To complete this Thomsen-style description of wave propagation in the symmetry-axis plane of HTI media, I need to specify two "isotropic" quantities—the vertical phase (and group) velocities of the P-wave and one of the S-waves:

$$V_{P\text{vert}} = \sqrt{\frac{c_{33}}{\rho}}, \tag{12}$$

$$V_{S^{\perp}\text{vert}} = \sqrt{\frac{c_{55}}{\rho}}. \tag{13}$$

The vertical velocity of the other (fast) shear wave is given by

$$V_{S^{\parallel}\text{vert}} = \sqrt{\frac{c_{44}}{\rho}} = \frac{V_{S^{\perp}\text{vert}}}{\sqrt{1 + 2\gamma^{(V)}}}. \tag{14}$$

To obtain kinematic seismic signatures (e.g., NMO velocities), polarizations, and plane-wave reflection coefficients in the symmetry-axis plane of HTI media, it is sufficient to substitute the Thomsen parameters defined above into the known VTI equations. $P - S^{\perp}$ velocities and polarizations are controlled by $V_{P\text{vert}}$, $V_{S^{\perp}\text{vert}}$, $\epsilon^{(V)}$, and $\delta^{(V)}$, while the $S^{\parallel}$-wave velocity depends on $V_{S^{\parallel}\text{vert}}$ and $\gamma^{(V)}$.

The relations between this set of Thomsen parameters and the generic Thomsen coefficients are given in Appendix A. Note that if the vertical and horizontal P-wave velocities are equal to each other [$\epsilon^{(V)} = 0$], the phase and group velocities of the P- and $S^{\perp}$-waves in the symmetry-axis plane are symmetric with respect to the 45° angle. As a result, in this case $P - S^{\perp}$ velocities do not change if I rotate the symmetry axis by 90°, and $\delta^{(V)}$ is equal to $\delta^{(R)}$ [$\epsilon^{(V)} = \epsilon^{(R)} = 0$]. The same holds for the $S^{\parallel}$-wave if $\gamma^{(R)} = 0$, but this is the trivial case of a medium with no $S^{\parallel}$-wave velocity anisotropy.

Although the direct analogy between HTI and VTI media is limited to the single symmetry plane that contains the symmetry axis, the anisotropic coefficients of the equivalent VTI medium turn out to be responsible for the azimuthal dependence of NMO velocity as well.

NORMAL MOVEOUT FROM A HORIZONTAL REFLECTOR

NMO velocity in symmetry planes

First, let us consider the influence of anisotropy on the normal-moveout velocity for survey lines in the planes of symmetry of an HTI layer (Figure 1). If the CMP line is perpendicular to the symmetry axis, the incident and reflected rays are confined to the isotropy plane, and the NMO velocities of each wave are just equal to the corresponding vertical velocities. Henceforth, therefore, I consider only the symmetry plane that contains the symmetry axis (the "symmetry-axis" plane).

The simplest way to obtain NMO velocities in the symmetry plane is to use the known NMO equations for vertical transverse isotropy (e.g.,Thomsen, 1986) and the analogy between VTI and HTI media described in the previous section. Alternatively, as shown in Appendix C, normal-moveout velocity can be found directly from the phase-velocity equations for horizontal transverse isotropy. Both approaches lead to the same expression for the NMO velocity of the P-wave, valid for any strength of the anisotropy:

$$V_{\text{nmo}}[P\text{-wave}] = V_{P\text{vert}}\sqrt{1+2\delta^{(V)}} = V_{P0}^{(R)}\sqrt{1-\frac{2\left[\epsilon^{(R)}-\delta^{(R)}\right]}{1+2\epsilon^{(R)}/f^{(R)}}}, \quad (15)$$

$$f^{(R)} \equiv 1-\left[V_{S0}^{(R)}\Big/V_{P0}^{(R)}\right]^2,$$

in accordance with the corresponding VTI equation (1) and the analogy between VTI and HTI media. Here, $\delta^{(V)}$ is expressed through the generic Thomsen coefficients in equation (A-9). As discussed below, the parameter $\delta^{(V)}$ for HTI media is typically negative, and the NMO velocity in the symmetry-axis plane usually is *smaller* than the vertical velocity. In contrast, for vertical transverse isotropy, δ may be both positive and negative, but most experimental data show that NMO velocity over VTI formations (such as shales) is *higher* than the rms vertical velocity (e.g., Banik, 1984), which indicates positive values of δ.

As discussed in the previous section, for $\epsilon^{(V)}=\epsilon^{(R)}=0$ the P-wave phase and group velocity in the plane containing the symmetry axis are symmetric with respect to the 45° angle, and the NMO velocities an terms of $\delta^{(R)}$ and $\delta^{(V)}$ become identical. Note that the symmetry-direction shear-wave velocity $V_{S0}^{(R)}$ contributes just to the term quadratic in the anisotropic coefficients $\epsilon^{(R)}$ and $\delta^{(R)}$ and, therefore, has only a small impact on the P-wave NMO velocity.

Clearly, the parameter $\delta^{(V)}$ is much more convenient in describing the P-wave NMO velocity than are the generic Thomsen coefficients defined with respect to the symmetry axis. Also note that not just the moveout velocities, but also non-hyperbolic moveout equations for vertical transverse isotropy (e.g., Tsvankin and Thomsen, 1994) remain entirely valid in the symmetry-axis plane of HTI media, if the parameters of the equivalent VTI medium are used.

For weak anisotropy ($|\delta^{(V)}| \ll 1$, $|\epsilon^{(R)}| \ll 1$, $|\delta^{(R)}| \ll 1$), I can simplify equation (15) by retaining only the terms linear in the anisotropic coefficients:

$$V_{\text{nmo}}[P\text{-wave}] \approx V_{P\text{vert}}\left[1+\delta^{(V)}\right] \approx V_{P0}^{(R)}\left[1+\delta^{(R)}-\epsilon^{(R)}\right]. \quad (16)$$

Equation (16) coincides with the expression given in Sena [1991, equation (A-10)], who calls the NMO velocity the "skewed" moveout velocity. Note that the corresponding equation (12a) in Thomsen (1988) is in error.

Similarly, the exact NMO velocity for the wave $S^\perp$ is given by equation (2) as

$$V_{\text{nmo}}[S^\perp\text{-wave}] = V_{S^\perp\text{vert}}\sqrt{1+2\sigma^{(V)}} = V_{S0}^{(R)}\sqrt{1+\frac{2\sigma^{(R)}}{1+2\epsilon^{(R)}/f^{(R)}}}, \quad (17)$$

with

$$\sigma^{(V)} = \left(\frac{V_{P\text{vert}}}{V_{S^\perp\text{vert}}}\right)^2\left[\epsilon^{(V)}-\delta^{(V)}\right]. \quad (18)$$

Since for the $S^\perp$-wave the velocities in the symmetry direction and in the perpendicular (isotropy) plane are identical, $V_{S^\perp\text{vert}} = V_{S0}^{(R)}$. As for P-waves, the $S^\perp$-wave NMO velocity in HTI media is given by the VTI equation in terms of the generic Thomsen parameters, if $\epsilon^{(V)}=\epsilon^{(R)}=0$. In the limit of weak anisotropy, equation (17) becomes

$$V_{\text{nmo}}[S^\perp\text{-wave}] \approx V_{S^\perp\text{vert}}\left[1+\sigma^{(V)}\right] \approx V_{S0}^{(R)}\left[1+\sigma^{(R)}\right]. \quad (19)$$

Since $V_{S^\perp\text{vert}} = V_{S0}^{(R)}$, for weak anisotropy the NMO velocity of the wave $S^\perp$ has the same form whether it is expressed through $\sigma^{(V)}$ or $\sigma^{(R)}$ (in the weak-anisotropy approximation, $\sigma^{(V)}=\sigma^{(R)}$). Equation (19) is equivalent to equation (12b) in Thomsen (1988) presented in a different form.

For the $S^\parallel$-wave, a 90° rotation of the symmetry axis amounts to interchanging the elliptical axes, and the NMO velocity remains equal to the horizontal shear-wave velocity as

$$V_{\text{nmo}}[S^\parallel\text{-wave}] = V_{S^\parallel\text{vert}}\sqrt{1+2\gamma^{(V)}} = V_{S0}^{(R)}. \quad (20)$$

Azimuthal dependence of NMO velocity

Normal-moveout velocity in symmetry planes of any anisotropic medium can be studied using the equation in Tsvankin (1995) derived under the assumption that the phase- and group-velocity vectors of the reflected waves lie in the incidence plane. For a survey line that is neither parallel nor perpendicular to the horizontal symmetry axis (Figure 3), the phase-velocity vectors may deviate from the incidence plane, thus making this equation inaccurate. A more general NMO expression for a horizontal HTI layer that fully honors the 3-D behavior of the phase- and group-velocity vectors is obtained in Appendix B:

$$V_{\text{nmo}}^2 = V_{\text{vert}}^2\frac{1+A}{1+A\sin^2\alpha}, \quad (21)$$

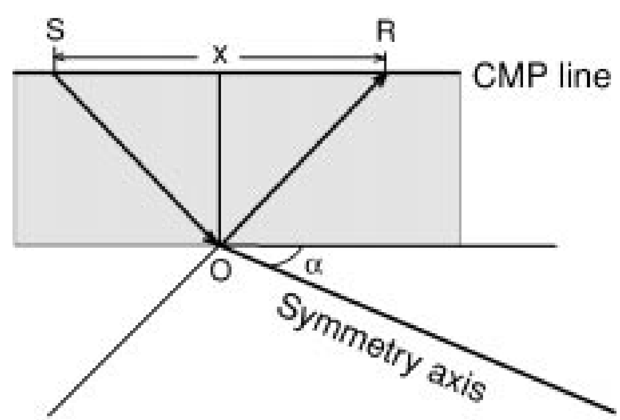

FIG. 3. Common-midpoint reflections over a transversely isotropic layer with a horizontal axis of symmetry. The symmetry axis makes the angle α with the survey (CMP) line. As shown in Appendix B, the incident and reflected rays of pure modes lie in the incidence (sagittal) plane, while the corresponding phase-velocity vectors may deviate from the plane.

where α is the angle between the common depth-point (CMP) line and the symmetry axis, and A is an anisotropic term given by

$$A = \frac{1}{V}\frac{d^2V}{d\theta^2}\bigg|_{\theta=90^\circ}. \tag{22}$$

Here, V is the phase velocity as a function of the phase angle θ with the symmetry axis; the phase velocity and its second derivative are evaluated at the vertical phase (and group) direction. Equation (21) is valid for HTI models with any strength of the anisotropy and can be used for all three pure modes (P, $S^{\perp}$, $S^{\parallel}$).

Two special cases considered in the previous section correspond to the survey line parallel ($\alpha = 0^\circ$) and perpendicular ($\alpha = 90^\circ$) to the symmetry axis. In the latter case ($\alpha = 90^\circ$), the incident and reflected rays lie in the isotropy plane, and the NMO velocity from equation (21) becomes equal to the vertical velocity,

$$V_{\text{nmo}}(\alpha = 90^\circ) = V_{\text{vert}}. \tag{23}$$

If the survey line is confined to the symmetry-axis plane (i.e., is parallel to the symmetry axis, $\alpha = 0$), equation (21) reduces to

$$V^2_{\text{nmo}}(\alpha = 0) = V^2_{\text{vert}}(1 + A) = V^2_{\text{vert}}\left(1 + \frac{1}{V}\frac{d^2V}{d\theta^2}\bigg|_{\theta=90^\circ}\right), \tag{24}$$

which is identical to the symmetry-plane NMO equation of Tsvankin (1995) for the special case of a horizontal reflector. Equation (24) has been discussed in detail for each wave type (P, $S^{\perp}$, $S^{\parallel}$) in the previous section.

The most interesting and somewhat surprising feature of equation (21) is that the influence of anisotropy on NMO velocity is absorbed just by the axis orientation and a single velocity term A that can be rewritten as

$$A = \frac{V^2_{\text{nmo}}(\alpha = 0)}{V^2_{\text{vert}}} - 1. \tag{25}$$

Using equations (25), (15), (17), and (20), I can identify A for different wave types as

$$A[P\text{-wave}] = 2\delta^{(V)}, \tag{26}$$

$$A[S^{\perp}\text{-wave}] = 2\sigma^{(V)}, \tag{27}$$

$$A[S^{\parallel}\text{-wave}] = 2\gamma^{(V)}. \tag{28}$$

Equations (26)–(28) demonstrate that the azimuthal dependence of NMO velocity for horizontal transverse isotropy is governed by the Thomsen parameters of the equivalent VTI medium. Therefore, the generic Thomsen coefficients are not needed even in the description of moveout velocity outside the symmetry planes.

In the weak-anisotropy approximation

$$\left|\frac{1}{V}\frac{d^2V}{d\theta^2}\bigg|_{\theta=90^\circ}\right| \ll 1,$$

equation (21) becomes simply

$$V^2_{\text{nmo}} = V^2_{\text{vert}}(1 + A\cos^2\alpha). \tag{29}$$

Equation (29) can be reduced to the expression for the "skewed" moveout velocity given in Sena (1991) in terms of the generic Thomsen parameters for each mode individually.

Thus, the exact NMO velocity from horizontal reflectors in HTI media is a relatively simple function of three parameters: the vertical velocity, the azimuthal angle between the survey line and the symmetry axis, and the anisotropic term A.

PARAMETER ESTIMATION USING NMO VELOCITY

The NMO equations given above can be used to invert the moveout velocities for the anisotropic parameters. One of the potential complications in this inversion is the distortion caused by nonhyperbolic moveout. Indeed, reflection moveout even in a homogeneous anisotropic medium is generally nonhyperbolic, and the moveout velocity on a finite-length spread may be different from the "zero-spread" NMO velocity (Hake et al., 1984). However, as shown by Tsvankin and Thomsen (1994) for horizontal reflectors beneath VTI media, anisotropy-induced deviations of P-wave moveout from hyperbolic for conventional spread lengths (close to the distance between the CMP and the reflector) usually are small. SV-wave moveout on these spreads is also close to hyperbolic for the most common, positive values of the difference $\epsilon - \delta$ (Tsvankin and Thomsen, 1994). [It should also be mentioned that the magnitude of P-wave nonhyperbolic moveout in VTI media usually decreases with reflector dip (Tsvankin, 1995).] These conclusions can be applied to moveout measurements in the vertical plane of HTI media that contains the symmetry axis by using the limited equivalence between VTI and HTI media. Note that the $S^{\parallel}$-wave moveout in this plane is purely hyperbolic for a single layer, both for horizontal and dipping reflectors (Uren et al., 1990), and deviates from a hyperbola only due to ray bending in stratified media. Obviously, reflection moveouts of all three waves are purely hyperbolic on the survey line in the isotropy plane of an HTI layer ($\alpha = 90^\circ$).

A synthetic example demonstrating the feasibility of recovering the P-wave NMO velocity *outside* the symmetry planes of a horizontal HTI layer is displayed in Figure 4. HTI models used in this test correspond to realistic cracked media (Thomsen, 1995) with the magnitude of nonhyperbolic moveout in the symmetry-axis plane ranging from moderate for model (a) to large for model (b). [The estimates of the nonhyperbolic moveout in the symmetry-axis plane are based on the analytic expressions given by Tsvankin and Thomsen (1994).] The moveout velocity was determined by least-squares fitting of the hyperbolic moveout equation to the exact reflection traveltimes calculated using a 3-D ray-tracing code. Despite some deviations caused by nonhyperbolic moveout, the moveout velocity on a conventional-length spread (equal to the reflector depth) is close to the analytic NMO velocity [equation (21)] for the full range of azimuthal angles. In fact, the maximum difference between the two velocities is observed in the symmetry-axis plane, where moveout is described by the known VTI equations. It should be emphasized that even if the moveout curve does deviate significantly from a hyperbola, the NMO velocity can still be recovered using nonhyperbolic (e.g., quartic) moveout correction.

Despite the fact that the NMO equations derived in the previous section are valid for a single HTI layer, they can be applied in a straightforward fashion to certain types of more realistic vertically inhomogeneous models. If the survey line lies in a vertical symmetry plane of any anisotropic medium, I can use the generalized Dix equation presented by Alkhalifah and Tsvankin (1995). In the case when all reflectors are horizontal, this equation reduces to the standard Dix (1995) formula that allows one to obtain the NMO velocity for a given layer from the NMO velocity for the reflections from the top and bottom of this layer. This implies that the Dix equation works in the two planes of symmetry of a stack of HTI layers with a uniform orientation of the symmetry axis. In a more general case, a model may be composed of vertically and horizontally transversely isotropic layers with the symmetry axis in HTI media pointing in either of two arbitrary but orthogonal directions. Then these directions will determine two thoroughgoing vertical symmetry planes of this anisotropic model in which the Dix equation is entirely valid. Thus, we can recover the interval (single-layer) NMO velocities in the symmetry planes of VTI–HTI horizontally layered media by the conventional Dix differentiation procedure. For a thorough discussion of reflection moveout in layered VTI media and numerical examples, see Tsvankin and Thomsen (1994); their results are fully applicable in the symmetry planes of horizontally layered VTI–HTI models.

Although the Dix equation does not work exactly outside the symmetry planes, it can still be expected to provide a good approximation in HTI models with weak and moderate azimuthal anisotropy, for which out-of-plane effects can be ignored (Sena, 1991). Performance of the Dix formula and the influence of nonhyperbolic moveout outside the vertical symmetry planes of stratified HTI media with pronounced anisotropy require a separate study that will be described in a sequel paper. Apart from problems caused by anisotropy, the Dix differentiation procedure in HTI media has the same major limitation as in isotropic media: the accuracy of the estimation of interval NMO velocities becomes inadequate for thin layers. For reservoirs with a relatively small thickness, the azimuthal variation of the reflection coefficient may be a more reliable diagnostic of azimuthal anisotropy (Rüger and Tsvankin, 1995).

In the following, it is assumed that the NMO velocity for a horizontal layer of interest [equation (21)] has been obtained using the reflections from the top and bottom of this layer. As mentioned above, in the case of a horizontal symmetry axis we can exploit the azimuthal dependence of reflection data by measuring NMO velocities on CMP lines with different orientation (e.g., using 3-D surveys). It should be mentioned that a small reflector dip does not pose a serious problem for this inversion because NMO velocity for mild dips (up to 5–10°) is close to the zero-dip value, even if the influence of anisotropy is pronounced (Tsvankin, 1995; Alkhalifah and Tsvankin, 1995).

In principle, we can expect to recover the three unknowns (the vertical velocity V_{vert}, the axis orientation, and A) from three NMO-velocity measurements at different azimuthal angles (i.e., on nonparallel lines) for any wave type. NMO velocity [equation (21)] can be represented as an ellipse in the horizontal plane:

$$V_{\text{nmo}}^2 = \frac{V_{\text{nmo1}}^2 V_{\text{nmo2}}^2}{V_{\text{nmo1}}^2 \sin^2\alpha + V_{\text{nmo2}}^2 \cos^2\alpha}, \tag{30}$$

where $V_{\text{nmo1}} = V_{\text{nmo}}(\alpha = 0^\circ)$ and $V_{\text{nmo2}} = V_{\text{nmo}}(\alpha = 90^\circ)$. Equation (30) shows that azimuthal moveout measurements make it possible to recover the NMO velocity in the symmetry planes and their orientation, but are not sufficient to distinguish between the isotropy plane and the symmetry-axis plane. To identify the symmetry direction and resolve all three parameters unambiguously, we have to take into account that NMO velocity is typically higher in the isotropy plane (parallel to the cracks).

Evidently, the parameter-estimation procedure becomes much more straightforward if the direction of the symmetry axis is known. For instance, for fractured reservoirs the crack orientation that determines the azimuth of the symmetry axis can sometimes be obtained from shear waves, geologic information, or borehole data (e.g., tiltmeter, breakouts). Then the NMO velocities in two azimuthal directions making different angles with the symmetry axis are sufficient to invert for the vertical velocity and the coefficient A. For instance, we may be able to determine the vertical velocities by performing moveout analysis on the survey line normal to the symmetry axis [equation (23)]. Then the NMO velocities on a line with any

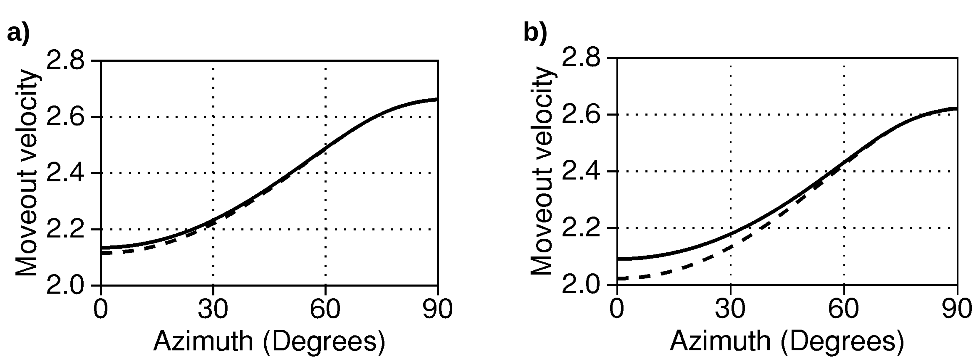

FIG. 4. The influence of nonhyperbolic moveout on the estimation of P-wave normal-moveout velocity. The solid curve represents the moveout velocity determined by fitting a straight line to the exact $t^2 - x^2$ curves on the spread length equal to the reflector depth (1.5 km); the traveltimes were generated using a 3-D ray-tracing code. The dashed curve is the normal-moveout (zero-spread) velocity from equation (21). The model parameters are (a) $V_{P\text{vert}} = 2.66$ km/s, $\epsilon^{(V)} = -0.14$, $\delta^{(V)} = -0.18$ [$V_{P0}^{(R)} = 2.25$ km/s, $\epsilon^{(R)} = 0.2$, $\delta^{(R)} = 0.1$]. (b) $V_{P\text{vert}} = 2.62$ km/s, $\epsilon^{(V)} = -0.045$, $\delta^{(V)} = -0.20$ [$V_{P0}^{(R)} = 2.5$ km/s, $\epsilon^{(R)} = 0.05$, $\delta^{(R)} = -0.15$].

other orientation make it possible to find the anisotropic parameter A. Finally, if both the axis orientation and the vertical velocity are known, a single value of NMO velocity (not too close to the isotropy plane) can be inverted for the parameter A. Once the moveout inversion has been carried out for one of the modes (e.g., for the P-wave), a single NMO velocity for any other mode (say, $S^{\perp}$) is sufficient to find the anisotropic parameter A for this wave since the vertical velocity ($V_{S^{\perp}\text{vert}}$) in this case can be found from $V_{P\text{vert}}$ and the vertical P and $S^{\perp}$ traveltimes.

Although A is determined by the parameters of the equivalent VTI model [$\delta^{(V)}, \sigma^{(V)}$, or $\gamma^{(V)}$, depending on the wave type], such an algorithm for moveout inversion cannot be devised for vertical transverse isotropy; obviously, in VTI media, moveout velocities do not vary with azimuth. As mentioned above, NMO velocities in VTI media are not sufficient to invert for the vertical velocities and anisotropic coefficients, even if both P and shear data are used.

Estimation of the shear-wave splitting parameter

It is believed that the most common physical reason for horizontal transverse isotropy is the presence of parallel vertical cracks (fractures) embedded in an isotropic medium (Gupta, 1973; Crampin, 1985). Thus, the question to be answered next is: what kind of information about the properties of crack systems can be obtained from moveout data?

An important parameter in the characterization of fractured reservoirs is the crack density (ζ) that is proportional to the product of the number of cracks per unit volume and their mean cubed diameter. Although all three generic anisotropic coefficients [$\epsilon^{(R)}, \delta^{(R)}, \gamma^{(R)}$] are dependent on ζ, the parameter most directly related to the crack density is $\gamma^{(R)}$, which governs the degree of shear-wave splitting at vertical incidence. For parallel, circular ellipsoidal (penny-shaped) cracks distributed in a porous isotropic rock, $\gamma^{(R)}$ is given by (Thomsen, 1995)

$$\gamma^{(R)} = \frac{8}{3}\frac{1-P}{2-P}\zeta, \tag{31}$$

where P is the Poisson's ratio of the dry isotropic porous medium.

It is easy to see that for plausible values of the Poisson's ratio the coefficient $8(1-P)/[3(2-P)]$ is close to unity, and $\gamma^{(R)} \approx \zeta$. Therefore, for penny-shaped cracks, measurements of $\gamma^{(R)}$ provide a good direct estimate of the crack density.

The parameter $\gamma^{(R)}$ in HTI media is usually obtained directly from the fractional difference between the vertical $S^{\perp}$ and $S^{\parallel}$ velocities using the traveltimes of split shear waves at vertical incidence (e.g., Crampin, 1985; Thomsen, 1988):

$$\gamma^{(R)} = \frac{1}{2}\left(\frac{V^2_{S^{\parallel}\text{vert}}}{V^2_{S^{\perp}\text{vert}}} - 1\right). \tag{32}$$

The above results suggest an alternative way of recovering $\gamma^{(R)}$ using just $S^{\parallel}$ moveout data. The azimuthal dependence of the $S^{\parallel}$-wave NMO velocities can be inverted for $\gamma^{(V)} = A/2$ and, consequently, for $\gamma^{(R)}$ [equations (21), (28), and [(A-10)]. If the symmetry direction is known, this inversion requires the $S^{\parallel}$-wave NMO velocities on two lines with different orientation. In the simplest case of the lines parallel and perpendicular to the symmetry axis,

$$\gamma^{(R)} = \frac{1}{2}\left[\frac{V^2_{\text{nmo}}(\alpha = 90^\circ)}{V^2_{\text{nmo}}(\alpha = 0)} - 1\right]. \tag{33}$$

Note that if both $S^{\parallel}$ and $S^{\perp}$ data were available, the ratio of the vertical shear-wave velocities could be obtained not only from the vertical traveltimes, but also from the respective NMO velocities (e.g., measured in the isotropy plane). Then $\gamma^{(R)}$ could be calculated from equation (32), which would provide useful redundancy in the evaluation of this parameter.

Next, I discuss the possibility of obtaining the shear-wave splitting parameter from the moveout velocities of the P and $S^{\perp}$-waves. As shown by Thomsen (1995), the relations between $\epsilon^{(R)}, \delta^{(R)}$ (and, consequently, $\epsilon^{(V)}$ and $\delta^{(V)}$) and the crack density are complicated by such quantities as the incompressibility of the solid grains and the fluid in the cracks, as well as by the so-called "fluid influence factor." This makes the inversion of $\epsilon^{(V)}$ and $\delta^{(V)}$ for the crack density an ambiguous procedure, unless detailed information about the physical properties of the rock is available. Since the moveouts of the P and $S^{\perp}$-waves are controlled by the vertical P and $S^{\perp}$ velocities, $\delta^{(V)}$, and $\epsilon^{(V)}$, it seems that there is no straightforward way to obtain an estimate of the crack density from the NMO velocities of the P- and $S^{\perp}$-waves.

However, the main difference between general transverse isotropy and TI media due to a system of thin parallel cracks is that in the latter case the elastic constants satisfy the following constraint that reduces the number of independent stiffness coefficients from five to four (Schoenberg and Sayers, 1995; Thomsen, 1995):

$$c_{11}c_{33} - c_{13}^2 = 2c_{44}(c_{11} + c_{13}). \tag{34}$$

Equation (34) is written for the cracks perpendicular to the x_1-axis. Other constraints for such a medium require that $\epsilon^{(R)} \geq 0$ [i.e., $\epsilon^{(V)} \leq 0$] and $\gamma^{(R)} \geq 0$ [i.e., $\gamma^{(V)} \leq 0$].

Replacing the stiffness coefficients with $\gamma^{(R)}$ [equations (11) and (A-10)] and the parameters $\epsilon^{(V)}$ and $\delta^{(V)}$ of the equivalent VTI medium [equations (9) and (10)], we find (assuming $c_{13} + c_{55} > 0$),

$$\zeta \approx \gamma^{(R)} = \frac{V^2_{P\text{vert}}}{2V^2_{S^{\perp}\text{vert}}} \frac{\epsilon^{(V)}\left[2 - 1/f^{(V)}\right] - \delta^{(V)}}{1 + 2\epsilon^{(V)}/f^{(V)} + \sqrt{1 + 2\delta^{(V)}/f^{(V)}}}, \tag{35}$$

with $f^{(V)} \equiv 1 - V^2_{S^{\perp}\text{vert}}/V^2_{P\text{vert}}$.

Equation (35) expresses $\gamma^{(R)}$ through the parameters that can be determined from P and $S^{\perp}$ moveout data. Although V_{vert} and A are the only two medium parameters (except for the axis orientation) that can be obtained for any single wave type, we can combine P and $S^{\perp}$-waves to resolve the required anisotropic coefficients individually. Indeed, the P-wave NMO velocity from horizontal reflectors can provide the parameters $V_{P\text{vert}}$ and $\delta^{(V)}$. By inverting $S^{\perp}$-wave NMO data, we can obtain the vertical $S^{\perp}$-wave velocity $V_{S^{\perp}\text{vert}}$ and the anisotropic term $A = 2\sigma^{(V)}$ [equation (18)]. In fact, it is sufficient to find one of the vertical velocities, since the second velocity can then be recovered from the ratio of the vertical P and $S^{\perp}$ traveltimes.Using the coefficients $V_{P\text{vert}}, \delta^{(V)}, V_{S^{\perp}\text{vert}}$, and $\sigma^{(V)}$, we can find the parameter $\epsilon^{(V)}$ from equation (18). Once

$V_{P\text{vert}}/V_{S^\perp\text{vert}}$, $\delta^{(V)}$, and $\epsilon^{(V)}$ have been recovered, the parameter $\gamma^{(R)}$ can be calculated directly from equation (35).

In general, P-wave NMO velocities from horizontal reflectors do not provide enough information to determine $\gamma^{(R)}$. However, P-wave moveout data, combined with an approximate value of the ratio of the vertical velocities ($V_{P\text{vert}}/V_{S^\perp\text{vert}}$), are sufficient to estimate the shear-wave splitting parameter if the P-wave velocity along the symmetry direction ($V_{P0}^{(R)}$) has been obtained (e.g., from head-wave traveltimes, crosshole tomography, or nonhyperbolic moveout). Then $\epsilon^{(V)}$ is given by

$$\epsilon^{(V)} = \frac{1}{2}\left[\left(\frac{V_{P0}^{(R)}}{V_{P\text{vert}}}\right)^2 - 1\right],$$

and all parameters in the right-hand side of equation (35) are known. Below, I suggest an alternative way of estimating $\epsilon^{(V)}$ using the dip-dependence of P-wave NMO velocity.

Of course, the value of $V_{P\text{vert}}/V_{S^\perp\text{vert}}$ cannot be found from P-wave moveout alone, but it is usually possible to obtain a rough estimate of this ratio from other information, such as well logs. [If, instead, the ratio $V_{P\text{vert}}/V_{S\|\text{vert}}$ is known, equation (35) for $\gamma^{(R)}$ can be modified in an obvious way using equation (32).] Since errors in $V_{P\text{vert}}/V_{S^\perp\text{vert}}$ directly propagate into $\gamma^{(R)}$ [as determined by equation (35)], evaluation of the shear-wave splitting parameter using P-wave data may not be quantitatively accurate. Nonetheless, the magnitude of the splitting parameter is relatively small, and even a substantial percentage error in $\gamma^{(R)}$ may not prevent this algorithm from producing reliable qualitative estimates. For instance, suppose $\gamma^{(R)}$ goes up from a value of 0.05 in one location to 0.15 in another. Even an error of, say, 30% in the squared velocity ratio would not prevent detection of this anomaly using P-wave moveout parameters and equation (35). Thus, the primary application of the method based on just P-wave data is to identify areas of high values of $\gamma^{(R)}$ that correspond to intensely fractured zones.

The inversion of P-wave data for the shear-wave splitting parameter becomes particularly simple in the special case of the equal vertical and horizontal (along the symmetry direction) velocities of the P-wave ($\epsilon^{(V)} = 0$), which corresponds to negligible equant porosity and "very thin" fluid-filled cracks (for quantitative estimates, see Thomsen, 1995). Such a model may be typical, for instance, for fractured coals, of primary importance in methane production. If $\epsilon^{(V)} = 0$, equation (35) reduces to

$$\gamma^{(R)} = \frac{V_{P\text{vert}}^2}{2V_{S^\perp\text{vert}}^2}\,\frac{-\delta^{(V)}}{1+\sqrt{1+2\delta^{(V)}/f^{(V)}}}. \tag{36}$$

Since the parameter $\delta^{(V)}$ can be found from P-wave NMO velocity, P-wave data in this case are sufficient to obtain the shear-wave splitting parameter given an approximate value of the $V_{P\text{vert}}/V_{S^\perp\text{vert}}$ ratio. Then the combination of P and $S^\perp$ data may be necessary only to get the exact ratio $V_{P\text{vert}}/V_{S^\perp\text{vert}}$.

In the limit of weak anisotropy [$|\epsilon^{(V)}| \ll 1$, $|\delta^{(V)}| \ll 1$], equation (35) becomes

$$\gamma^{(R)} = \frac{V_{P\text{vert}}^2}{4V_{S^\perp\text{vert}}^2}\left[\epsilon^{(V)}\left(2-\frac{1}{f^{(V)}}\right) - \delta^{(V)}\right]. \tag{37}$$

If $V_{P\text{vert}}/V_{S^\perp\text{vert}} = 2$, equation (37) further simplifies to

$$\gamma^{(R)} = 0.67\epsilon^{(V)} - \delta^{(V)}. \tag{38}$$

Equation (38) suggests that for weakly anisotropic HTI models the parameter $\gamma^{(R)}$ is close to the difference between $\epsilon^{(V)}$ and $\delta^{(V)}$. As demonstrated below, this difference can be evaluated using the dip-dependence of the P-wave NMO velocity. Also, $\gamma^{(R)}$ can be roughly estimated using just the $S^\perp$-wave NMO velocity, which provides the parameter $\sigma^{(V)} = V_{P\text{vert}}^2/V_{S^\perp\text{vert}}^2[\epsilon^{(V)} - \delta^{(V)}]$.

In terms of the generic Thomsen coefficients, the weak-anisotropy approximation for $\gamma^{(R)}$ can be rewritten as (using the results of Appendix A)

$$\gamma^{(R)} = \left[\frac{V_{P0}^{(R)}}{2V_{S0}^{(R)}}\right]^2\left[\frac{\epsilon^{(R)}}{1-\left[V_{S0}^{(R)}/V_{P0}^{(R)}\right]^2} - \delta^{(R)}\right], \tag{39}$$

in agreement with Thomsen (1995). Since $\gamma^{(R)}$ is nonnegative, it is clear from equation (39) that the difference between $\epsilon^{(R)}$ and $\delta^{(R)}$ for TI media caused by parallel cracks is typically positive.

Once the crack density has been estimated from the parameter $\gamma^{(R)}$ (for ellipsoidal cracks), the coefficients $\epsilon^{(V)}$ and $\delta^{(V)}$ can be used to obtain more information about the properties of the crack system (e.g., about the fluid filling the cracks).

Determination of phase velocity

The phase-velocity function governs group velocities, traveltimes, and other kinematic properties of body waves. It is also needed in depth imaging, such as prestack depth migration for HTI media. Hence, a question that arises is what information about phase velocity can be obtained from the vertical velocities and parameters A determined using moveout data for different modes.

The phase velocity of P- and $S^\perp$-waves in the symmetry-axis plane (that contains the symmetry axis) can be found using equation (C-2) for vertical transverse isotropy (Tsvankin, 1996):

$$\frac{V^2(\bar\theta)}{V_{P\text{vert}}^2} = 1 + \epsilon^{(V)}\sin^2\bar\theta - \frac{f^{(V)}}{2} \pm \frac{f^{(V)}}{2} \times\sqrt{\left[1+\frac{2\epsilon^{(V)}\sin^2\bar\theta}{f^{(V)}}\right]^2 - \frac{8\left[\epsilon^{(V)}-\delta^{(V)}\right]\sin^2\bar\theta\cos^2\bar\theta}{f^{(V)}}}, \tag{40}$$

where the plus sign corresponds to the P-wave, and the minus to the $S^\perp$-wave; $\bar\theta$ is the phase angle with respect to vertical (symmetry direction of the equivalent VTI medium). Replacing $\bar\theta$ with the phase angle θ with respect to the symmetry axis ($\bar\theta = 90^\circ - \theta$) yields

$$\frac{V^2(\theta)}{V_{P\text{vert}}^2} = 1 + \epsilon^{(V)}\cos^2\theta - \frac{f^{(V)}}{2} \pm \frac{f^{(V)}}{2} \times\sqrt{\left[1+\frac{2\epsilon^{(V)}\cos^2\theta}{f^{(V)}}\right]^2 - \frac{8\left[\epsilon^{(V)}-\delta^{(V)}\right]\sin^2\theta\cos^2\theta}{f^{(V)}}}. \tag{41}$$

Although equation (41) has been derived just for the symmetry-axis plane, it expresses phase velocity in HTI media through the phase angle θ with the symmetry axis for any value of θ. Since phase velocity in TI media depends only on the phase angle with the symmetry axis, equation (41) is valid for any phase direction, not necessarily confined to the symmetry-axis plane.

Equation (41) makes it possible to calculate phase velocity directly from the parameters responsible for the reflection moveout of the P- and $S^{\perp}$-waves in HTI media. As shown above, the orientation of the symmetry axis can be determined from the azimuthal dependence of NMO velocities. All other unknown quantities needed to obtain phase velocity using equation (41) can be found by combining P and $S^{\perp}$ NMO velocities from horizontal reflectors (see the analysis in the previous section).

However, the question of most practical importance is whether P-wave NMO velocity alone contains enough information to build the P-wave phase-velocity function. Note that, in accordance with results for VTI media (Tsvankin and Thomsen, 1994), the shear-wave vertical velocity hidden in the coefficient $f^{(V)}$ has only a small influence on the P-wave phase velocity. Thus, the three parameters to be determined are the P-wave vertical velocity $V_{P\text{vert}}$ and the anisotropic coefficients $\epsilon^{(V)}$ and $\delta^{(V)}$. In general, just two of them ($V_{P\text{vert}}$ and $\delta^{(V)}$) can be recovered from P-wave NMO velocities from horizontal reflectors. Nonetheless, the symmetry–direction velocity and, consequently, the parameter $\epsilon^{(V)}$ may be known from additional data or, for some formations, $\epsilon^{(V)}$ is equal to zero (see the discussion in the previous section). As shown below, the presence of dipping events makes it possible to obtain $\epsilon^{(V)}$ from the dip dependence of P-wave NMO velocities.

In the weak-anisotropy approximation, P-wave phase velocity from equation (41) reduces to a simple expression

$$V(\theta) = V_{P\text{vert}}\big[1 + \delta^{(V)} \sin^2\theta \cos^2\theta + \epsilon^{(V)} \cos^4\theta\big].$$

For the wave $S^{\parallel}$ the anisotropy is elliptical, and phase velocity is given by

$$V(\theta)[S^{\parallel}\text{-wave}] = V_{S^{\parallel}\text{vert}}\sqrt{1 + 2\gamma^{(V)}\cos^2\theta}.$$

Both the vertical velocity $V_{S^{\parallel}\text{vert}}$ and the coefficient $\gamma^{(V)}$ can be found from the azimuthally dependent NMO velocity of the $S^{\parallel}$-wave [equations (21) and (28)].

NMO VELOCITY FROM DIPPING REFLECTORS

Normal-moveout velocity from both horizontal and dipping reflectors in symmetry planes of any homogeneous anisotropic medium, including horizontal transverse isotropy, can be studied using the following equation given in Tsvankin (1995):

$$V_{\text{nmo}}(\phi) = \frac{V(\phi)}{\cos\phi} \frac{\sqrt{1 + \frac{1}{V(\phi)} \frac{d^2V}{d\bar{\theta}^2}\Big|_{\bar{\theta}=\phi}}}{1 - \frac{\tan\phi}{V(\phi)} \frac{dV}{d\bar{\theta}}\Big|_{\bar{\theta}=\phi}}, \qquad (42)$$

where V is phase velocity, $\bar{\theta}$ is the phase angle with vertical, and ϕ is the dip angle of the reflector. Equation (42) is strictly valid for 2-D wave propagation, with phase and group velocities of the reflected waves confined to the incidence plane. This implies that the incidence plane should represent both a symmetry plane of the medium and the dip plane of the reflector.

Equation (42) is applied below to the symmetry-axis plane of HTI media. To comply with the assumptions behind equation (42), the incidence plane is taken to be the dip plane of the reflector, i.e., the strike of the reflector is perpendicular to the symmetry axis. The dip dependence of NMO velocity in the isotropy plane is trivial because velocity is independent of propagation angle. For any other survey (CMP) line making an arbitrary angle with the symmetry axis, equation (42) can be used only under the assumption of weak azimuthal anisotropy.

For homogeneous, isotropic media, equation (42) reduces to the simple cosine-of-dip relationship (Levin, 1971),

$$V_{\text{nmo}}(\phi) = \frac{V_{\text{nmo}}(0)}{\cos(\phi)}. \qquad (43)$$

Velocity variations with the phase angle $\bar{\theta}$ [represented by the derivatives in equation (42)] lead to deviations from the cosine-of-dip formula (43). Therefore, NMO velocities from dipping reflectors can provide useful information about anisotropy (Alkhalifah and Tsvankin, 1995). Description of the dip-dependent NMO velocity is also important in developing dip-moveout (DMO) algorithms, as well as other seismic processing methods for anisotropic media (Anderson et al., 1996). These observations, discussed originally for VTI media, pertain to the symmetry-axis plane of HTI media as well.

NMO velocity as a function of dip

Although wave propagation in the symmetry-axis plane of HTI media can be described using VTI equations, the parameters of the equivalent VTI medium are different from those conventionally used for real vertical transverse isotropy. For instance, since in HTI media the vertical P-wave velocity is *higher* than the velocity in the symmetry (horizontal) direction, the parameter $\epsilon^{(V)}$ of the equivalent VTI medium is *negative*, an extremely unusual case for actual media with vertical transverse isotropy. Also, the parameter $\delta^{(V)} \approx \delta^{(R)} - 2\epsilon^{(R)}$ [equation (A-9)] is typically negative [since $\epsilon^{(R)}$ and $\epsilon^{(R)} - \delta^{(R)}$ are usually positive] and its magnitude may be larger than that for Thomsen's δ in VTI media.

To understand the influence of the anisotropic parameters on NMO velocity, it is convenient to apply the weak-anisotropy approximation ($|\epsilon^{(V)}| \ll 1, |\delta^{(V)}| \ll 1, |\gamma^{(V)}| \ll 1$) to equation (42). Using the equivalence with VTI media, the P-wave NMO velocity in the symmetry-axis plane of HTI media can be represented (Tsvankin, 1995) as

$$\frac{V_{\text{nmo}}(\phi)\cos\phi}{V_{\text{nmo}}(0)} = 1 + \delta^{(V)}\sin^2\phi + 3\big[\epsilon^{(V)} - \delta^{(V)}\big]\sin^2\phi(2 - \sin^2\phi). \qquad (44)$$

To separate the influence of the anisotropy, the NMO velocity in equation (44) has been normalized by the isotropic cosine-of-dip relationship. Since for weak anisotropy, as follows from equations (A-8) and (A-9),

$$\epsilon^{(V)} - \delta^{(V)} = \epsilon^{(R)} - \delta^{(R)},$$

the main (last) term in equation (44) is typically positive, and the NMO velocity increases with dip faster than in isotropic media. However, the δ-term, which is usually small in VTI media, is relatively large and negative in the HTI case. As a result, for typical positive values of $\epsilon^{(V)} - \delta^{(V)}$ the cosine-of-dip corrected NMO velocity increases with dip much slower than in typical VTI media with the same value of the difference between ϵ and δ, and the accuracy of the isotropic cosine-of-dip relationship tends to be higher in HTI than in VTI media. Also, in contrast with the results for vertical transverse isotropy described in Tsvankin (1995), the dependence of P-wave NMO velocity on dip is not tightly controlled by the difference between $\epsilon^{(R)}$ and $\delta^{(R)}$ or $\epsilon^{(V)}$ and $\delta^{(V)}$ (Figure 5).

The dip-dependence of NMO velocity for the waves $S^{\perp}$ and $S^{\parallel}$ can be treated in the same fashion, using the results obtained by Tsvankin (1995) for vertical transverse isotropy.

Parameter η and time processing in HTI media

For purposes of seismic processing, it is more convenient to treat NMO velocity as a function of the ray parameter corresponding to zero-offset reflection. Then, as shown in Alkhalifah and Tsvankin (1995), the P-wave NMO velocity in VTI media is governed by just two parameters: the zero-dip NMO velocity $V_{\text{nmo}}(0)$ [equation (1)] and the anisotropic coefficient η:

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}. \tag{45}$$

Both parameters $V_{\text{nmo}}(0)$ and η can be reliably recovered from P-wave surface data using NMO velocities and ray parameters for two distinctly different dips. In VTI media, the parameters $V_{\text{nmo}}(0)$ and η are sufficient to perform all time-related processing steps including NMO correction, dip-moveout removal, and prestack and poststack time migration (Alkhalifah and Tsvankin, 1995). (Note that the contribution of the shear-wave vertical velocity to P-wave traveltimes is small.)

In essence, η is responsible for the influence of vertical transverse isotropy on P-wave NMO velocity and time-related processing in general. For elliptical anisotropy, $\eta = 0$, and NMO equation (42) reduces to the well-known expression valid for isotropic media:

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}}, \tag{46}$$

where p is the ray parameter. The contribution of anisotropy in equation (46) is hidden in the values of the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and the ray parameter p. All isotropic time-processing methods remain valid for elliptical models, irrespective of the strength of velocity anisotropy.

Despite the uncommon values of the parameters of the equivalent VTI model, time-related processing of P-wave data in the symmetry-axis plane of HTI media can be carried out using the algorithms developed for vertical transverse isotropy. Indeed, the P-wave NMO velocities for HTI models with the same value of $\eta^{(V)}$ practically coincide with each other (Figure 6a). This proves that the P-wave NMO velocity as a function of the ray parameter is entirely controlled by the zero-dip value $V_{\text{nmo}}(0)$ and the parameter $\eta^{(V)}$, whether the medium has a vertical or horizontal symmetry axis. Also, as demonstrated by Figure 6b, the resolution in $\eta^{(V)}$ is high enough for stable recovery of this parameter from the dip-dependence of P-wave NMO velocity.

It is interesting that the parameter $\eta^{(V)}$ turns out to be close to the coefficient $\eta^{(R)}$ defined through the generic Thomsen parameters. Indeed, using equations (A-8) and (A-9), I find

$$\eta^{(V)} = \frac{\epsilon^{(R)} - \delta^{(R)}}{1 + 2\delta^{(R)}} \left[1 - \frac{2\epsilon^{(R)}\left(1 - 1/f^{(R)}\right)}{1 + 2\delta^{(R)}} \right]^{-1}$$

$$= \eta^{(R)} \left[1 - \frac{2\epsilon^{(R)}\left(1 - 1/f^{(R)}\right)}{1 + 2\delta^{(R)}} \right]^{-1}. \tag{47}$$

Thus, if we apply VTI inversion algorithms to the dip-dependence of P-wave NMO velocity in the symmetry-axis plane of HTI media, we get the value of $\eta^{(V)}$. Alternatively, in principle the parameter $\eta^{(V)}$ can also be obtained from the P-wave velocity in the symmetry direction (which may be known from head waves, crosshole tomography, etc.) and the zero-dip NMO velocity in the symmetry-axis plane without using dipping events (Alkhalifah and Tsvankin, 1995).

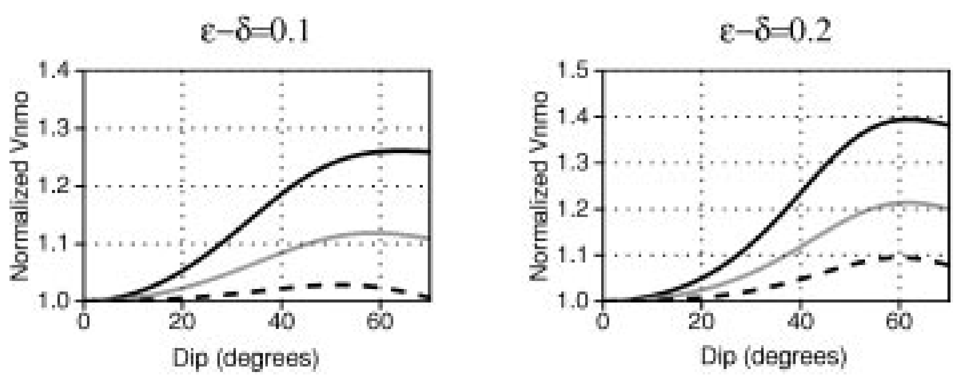

FIG. 5. The dip-dependence of P-wave normal-moveout velocity in the symmetry plane of HTI media that contains the symmetry axis. NMO velocity is calculated from equation (42) and divided by the isotropic equation (43). The curves on the left plot correspond to models with $\epsilon^{(R)} - \delta^{(R)} = 0.1$: $\epsilon^{(R)} = 0, \delta^{(R)} = -0.1$ (solid black); $\epsilon^{(R)} = 0.1, \delta^{(R)} = 0$ (gray); $\epsilon^{(R)} = 0.2, \delta^{(R)} = 0.1$ (dashed). On the right plot, $\epsilon^{(R)} - \delta^{(R)} = 0.2$: $\epsilon^{(R)} = 0.1, \delta^{(R)} = -0.1$ (solid black); $\epsilon^{(R)} = 0.2, \delta^{(R)} = 0$ (gray); $\epsilon^{(R)} = 0.3, \delta^{(R)} = 0.1$ (dashed).

The value of $\eta^{(V)}$ adds new information to the inversion procedure described in the previous section and makes it possible, in the following way, to estimate the shear-wave splitting parameter $\gamma^{(R)}$ just from P-wave moveout data. As discussed above, the parameter $\delta^{(V)}$ can be found using the P-wave NMO velocity from horizontal reflectors. The presence of dipping events makes it possible to recover $\epsilon^{(V)}$ from the parameter $\eta^{(V)}$. Then, given an estimate of the ratio of the P-to-$S^{\perp}$ vertical velocities, $\gamma^{(R)}$ can be calculated from equation (35).

Even if no other data are available, just the value of $\eta^{(V)}$ can be used to constrain the shear-wave splitting parameter. First, in the special case of $\epsilon^{(V)} = 0$, the parameter $\eta^{(V)}$ is sufficient to obtain $\delta^{(V)}$ and compute $\gamma^{(R)}$ from equation (36) (provided an approximate ratio of the vertical P and $S^{\perp}$ velocities is available). In the more general case of nonzero $\epsilon^{(V)}$, we can use the weak-anisotropy approximation (38) and make a crude estimate of $\gamma^{(R)}$ by further assuming $\gamma^{(R)} \approx \epsilon^{(V)} - \delta^{(V)} \approx \eta^{(V)}$. This approximation works better for substantially different $\epsilon^{(V)}$ and $\delta^{(V)}$ than for media close to elliptically anisotropic with close values of $\epsilon^{(V)}$ and $\delta^{(V)}$.

DISCUSSION AND CONCLUSIONS

Horizontal transverse isotropy is usually associated with parallel vertical penny-shaped cracks (fractures) embedded in an otherwise isotropic matrix. This work provides an analytic basis for estimating the anisotropic parameters of HTI media from normal-moveout information. The methodology is based on a new exact equation for NMO velocities from horizontal reflectors that is valid for arbitrary direction of the survey line with respect to the axis of symmetry. Normal-moveout velocity for any pure mode is controlled by the vertical velocity, the angle between the symmetry axis and the survey line, and a single effective anisotropic parameter.

Therefore, information about the true vertical velocity (hence the reflector depth) and the principal directions of the anisotropy can be obtained from the azimuthal dependence of P-wave NMO velocity without using converted and shear modes. On the other hand, if shear data are available, the symmetry direction can be determined from S-wave polarizations, which simplifies the inversion of P-wave data for the vertical velocity and anisotropy. In general, it is highly beneficial to combine different types of data, such as moveout velocities, the azimuthal dependence of AVO response (Rüger and Tsvankin, 1995), and polarizations of shear waves.

Analysis of the Christoffel equation for HTI media shows that velocities and polarizations in the symmetry-axis plane can be found using the known equations for vertical transverse isotropy. Hence, not only NMO velocities but also nonhyperbolic moveout in the symmetry-axis plane of HTI media can be studied using the formalism developed for VTI media (e.g., Tsvankin and Thomsen, 1994). Note, however, that point-source radiation patterns (and body-wave amplitudes in general) in the symmetry-axis plane depend on the azimuthal velocity variations (Tsvankin and Chesnokov, 1990) and, therefore, would be different in the actual HTI medium and the equivalent VTI model. Radiation patterns in HTI media should be calculated using the phase and group angles with the symmetry axis.

The analogy between HTI and VTI media makes it possible to introduce the Thomsen coefficients of the "equivalent" VTI medium, which are more convenient in describing reflection seismic signatures in HTI media than are the stiffness coefficients or the generic Thomsen parameters defined with respect to the symmetry axis. For instance, the anisotropic coefficient that controls the azimuthally dependent P-wave NMO velocity is equal to the parameter δ of the equivalent VTI medium.

The limited equivalence with vertical transverse isotropy also implies that time-related 2-D processing (NMO, DMO, time migration) of P-waves in the symmetry-axis plane of HTI media is governed by the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and the parameter η introduced by Alkhalifah and Tsvankin (1995) for VTI media. Though the values of the Thomsen parameters of the equivalent VTI medium are extremely uncommon for truly VTI media, time-related processing of P-wave data can still be performed by means of NMO, DMO, and migration algorithms developed for vertical transverse isotropy. The anisotropic parameters recovered from moveout data make it possible to reconstruct the phase-velocity function and process data in off-symmetry planes as well. However, this processing

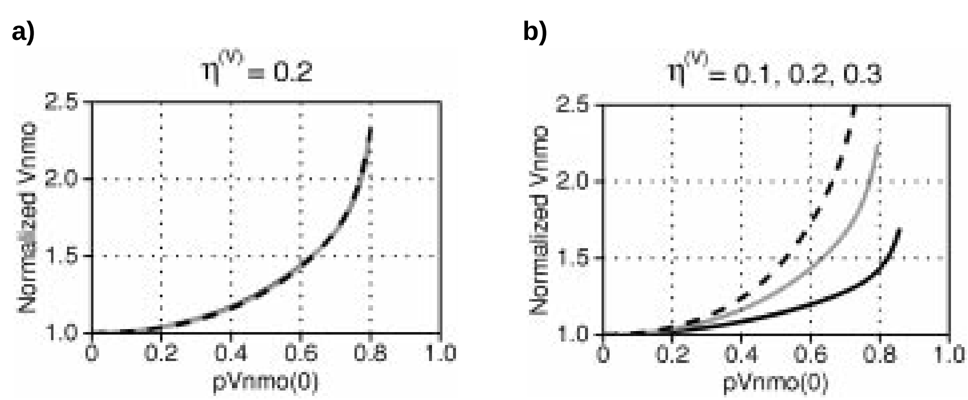

FIG. 6. P-wave normal-moveout velocity in the symmetry-axis plane of HTI media calculated from equation (42) and normalized by the expression for isotropic media (46). The dips range between 0° and 70°. (a) Models with the same $\eta^{(V)} = 0.2$: $\epsilon^{(R)} = 0.1, \delta^{(R)} = -0.0838$ (solid black); $\epsilon^{(R)} = 0.2, \delta^{(R)} = -0.0248$ (gray); $\epsilon^{(R)} = 0.3, \delta^{(R)} = 0.0343$ (dashed)—the curves practically coincide with each other [the ratio $V_{S0}^{(R)}/V_{P0}^{(R)} = 0.55$]. (b) Models with different $\eta^{(V)}$: $\eta^{(V)} = 0.1$ (solid black); $\eta^{(V)} = 0.2$ (gray); $\eta^{(V)} = 0.3$ (dashed).

cannot be carried out without a proper treatment of the 3-D relation between phase and group velocities, which is not accounted for by VTI algorithms.

If the anisotropy is caused by parallel ellipsoidal (penny-shaped) cracks, the coefficients of the equivalent VTI model are related to the crack density—an important parameter of fractured reservoirs. The crack density is close to the shear-wave splitting parameter γ, which determines the fractional difference between the velocities of split shear waves at vertical incidence. The shear-wave methods are designed to obtain γ directly from the shear-wave traveltimes and reflection amplitudes. That technology, however, has drawbacks associated with the cost of multicomponent surveys and the need to acquire high-quality shear data suitable for reliable polarization analysis. Also, shear-wave splitting yields an estimate of a single anisotropic parameter γ, which is not sufficient for processing of P-wave data in HTI media.

The simplest way of inverting normal-moveout velocities for the shear-wave splitting parameter is to use the $S^{\parallel}$-wave NMO velocity on lines parallel and perpendicular to the symmetry axis. However, it is also possible to infer γ from P- and $S^{\perp}$-wave NMO velocities from horizontal reflectors by using a constraint on the elastic constants for a medium with a system of thin parallel cracks. P-wave NMO velocity from horizontal reflectors is sufficient to estimate the parameter γ only in the special case of P-wave velocities that are equal in the vertical and symmetry directions. Such a model, which corresponds to negligible equant porosity and "very thin" fluid-filled cracks, may be relevant for coalbed methane plays with gas production from low-porosity fractured coals. In the general case, P-wave NMO velocity from horizontal reflectors can be supplemented with moveout from dipping events or the velocity in the symmetry direction (obtained from cross hole tomography, head waves, etc.) to constrain the shear-wave splitting parameter. The equation for γ, however, includes the ratio of the P-to-$S^{\perp}$ vertical velocities that cannot be determined from P-wave NMO velocities alone and should be estimated using well logs or surface shear-wave data. The main application of the P-wave inversion for the shear-wave splitting parameter is to identify pronounced anomalies of γ and the crack density corresponding to "sweet spots" in fractured reservoirs. Once the crack density has been estimated from the coefficient γ, the other anisotropic parameters of the equivalent VTI model can be used to obtain additional information about the physical properties of the cracks.

While the above results provide an analytic foundation for moveout inversion in HTI media, practical difficulties in the implementation of the algorithms outlined above should not be underestimated. The behavior of reflection moveout outside the symmetry planes of strongly anisotropic *layered* HTI media requires a separate study that will be reported in a sequel paper. As is true for isotropic, layered media, the accuracy of the Dix differentiation, needed to obtain the interval NMO velocity, reduces with decreasing thickness of the layer of interest. More reliable estimates of azimuthal anisotropy in thin layers can be obtained using amplitude methods (e.g., the azimuthal variation of the reflection coefficient). Also, the presence of anisotropy in the uncracked medium or deviation of the crack shapes from circular ellipsoids lead to more complicated types of symmetry than horizontal transverse isotropy. For instance, if vertical cracks are embedded in a transversely isotropic material with a vertical symmetry axis, the medium will have orthorhombic symmetry, which requires a special treatment.

Although the moveout equation derived here is limited to horizontal transverse isotropy, the same approach can be used to study the azimuthal dependence of NMO velocities from both horizontal and dipping reflectors in more complicated azimuthally anisotropic media semianalytically, without the need to perform ray tracing.

ACKNOWLEDGMENTS

I am grateful to Leon Thomsen (Amoco), Andreas Rüger, Ken Larner, and Tariq Alkhalifah (all of CSM) for useful discussions and their reviews of the manuscript and to Abdulfattah Al-Dajani (CSM) for his contribution to the inversion part of the paper. The 3-D anisotropic ray-tracing code was provided to the Center for Wave Phenomena (CWP) by Dirk Gajewski (University of Hamburg). This work was supported by the members of the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP, Colorado School of Mines, and by the United States Department of Energy, grant no. DE-FG02-89ER14079 (this support does not constitute an endorsement by DOE of the views expressed in this paper).

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APPENDIX A

RELATIONS BETWEEN THE TWO SETS OF THOMSEN PARAMETERS

The "generic" Thomsen parameters are defined in the rotated coordinate system in which the symmetry direction coincides with the x_3-axis. The relations between these Thomsen parameters and the corresponding stiffness coefficients $c_{ij}^{(R)}$ have the same form as for vertical transverse isotropy:

$$V_{P0}^{(R)} \equiv \sqrt{\frac{c_{33}^{(R)}}{\rho}}, \tag{A-1}$$

$$V_{S0}^{(R)} \equiv \sqrt{\frac{c_{55}^{(R)}}{\rho}}, \tag{A-2}$$

$$\epsilon^{(R)} \equiv \frac{c_{11}^{(R)} - c_{33}^{(R)}}{2c_{33}^{(R)}}, \tag{A-3}$$

$$\delta^{(R)} \equiv \frac{\left[c_{13}^{(R)} + c_{55}^{(R)}\right]^2 - \left[c_{33}^{(R)} - c_{55}^{(R)}\right]^2}{2c_{33}^{(R)}\left[c_{33}^{(R)} - c_{55}^{(R)}\right]}, \tag{A-4}$$

$$\gamma^{(R)} \equiv \frac{c_{66}^{(R)} - c_{44}^{(R)}}{2c_{44}^{(R)}}, \tag{A-5}$$

where ρ is the density $[c_{44}^{(R)} = c_{55}^{(R)}]$. The parameters $V_{P0}^{(R)}$ and $V_{S0}^{(R)}$, which in VTI models correspond to the vertical P and S-wave velocities respectively, in this case represent the P and S-wave velocities in the symmetry (horizontal) direction. The velocities and traveltimes of the $P - S^{\perp}$-waves for horizontal transverse isotropy are determined by the azimuth of the axis and the same four generic Thomsen coefficients as for $P-SV$-waves in VTI media $[V_{P0}^{(R)}, V_{S0}^{(R)}, \epsilon^{(R)}$, and $\delta^{(R)}]$. Furthermore, P-wave velocities and traveltimes depend mostly just on $V_{P0}^{(R)}$, $\epsilon^{(R)}$, and $\delta^{(R)}$, even for strong anisotropy (Tsvankin and Thomsen, 1994; Tsvankin, 1996).

To relate the coefficients of the equivalent VTI medium introduced in the main text to the generic Thomsen parameters, we have to express the stiffnesses $c_{ijk\ell}$ of the HTI model through the components of the VTI tensor $c_{ijk\ell}^{(R)}$ by interchanging the indices 1 and 3. Using the matrix notation (Voigt recipe), we find the following transformation rule for the nonzero stiffness components responsible for wave propagation in the $[x_1, x_3]$ plane:

$$c_{11} = c_{33}^{(R)}; \quad c_{33} = c_{11}^{(R)}; \quad c_{13} = c_{13}^{(R)}; \quad c_{55} = c_{55}^{(R)}, \tag{A-6}$$

and

$$c_{44} = c_{66}^{(R)}; \qquad c_{66} = c_{44}^{(R)}. \tag{A-7}$$

Using equations (A-6) and (A-7) and the definition of both sets of Thomsen parameters [equations (9)–(14), (A-1)–(A-5)], we obtain

$$\epsilon^{(V)} = -\frac{\epsilon^{(R)}}{1 + 2\epsilon^{(R)}}, \tag{A-8}$$

$$\delta^{(V)} = \frac{\delta^{(R)} - 2\epsilon^{(R)}\left[1 + \epsilon^{(R)}/f^{(R)}\right]}{\left[1 + 2\epsilon^{(R)}\right]\left[1 + 2\epsilon^{(R)}/f^{(R)}\right]}, \tag{A-9}$$

$$\gamma^{(V)} = -\frac{\gamma^{(R)}}{1 + 2\gamma^{(R)}}, \tag{A-10}$$

$$V_{P\text{vert}} = V_{P0}^{(R)}\sqrt{1 + 2\epsilon^{(R)}}, \tag{A-11}$$

$$V_{S^{\perp}\text{vert}} = V_{S0}^{(R)}, \tag{A-12}$$

$$V_{S\parallel\text{vert}} = V_{S0}^{(R)}\sqrt{1 + 2\gamma^{(R)}}, \tag{A-13}$$

where

$$f^{(R)} \equiv 1 - \left[V_{S0}^{(R)}\big/V_{P0}^{(R)}\right]^2. \tag{A-14}$$

The $S^{\perp}$-wave coefficient σ should be transformed according to

$$\sigma^{(V)} = \frac{\sigma^{(R)}}{1 + 2\epsilon^{(R)}/f^{(R)}}. \tag{A-15}$$

Equations (A-8)–(A-13) can be used to describe velocities and polarizations in the symmetry-axis plane of HTI media, as well as NMO velocity as a function of azimuth, in terms of the generic Thomsen parameters.

APPENDIX B

AZIMUTHALLY DEPENDENT NMO VELOCITY IN HTI MEDIA

Here the approach suggested in Tsvankin (1995) for moveout analysis in symmetry planes of anisotropic models is extended to an arbitrary incidence plane in transversely isotropic media with a horizontal symmetry axis.

Suppose the symmetry axis makes the azimuthal angle α with the common-midpoint (CMP) line (Figure B-1). Normal-moveout (NMO) velocity is defined on CMP gathers as

$$V_{\text{nmo}}^2 \equiv \lim_{x \to 0} \frac{d(x^2)}{d(t^2)}, \tag{B-1}$$

where x is the source-receiver offset and t is the two-way traveltime.

The derivation below is limited to the relatively simple case of horizontal reflectors, but the same approach can be used to find NMO velocity for reflections from dipping interfaces. Since a horizontal reflector represents a symmetry plane in HTI media, the group-velocity (ray) vector of any pure (nonconverted) reflected wave is the mirror image of the incident ray with respect to the horizontal plane. This means that the incident and reflected *rays* (SO and OR in Figure B-1) are confined to the incidence (sagittal) plane, even if this plane is not a plane of symmetry. Furthermore, since the incident and reflected rays lie in the incidence plane and make the same angle with the reflector, they also make the same angle with vertical, and there is no reflection point dispersal on CMP gathers. However, the *phase-velocity* vectors of the incident and reflected waves may deviate from the incidence plane, while still being symmetric with respect to the reflector.

Hale et al. (1992) gave a convenient form of equation (B-1) in terms of the one-way reflection traveltime:

$$V_{\mathrm{nmo}}^2 = \frac{2}{t_0} \lim_{h\to 0} \left[\frac{d}{dh}\left(\frac{dt}{dh}\right)\right]^{-1}, \qquad \text{(B-2)}$$

where $h = x/2$ is half the source-receiver offset, t is the one-way traveltime from the zero-offset reflection point to the receiver, and t_0 is the two-way traveltime along the zero-offset ray. In the case of a horizontal reflector beneath HTI media, both the phase- and group-velocity (ray) vectors of the zero-offset reflection are vertical. Note that the zero-offset ray is not necessarily vertical for other azimuthally anisotropic models, even for horizontal reflectors.

Equation (B-2) was derived under the assumption that the specular reflection point does not change with offset. As discussed above, this assumption is satisfied for our model; moreover, reflection point dispersal has no influence on NMO velocity anyway because it contributes only to the quartic and higher-order terms of the traveltime series (Hubral and Krey, 1980, Appendix D; Tsvankin, 1995).

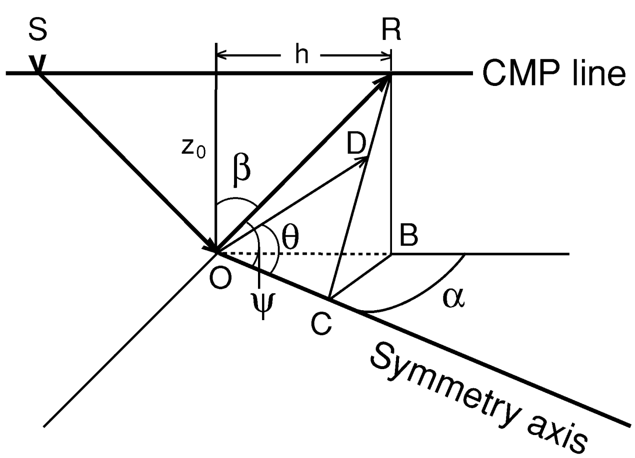

FIG. B-1. Geometry of the group- and phase-velocity vectors reflected waves in HTI media. The incident (SO) and reflected (OR) group-velocity vectors (rays) are confined to the incidence plane. The phase-velocity vector (direction OD) corresponding to reflected ray OR lies in the plane formed by OR and the axis of symmetry. Triangle RCB defines a plane normal to the symmetry axis.

Since the derivative dt/dh represents the apparent slowness on the CMP gather, it is equal to the projection of the slowness vector on the CMP line:

$$p_h = \frac{dt}{dh},$$

and the NMO velocity [equation (B-2)] can be rewritten as

$$V_{\mathrm{nmo}}^2 = \frac{2}{t_0} \lim_{h\to 0} \frac{dh}{dp_h}, \qquad \text{(B-3)}$$

Equation (B-3) remains valid for the case when the rays, as well as the slowness vectors of the incident and reflected waves, diverge from the incidence plane. Thus, it can be applied to much more complicated problems than the one considered here.

Introducing the group angle β in the incidence plane (Figure B-1) and substituting $h = z_0 \tan\beta$ and $z_0 = V_{\mathrm{vert}}\, t_0/2 (V_{\mathrm{vert}}$ is the vertical velocity) yield

$$V_{\mathrm{nmo}}^2 = V_{\mathrm{vert}} \lim_{\beta\to 0} \frac{d\tan\beta}{dp_h}. \qquad \text{(B-4)}$$

It is convenient to represent β and p_h as function of the phase angle θ with the symmetry axis (Figure B-1). Note that the phase-velocity vector in transversely isotropic media always lies in the plane formed by the symmetry axis and the group-velocity vector. Equation (B-4) then becomes

$$V_{\mathrm{nmo}}^2 = V_{\mathrm{vert}} \lim_{\theta\to 90^\circ} \frac{d\tan\beta}{d\theta}\left(\frac{dp_h}{d\theta}\right)^{-1}. \qquad \text{(B-5)}$$

Next, it is necessary to estimate both derivatives in equation (B-5). From simple trigonometry (Figure B-1),

$$\sin\beta = \frac{\cos\psi}{\cos\alpha},$$

where ψ is the group angle of ray OR with the symmetry axis. Then

$$\tan\beta = \frac{1}{\tan\psi\sqrt{1 - \dfrac{\sin^2\alpha}{\sin^2\psi}}}. \qquad \text{(B-6)}$$

The group angle ψ can be expressed through the phase angle θ and phase velocity $V(\theta)$ (Thomsen, 1986) as

$$\tan\psi = \frac{\tan\theta + \dfrac{1}{V}\dfrac{dV}{d\theta}}{1 - \dfrac{\tan\theta}{V}\dfrac{dV}{d\theta}}. \qquad \text{(B-7)}$$

Differentiating $\tan\psi$ with respect to θ yields (Tsvankin, 1995)

$$\frac{d\tan\psi}{d\theta} = \frac{1 + \dfrac{1}{V}\dfrac{d^2V}{d\theta^2}}{\cos^2\theta\left(1 - \dfrac{\tan\theta}{V}\dfrac{dV}{d\theta}\right)^2}. \qquad \text{(B-8)}$$

Using equations (B-6) and (B-8), we obtain the first derivative in equation (B-5):

$$\left.\frac{d\tan\beta}{d\theta}\right|_{\theta=\psi=90^\circ} = -\frac{1}{\cos\alpha}\left(1 + \frac{1}{V}\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ}\right). \qquad \text{(B-9)}$$

Now we have to find the relation between the projection of the slowness vector on the CMP line (p_h) and the phase angle θ. The slowness vector (which is parallel to OD in Figure B-1) can be decomposed into two vectors parallel to sides OC and CD of triangle OCD. Taking into account that

$$\cos(\angle RCB) = \frac{\tan\beta \sin\alpha}{\sqrt{1+\tan^2\beta\,\sin^2\alpha}} = \frac{\tan\alpha}{\tan\psi},$$

and projecting each of the two components onto the CMP line, we get

$$p_h = \frac{1}{V}(\cos\theta\cos\alpha + \sin\theta\sin\alpha\tan\alpha/\tan\psi), \quad \text{(B-10)}$$

with $\tan\psi$ given by equation (B-7).

Evaluating the derivative of equation (B-10) with respect to θ yields

$$\left.\frac{dp_h}{d\theta}\right|_{\theta=\psi=90^\circ} = -\frac{1}{V_{\text{vert}}\cos\alpha} \quad \text{(B-11)}$$

$$\times\left[1+\sin^2\alpha\left(\frac{1}{V}\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ}\right)\right]. \quad \text{(B-12)}$$

Finally, we obtain NMO velocity by substituting equations (B-9) and (B-11) into equation (B-5):

$$V^2_{\text{nmo}} = V^2_{\text{vert}}\frac{1+\frac{1}{V}\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ}}{1+\sin^2\alpha\left[\frac{1}{V}\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ}\right]}. \quad \text{(B-13)}$$

APPENDIX C

P-WAVE NMO VELOCITY ALONG THE SYMMETRY DIRECTION

Here, I derive the P-wave normal-moveout velocity on a line parallel to the symmetry axis directly from the phase-velocity equation for HTI media, without using the analogy between vertical and horizontal transverse isotropy discussed in the main text. For a CMP gather aligned with the symmetry axis ($\alpha = 0$), equation (B-13) reduces to

$$V^2_{\text{nmo}}(\alpha=0) = V^2_{\text{vert}}\left(1+\frac{1}{V}\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ}\right). \quad \text{(C-1)}$$

Since the term $(1/V)(d^2V/d\theta^2)$ should be calculated at vertical incidence, equation (C-1) coincides with NMO formula (42) of Tsvankin (1995) for the special case of horizontal reflectors. To evaluate NMO velocity (C-1) for the P-wave, it is convenient to use the exact expression for P-wave phase velocity in Thomsen notation (Tsvankin, 1996),

$$\left[\frac{V(\theta)}{V^{(R)}_{P0}}\right]^2 = 1+\epsilon^{(R)}\sin^2\theta - \frac{f^{(R)}}{2}$$

$$+\frac{f^{(R)}}{2}\sqrt{\left(1+\frac{2\epsilon^{(R)}\sin^2\theta}{f^{(R)}}\right)^2 - \frac{8\left[\epsilon^{(R)}-\delta^{(R)}\right]\sin^2\theta\cos^2\theta}{f^{(R)}}}. \quad \text{(C-2)}$$

Differentiating $V(\theta)$ from equation (C-2) twice with respect to θ, leads to

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90^\circ} = -\frac{2V^{(R)}_{P0}}{\sqrt{1+2\epsilon^{(R)}}}\left[\epsilon^{(R)}+\frac{\epsilon^{(R)}-\delta^{(R)}}{1+2\epsilon^{(R)}/f^{(R)}}\right]. \quad \text{(C-3)}$$

Substitution of equation (A-11) for the P-wave vertical velocity and equation (C-3) into NMO expression (C-1) yields

$$V_{\text{nmo}}(\alpha=0) = V^{(R)}_{P0}\sqrt{1-\frac{2\left[\epsilon^{(R)}-\delta^{(R)}\right]}{1+2\epsilon^{(R)}/f^{(R)}}}. \quad \text{(C-4)}$$