Using AVO for fracture detection: Analytic basis and practical solutions

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Tight, low-porosity reservoirs can produce significant amounts of hydrocarbons if fracture networks provide the necessary permeability. Characterization of naturally fractured reservoirs using surface seismic data is an important exploration problem that has attracted much attention in the literature. *TLE*, for example, had an entire issue (August 1996) on this subject. While the data and modeling in existing publications demonstrate that fracturing causes significant azimuthal variation in recorded seismic signatures, the problem is how to infer fracture parameters from these data.

The goal of this paper is to complement recent experimental results with an analytic and modeling study of the amplitude-variation-with-offset (AVO) response of fractured reservoirs. The question we address is: What does *P*-wave reflectivity really tell us about the fractures? To find the answer, we introduce a convenient notation for azimuthally anisotropic models and present concise approximations for reflection coefficients in terms of the anisotropy parameters. Our analysis shows that *P*-wave amplitudes may be sensitive even to relatively weak anisotropy of the rock mass. We also discuss how the azimuthally dependent AVO signature can be interpreted and combined with normal-moveout and shear-wave-splitting analyses to constrain the crack density and other medium parameters. The simple relationships between reflection amplitudes and anisotropic coefficients can be regarded as rules of thumb to quickly evaluate the impact of anisotropy in a particular play, plan acquisition, and guide more advanced numerical inversion.

Reflection coefficients and AVO. It is relatively straightforward, albeit tedious, to compute reflection coefficients for interfaces between anisotropic media. Inverting the reflection response for the physical properties of the medium,
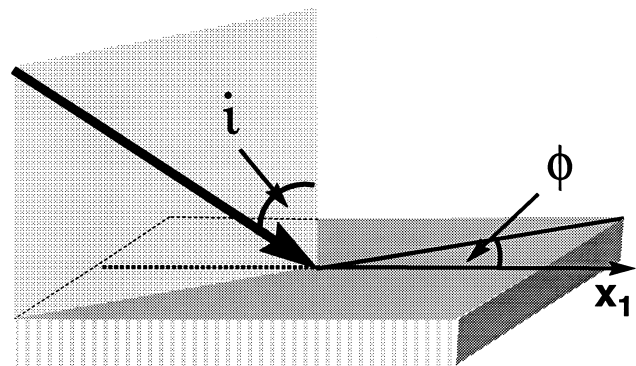
Figure 2. The reflection response from fractured reservoirs depends on two angles: (1) the polar (incidence) angle i between the vertical and the slowness vector of the incident wave and (2) the azimuthal angle ϕ defined with respect to the symmetry axis.

however, is much more involved due to the algebraic complexity of the reflection coefficients and the number of relevant medium parameters. To gain insight into anisotropic reflectivity, it is helpful to derive approximate scattering coefficients by assuming that both the contrast in the elastic properties across the interface and the anisotropic parameters are small. Shuey used the weak-contrast assumption for isotropic media to provide simple approximations illustrating the dependence of the AVO gradient (the initial slope of the reflection-coefficient curves) on the jumps in the *P*- and *S*-wave velocities and density (see "A simplification of the Zoeppritz equations," GEOPHYSICS, 1985). Shuey's relationship is widely used for lithologic interpretation in modern (isotropic) AVO analysis.

To understand the reflectivity of fractured reservoirs in a similar way, we follow the approach of Stuart Crampin, Leon Thomsen and others and initially consider the first-order model of azimuthal anisotropy conventionally used in shear-wave birefringence experiments. Assuming that parallel vertical cracks are embedded in a homogeneous isotropic matrix, we obtain the transversely isotropic model with a horizontal axis of symmetry (HTI media), sketched in Figure 1. As indicated by the arrows, azimuthal anisotropy has a first-order influence on shear waves that split into two components traveling with different velocities. The fractional difference between the velocities of split shear waves at vertical incidence (Thomsen's coefficient γ) is close to the crack density, an important parameter responsible for the permeability of the fracture network. Shear-wave methods, developed extensively during the last decade, are designed to obtain γ from the difference in shear-wave traveltimes and normal-incidence reflection amplitudes. This technology, however, has drawbacks (e.g., the cost and the difficulty in acquiring high-quality shear data). Hence, it is extremely important to obtain more information about fractured reservoirs from 3-D *P*-wave data. Amplitude variation with offset (AVO) analysis is particularly well-suited for frac-
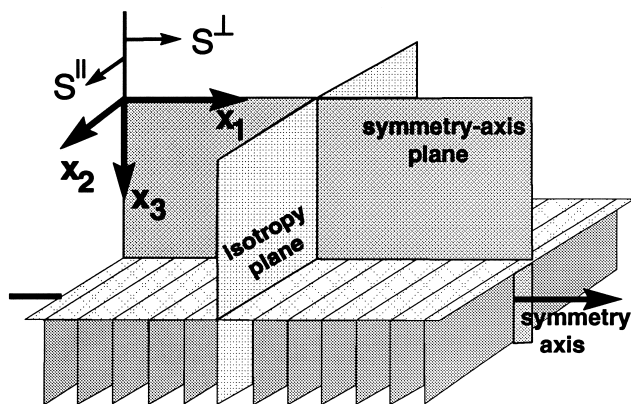

Figure 1. Sketch of an HTI model. Different *P*-wave reflection amplitudes in the two vertical symmetry planes, here called the symmetry-axis plane and the isotropy plane, lead to azimuthally-dependent AVO response. As indicated by the arrows, shear waves polarized parallel and normal to the isotropy plane have different vertical velocities.
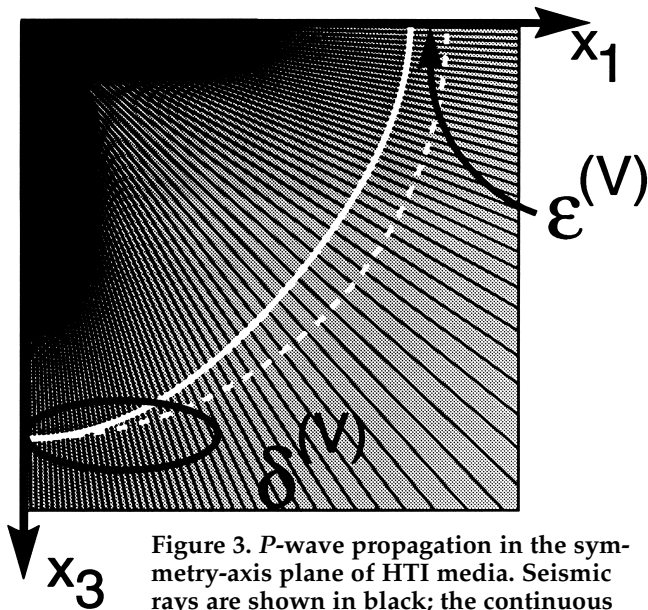
Figure 3. *P*-wave propagation in the symmetry-axis plane of HTI media. Seismic rays are shown in black; the continuous and dashed white curves are the anisotropic and isotropic wavefronts, respectively.

ture characterization because it provides local information about the anisotropy at the target horizon.

P-waves in HTI media. The influence of azimuthal anisotropy on *P*-wave propagation is not as dramatic as for shear waves, but it can be substantial. Let us discuss *P*-wave reflectivity for the HTI model introduced above. Waves confined to the plane normal to the symmetry axis (the "isotropy plane") do not experience any angular velocity variation. For all other vertical planes, the velocity does change with incidence angle; it also varies with azimuth (Figure 2), significantly complicating interpretation of reflection data.

Figure 3 illustrates *P*-wave propagation in the vertical plane containing the symmetry axis (the symmetry-axis plane). The continuous white line marks the points of equal *P*-wave traveltime on the seismic rays (wavefront), while the dashed white circle represents the reference isotropic wavefront. Conventionally, anisotropic media are described by the elements of the elastic stiffness tensor (a maximum of 21). While helpful in numerical modeling, this tensor notation is not optimal for analyzing and inverting seismic signatures. For example, it would benefit surface seismic applications to define a coefficient responsible for *P*-wave near-vertical velocity variations. We found that AVO and normal moveout in HTI are best described by adapting Thomsen's notation for transverse isotropy with a vertical symmetry axis (VTI media). Instead of using the generic Thomsen's coefficients specified with respect to the horizontal symmetry axis, however, we introduce these parameters with respect to the vertical by taking advantage of the equivalence between the symmetry-axis plane of HTI media and VTI media. Thus, five elastic coefficients can be replaced with the vertical velocities of the *P*-wave (α) and the fast *S*-wave (β), the shear-wave splitting parameter γ, and two more Thomsen-style anisotropic coefficients, $\varepsilon^{(v)}$ and $\delta^{(v)}$. The superscript "V" emphasizes that the coefficients are computed with respect to the vertical and correspond to the "equivalent" VTI model that describes wave propagation in the symmetry-axis plane.

For the particular example in Figure 3, the continuous white wavefront falls behind the corresponding isotrop-

ic (dashed) wavefront as the ray deviates from vertical, indicating that the *P*-wave velocity near vertical decreases with angle (i.e., $\delta^{(v)} < 0$). Similarly, the isotropic wavefront travels faster than the anisotropic in the horizontal direction. This implies that the fractional difference between horizontal and vertical *P*-wave speeds (close to $\varepsilon^{(v)}$) is negative as well. Small negative values of $\delta^{(v)}$ and $\varepsilon^{(v)}$ are typical for fractured reservoirs.

Analysis of *P*-wave reflectivity. Analytic expressions for the reflection coefficients at HTI/HTI interfaces with the same orientation of the symmetry axis above and below the interface can be obtained using a perturbation technique similar to that applied by Thomsen to (azimuthally isotropic) VTI media. Linearized in the small relative differences in elastic parameters (essentially Shuey's approximation) and small anisotropy coefficients, the *P*-wave plane-wave reflection coefficient has the following form:

$$R_P(i,\phi) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\left\{ \frac{\Delta\alpha}{\bar{\alpha}} - \left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^2 \frac{\Delta G}{\bar{G}} + \right. \tag{1}$$
$$\left[\Delta\delta^{(V)} + 2\left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^2 \Delta\gamma\right]\cos^2\phi \left.\right\}\sin^2 i +$$
$$\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}} + \Delta\varepsilon^{(V)}\cos^4\phi + \Delta\delta^{(V)}\sin^2\phi\cos^2\phi\right\}\times\sin^2 i\,\tan^2 i,$$

where i is the incidence (polar) phase angle, ϕ is the azimuthal phase angle with the symmetry axis, $Z = \rho\alpha$ is the vertical *P*-wave impedance and $G = \rho\beta^2$ is the vertical shear modulus. The elastic parameters are expressed through their average values and relative differences across the interface. The vertical *P*-wave velocities in the upper and lower layers, for example, can be written as functions of the average velocity $\bar{\alpha} = 1/2\,(\alpha_2 + \alpha_1)$ and the difference $\Delta\alpha = (\alpha_2 - \alpha_1)$:

$$\alpha_1 = \bar{\alpha}\left(1 - \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\right); \quad \alpha_2 = \bar{\alpha}\left(1 + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\right)$$

Corresponding expressions are defined for the shear modulus, the density and the *P*-wave impedance.

The simplicity of approximation (1), which is valid for incidence angles not too close to the critical angle, is striking, especially because no angular terms have been neglected; the azimuthally dependent contribution of anisotropy is described by simple trigonometrical functions. For azimuth $\phi = 90°$, equation (1) yields the reflection coefficient in the isotropy plane, which is identical to Shuey's approximation for isotropic media. While *P*-waves incident in the symmetry-axis plane excite $S^\perp$ waves with the vertical velocity $\beta^\perp$, *P*-waves in the isotropy plane are coupled with the faster $S^\parallel$ shear wave. The difference in the vertical shear-wave velocities can be described by the shear-wave splitting parameter, i.e., $\beta^\perp \approx \beta(1 - \gamma)$. That is why the parameter γ influences the gradient ($\sin^2 i$) AVO term in the symmetry-axis plane ($\phi = 0°$) and elsewhere outside the isotropy plane. Since $\delta^{(v)}$ describes the near-vertical *P*-wave velocity in the symmetry-axis plane, it is not surprising that it also enters the gradient term at all azimuths except for the isotropy plane. Likewise, the definition of $\varepsilon^{(v)}$ implies that it should contribute to the reflection coefficient at higher angles i (see the $\sin^2 i\,\tan^2 i$ term).

P-wave AVO inversion. Equation (1) provides a comprehensive insight into the AVO inversion. Rewriting it as

$$R_P(i,\phi) = A + \left\{B^{iso} + B^{ani}\cos^2\phi\right\}\sin^2 i + \tag{2}$$
$$\left\{C^{iso} + C^{ani_1}\cos^4\phi + C^{ani_2}\sin^2\phi\,\cos^2\phi\right\}\sin^2 i\ \tan^2 i,$$

reveals that a maximum of six coefficients can be extracted from the dependence of Rp on the azimuthal and incidence angles. Since it is often difficult to extract reliable estimates of the high-angle reflectivity term $\sin^2 i\ \tan^2 i$, we concentrate on the AVO gradient that determines the low-angle reflection response.

One of the most important messages of our approximation is that the contrasts in anisotropy parameters $\delta^{(V)}$ and γ have a non-negligible, first-order influence on the AVO gradient (see the term B^{ani}). For example, for azimuth $\phi = 0$ and a typical $\beta/\alpha \approx 1/2$, the contrast in the shear-wave splitting parameter ($\Delta\gamma = \gamma_2 - \gamma_1$) at the boundary has a weighting factor twice as large as that of the P-wave velocity contrast $\frac{\Delta\alpha}{\alpha}$. Hence, B^{ani} introduces a measurable azimuthal variation in the AVO response.

The AVO gradient term $B = B^{iso} + B^{ani}\cos^2\phi$ in equations (1) and (2) is composed of the azimuthally invariant part B^{iso} and the anisotropic contribution B^{ani} dependent on the azimuthal angle ϕ with the symmetry axis. If the symmetry-axis orientation is unknown, ϕ can be expressed as the difference between the azimuthal direction ϕ_j of the jth observed azimuth and the direction of the symmetry-axis plane ϕ_{sym}. The AVO gradient measured at azimuth ϕ_j can then be written as

$$B(\phi_j) = B^{iso} + B^{ani}\cos^2(\phi_j - \phi_{sym}). \tag{3}$$

This equation is nonlinear with three unknowns (B^{iso}, B^{ani}, and ϕ_{sym}), so a minimum of three azimuthal measurements of the AVO gradient is needed to find the orientation of the symmetry planes and reconstruct the low-angle reflection coefficient at all azimuths. If the direction of the symmetry axis is known (for example from S-wave splitting analysis), the equation becomes linear and two independent measurements are sufficient to recover B^{iso} and B^{ani}. Plotting B as a function of ϕ_j, we find that the extrema correspond to the symmetry-plane azimuths, even though more information is needed to distinguish between the symmetry-axis and the isotropy plane. The *difference* between the symmetry-plane AVO gradients yields the value B^{ani} that contains information about anisotropy in the form of a combination of the coefficients $\delta^{(v)}$ and γ. In general, $\delta^{(v)}$ and γ are independent; however, for a tight formation with no equant porosity and thin fluid-filled cracks, $\delta^{(v)} \approx -\gamma$. In this case, the azimuthal variation in the P-wave AVO gradient can be inverted directly for γ and the crack density.

Combination of AVO and moveout data. A complication in the AVO inversion is the anisotropic parameter $\delta^{(v)}$ in the gradient term B^{ani} that prevents (except for the special case discussed above) obtaining the shear-wave splitting parameter and estimating the crack density using the P-wave AVO response. However, $\delta^{(v)}$ can be found from azimuthally dependent P-wave moveout data. The P-wave normal moveout (short-spread) velocity in a horizontal HTI layer is given by

$$V_{nmo}^2 = \alpha^2 \frac{1 + 2\delta^{(V)}}{1 + 2\delta^{(V)}\sin^2\phi}, \tag{4}$$

where ϕ is the azimuth of the CMP line with respect to the symmetry axis. Equation (4) describes an *ellipse* in the horizontal plane and is valid for HTI models with any strength of the anisotropy. Since the NMO velocity con-
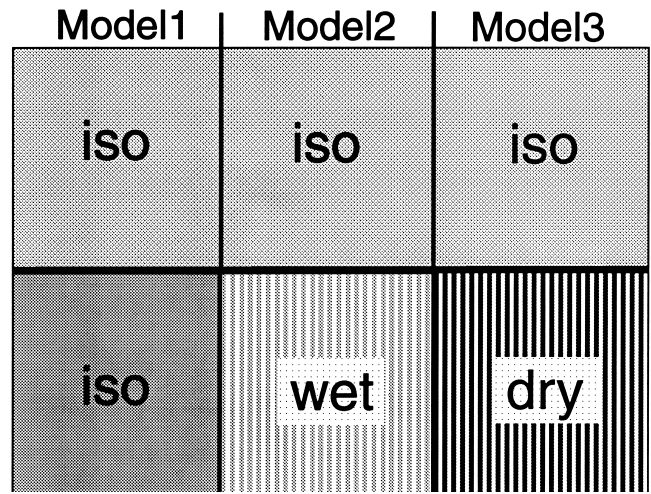

Figure 4. Three models used to study the differences in the AVO response for fluid-filled and dry cracks. Table 1 lists the model parameters.

Table 1. Parameters of the reflecting layers used to investigate the azimuthally varying AVO response. Model 1 is isotropic; Models 2 (wet cracks) and 3 (dry cracks) have the same host rock, but also contain vertical cracks with different contents.

	isotropic	wet cracks	dry cracks
α	4.500	4.498	4.388
β	2.530	2.530	2.530
ρ	2.800	2.800	2.800
γ	0.000	0.085	0.085
$\varepsilon^{(v)}$	0.000	-0.003	-0.150
$\delta^{(v)}$	0.000	-0.088	-0.155

Table 2. Percentage changes in the elastic parameters for reflection from the top of the target layer. The parameters of the isotropic overburden are α = 3.67 km/s, β = 2.0 km/s, ρ = 2.41 g/cm³.

	isotropic	wet cracks	dry cracks
$\frac{\Delta\alpha}{\alpha}$	0.203	0.203	0.180
$\frac{\Delta Z}{Z}$	0.350	0.350	0.328
$\frac{\Delta G}{G}$	0.601	0.601	0.601

tains just three unknowns, three measurements of V_{nmo} at different azimuthal angles can be inverted for the vertical velocity, the axis orientation, and the parameter $\delta^{(v)}$ (provided the fractured interval is not too thin). Hence, P-wave moveout data can identify the crack orientation and recover the parameter $\delta^{(v)}$ that is required for the AVO inversion. Then two azimuthal measurements of the AVO gradient are sufficient to obtain the AVO gradient in the symmetry planes and estimate the shear-splitting parameter γ.

Fluid-filled vs. dry cracks: a modeling example. Figure 4 shows three models with a different target layer and the same isotropic overburden. The first target layer is isotrop-
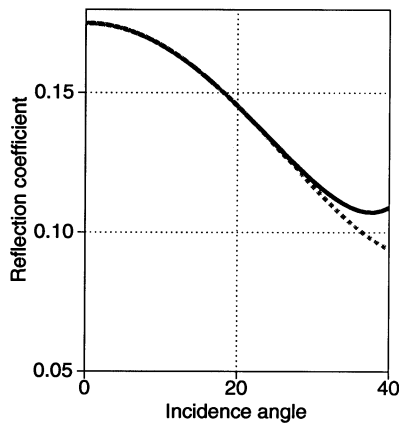
Figure 5. The exact reflection coefficient (solid line) and the approximation (dashed) for the unfractured target layer. Model parameters are given in the left column of Tables 1 and 2.
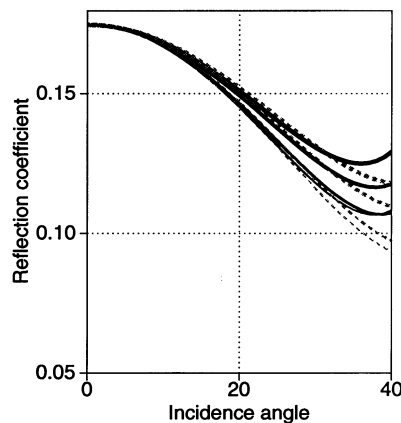
Figure 6. Same as Figure 5, but the reflecting layer contains water-filled cracks (the central column in Tables 1 and 2). The reflection coefficients are shown for azimuths of 0°, 30°, 60°, and 90° measured from the symmetry axis, with decreasing line thickness (i.e., the thickest line corresponds to 0°).
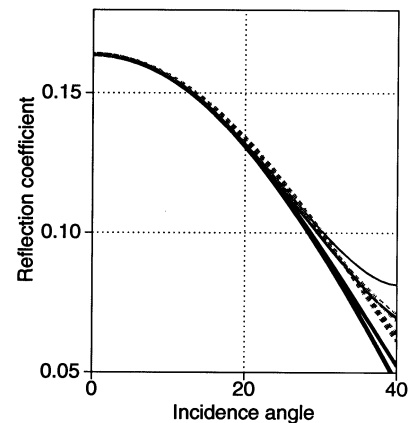
Figure 7. Same as Figures 5 and 6, but the reflecting layer contains dry cracks (the right column in Tables 1 and 2) .

ic with the elastic parameters given in the first column of Table 1; the other two layers have the same host rock, but also contain vertical aligned cracks (causing HTI symmetry) with an aspect ratio of 0.001 and a crack density of 7%. The second column is computed for water-filled cracks, the third for dry cracks.

Before evaluating the reflection response for the three models, let us discuss the difference between the model parameters and use equation (1) to predict the behavior of the three AVO responses. All three models have the same density and the same fast vertical shear-wave velocity (the shear wave polarized within the isotropy plane is not influenced by the fracturing). Also, the fractures are too thin and their density is too small to influence the bulk density of the host rock. As a result, the vertical P-wave velocity is almost the same for the unfractured rock and the medium with water-filled cracks, and is only slightly smaller (by about 2.5 %) for the medium with dry cracks. The anisotropy parameters, however, are substantially influenced by the presence and contents of the cracks. For water-filled cracks, the coefficient $\varepsilon^{(v)}$ is negligibly small, while the shear-wave-splitting parameter γ and coefficient $\delta^{(v)}$ have almost the same magnitude but opposite sign. If the cracks are dry, both $\varepsilon^{(v)}$ and $\delta^{(v)}$ are negative and $\varepsilon^{(v)} \approx \delta^{(v)}$; the absolute value of $\varepsilon^{(v)}$ and $\delta^{(v)}$ is approximately twice as large as the shear-wave splitting parameter γ.

These differences in the anisotropy parameters lead to a different azimuthal variation in the reflection response for each of the models. Taking into account the percentage changes in the elastic parameters (see Table 2), equation (1) predicts that the initial slope of the reflection coefficient (AVO gradient) for the isotropic Model 1 is identical to the AVO gradient in the isotropy plane of Model 2, and is slightly less negative than the isotropy-plane AVO gradient in Model 3. For water-filled cracks, equation (1) predicts a substantial azimuthal difference in the AVO gradients, with a more negative value in the isotropy plane [recall that the difference in the symmetry plane gradients is approximated by $(\delta^{(v)} + 2\gamma)$]. In contrast, for dry cracks the combination $\delta^{(v)} + 2\gamma \approx 0$, and we cannot expect a measurable azimuthal variation in the

AVO gradient. In this case, reflection coefficients computed at different azimuths start to diverge only at large angles of incidence.

Figures 5-7 show the exact and approximate reflection coefficients for the three models (Tables 1 and 2). To facilitate the comparison, the vertical scale is the same on all three plots. Approximation (1) is sufficiently close to the exact reflection coefficients for all three models; specifically, it predicts the different azimuthal variation in the small- and large-angle reflection response for the dry and water-filled cracks.

Hence, this study confirms the physical insight provided by equation (1) and shows that P-wave AVO has the potential of discriminating between gas- and fluid-filled crack systems. Also, if the AVO response at different azimuths diverges substantially only at large angles of incidence, it is a strong indication of a large value of the parameter $\varepsilon^{(v)}$. In this case, the less negative AVO gradient corresponds to the isotropy-plane reflection because $\Delta\varepsilon^{(v)}$ is negative for the interface between an isotropic layer and a fractured medium with the HTI symmetry.

Another important message is that the azimuthal difference in AVO gradients cannot be related directly to the crack density (it may even be close to zero for dry cracks), and that knowledge of the anisotropy parameter $\delta^{(v)}$ is crucial to any meaningful azimuthal AVO analysis. As mentioned above, $\delta^{(v)}$ can be obtained from P-wave NMO velocity; alternatively, if γ is available from S-wave data, this study suggests combining the shear-wave-splitting and azimuthal-AVO analysis to obtain $\delta^{(v)}$ for study of the contents of the cracks.

Discussion. This analysis was limited to interpretation and inversion of the reflection coefficients. In practice, we record the amplitude-versus-offset signature at the surface rather than the reflection coefficients at the target horizon. Various methods to correct for the propagation phenomena have been discussed in the literature, but most are approximate even for isotropic media. The situation becomes more complex if the reflected wave propagates through an *anisotropic* overburden, which may happen, for

instance, if the cracks are not confined to the reservoir layer. The presence of anisotropy above the reflector leads to the so-called wavefront focusing phenomena, or distortions in the amplitude distribution along the wavefronts of the incident and reflected waves. This can be seen in Figure 3, where focusing and defocusing of seismic rays mark areas of high and low energy, respectively. Azimuthal variation in the transmission coefficients in the anisotropic overburden may also contribute to the overall amplitude response at the surface. If not corrected, these distortions can propagate into the AVO signature and distort the inversion results.

Another complication is the difference between group (ray) and phase angles in anisotropic media. Analyzing the extracted reflection coefficient as a function of the source-receiver offset (rather than as a function of the incident phase angle) can lead to a misleading interpretation of the azimuthal AVO variation. In HTI media, the group and phase angles coincide for waves propagating in the isotropy plane, but may differ significantly in the symmetry-axis plane. In other words, the rays connecting sources and receivers at any fixed offset have an azimuthally dependent incidence phase angle at the reflector. To correctly interpret the AVO signatures in anisotropic media using the analytic reflection coefficients given here, it is essential to represent them as a function of phase angle. Unfortunately, in practice the anisotropic parameters of the overburden may not be known with sufficient accuracy to account for various propagation phenomena.

Conclusions. Linearized approximations can provide valuable insight into the otherwise incomprehensibly complex reflection coefficients in anisotropic media. The main assumptions used in this study — small jumps in the elastic parameters across the reflecting interface, subcritical incidence, and weak anisotropy — are geologically and geophysically reasonable and have proved useful in many exploration contexts. We showed that the azimuthal variation in the P-wave AVO gradient is controlled by two anisotropic parameters — the shear-wave splitting coefficient γ and the "moveout" parameter $\delta^{(v)}$. Both analytic and modeling results indicate that P-wave reflectivity can detect fracture orientation, estimate crack density, and discriminate between fluid-filled and dry cracks. However, unambiguous interpretation of P-wave AVO results in terms of the medium parameters requires combination with moveout data or shear-wave splitting analysis. A big challenge in the implementation of anisotropic AVO is reliable recovery of reflection coefficients from surface data by amplitude-preserving anisotropic processing.

Certainly, the simple HTI model discussed here may be inadequate for realistic models of fractured reservoirs, and it is important to study whether the reflection response in media of lower symmetry (e.g., orthorhombic) differs significantly from the HTI case. However, it has already been shown that although wave propagation in orthorhombic media is significantly more complex, the reflection coefficients in the symmetry planes have essentially the same form as in HTI models and vary smoothly with azimuth.

Suggestions for further reading. Papers of Stuart Crampin ("Evidence for aligned cracks in the Earth's crust", *First Break*, 1985, and many others) and Leon Thomsen ("Reflection seismology over azimuthally anisotropic media", GEOPHYSICS, 1988) give a good exposition of the implications of azimuthal anisotropy in seismology. Thomsen's work ("Weak elastic anisotropy," GEOPHYSICS, 1986; "Weak anisotropic reflections" in *Offset Dependent Reflectivity*, SEG, 1993) provided a basis for defining anisotropy parameters and deriving linearized scattering coefficients. The models shown in this paper are computed using Hudson's fracture theory ("A higher order approximation to the wave propagation constants for a cracked solid," *Geophysical Journal of the Royal Astronomical Society*, 1986). Derivations of the AVO equations and extensions to P- and S-wave AVO in orthorhombic media are given in Rüger's articles "Variation of P-wave reflectivity with offset and azimuth in anisotropic media" (GEOPHYSICS, accepted for publication) and "Analytic insight into shear-wave AVO for fractured reservoirs" (to appear in *Proceedings of 71WSA*, special SEG volume on seismic anisotropy, Miami, 1996). Reflection from the bottom of a fractured layer was studied by Sayers and Rickett in "Azimuthal variation in AVO response for fractured gas sands" (*Geophysical Prospecting*, 1997). The anisotropic energy-focusing phenomena and normal-moveout velocity in HTI media were described by Tsvankin in "Body-wave radiation patterns and AVO in transversely isotropic media" (GEOPHYSICS, 1995) and "Reflection moveout and parameter estimation for horizontal transverse isotropy" (GEOPHYSICS, 1997). LE

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