Fluid Flow Considerations (Supplemental Chapter 7)

Bernoulli's equation for fluid flow

Bernoulli's equation for fluid flow

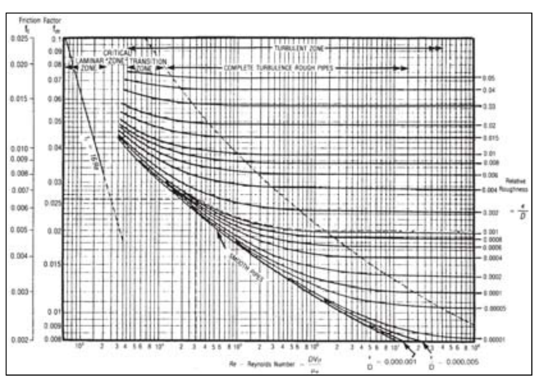
$$Z_1 + \frac{P_1}{\rho_1} + \frac{v_1^2}{2g} = Z_2 + \frac{P_2}{\rho_2} + \frac{v_2^2}{2g} + h_L$$

h_L is the frictional head loss

$$h_L = \frac{f_M \, L \, v^2}{2 \, g \, D}$$

- Two friction factors, Moody & Fanning – different by a factor of 4

$$f_M = 4 \, f_F$$



Reynold's number

Reynold's number is an expression for the relationship of inertial to frictional forces

Have typically used expression for flow though a circular conduit (i.e., a pipe)

$$N_{Re} = \frac{D\,v\,\rho}{\mu} \quad \Rightarrow \quad N_{Re} = \frac{4\,\dot{V}\,\rho}{\pi\,D^2\,\mu} = \frac{4\,\dot{m}}{\pi\,D^2\,\mu}$$

There can be other expressions for different geometries. For example, for a stirred tank:

$$N_{Re} = \frac{N_i\,D_i^2\,\rho}{\mu}$$

Reynold's number

Reynold's numbers can give a guideline for the transition between laminar & turbulent flow – dependent on geometry

- Flow through circular pipes – transition 2000 to 4000
- Stirred tanks depend on geometry of impeller, tank, etc. Typically laminar below 10 & turbulent greater than 10000

Relationship between sheer stresses & viscosity

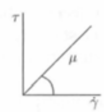
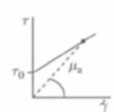
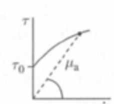
The shear stress experienced by a flowing fluid is the shear force applied by an area. For flow in the x-direction & a flow gradient in the y-direction:

$$\tau = \frac{F}{A} \quad \Rightarrow \quad \tau = -\mu \frac{dv}{dy} \quad \Rightarrow \quad \tau = \mu\dot{\gamma} \text{ where } \dot{\gamma} = -\frac{dv}{dy}$$

Here τ is the shear stress & γ is the shear rate

If the viscosity is not dependent on the shear stress or rate then the fluid is said to be Newtonian

Newtonian vs. Non-Newtonian Fluids

Fluid	Flow curve	Equation	Apparent viscosity μ_a
Newtonian		$\tau = \mu\dot{\gamma}$	Constant $\mu_a = \mu$
Pseudoplastic (power law)		$\tau = K\dot{\gamma}^n$ $n<1$	Decreases with increasing shear rate $\mu_a = K\dot{\gamma}^{n-1}$
Bingham plastic		$\tau = \tau_0 + K_p\dot{\gamma}$	Decreases with increasing shear rate when yield stress τ_0 is exceeded $\mu_a = \frac{\tau_0}{\dot{\gamma}} + K_p$
Casson plastic		$\tau^{1/2} = \tau_0^{1/2} + K_p\dot{\gamma}^{1/2}$	Decreases with increasing shear rate when yield stress τ_0 is exceeded $\mu_a = \left[\left(\frac{\tau_0}{\dot{\gamma}}\right)^{1/2} + K_p\right]^2$

Rheological Properties of Fermentation Broths

TABLE 7.2 Rheological Properties of Microbial and Plant Cell Suspensions

Culture	Shear rate (s^{-1})	Viscometer	Comments	Reference
Saccharomyces cerevisiae (pressed cake diluted with water)	2–100	rotating spindle	Newtonian below 10% solids ($\mu < 4-5$ cP); pseudoplastic above 10% solids	[12]
Aspergillus niger (washed cells in buffer)	0–21.6	rotating spindle (guard removed)	pseudoplastic	[13]
Penicillium chrysogenum (whole broth)	1–15	turbine impeller	Casson plastic	[8]
Penicillium chrysogenum (whole broth)	not given	coaxial cylinder	Bingham plastic	[14]
Penicillium chrysogenum (whole broth)	not given	coaxial cylinder	pseudoplastic; *K* and *n* vary with CO_2 content of inlet gas	[15]
Endomyces sp. (whole broth)	not given	coaxial cylinder	pseudoplastic; *K* and *n* vary during batch culture	[16]
Streptomyces noursei (whole broth)	4–28	rotating spindle (guard removed)	Newtonian in batch culture; viscosity 40 cP after 96 h	[17]
Streptomyces aureofaciens (whole broth)	2–58	rotating spindle/ coaxial cylinder	initially Bingham plastic due to high starch concentration in the medium; becomes Newtonian as starch is broken down; increasingly pseudoplastic as mycelium concentration increases	[18]

Updated: November 13, 2017
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Rheological Properties of Fermentation Broths

TABLE 7.2 Rheological Properties of Microbial and Plant Cell Suspensions (Continued)

Culture	Shear rate (s^{-1})	Viscometer	Comments	Reference
Aureobasidium pullulans (whole broth)	10.2–1020	coaxial cylinder	Newtonian at beginning of culture; increasingly pseudoplastic as concentration of product (exopolysaccharide) increases	[19]
Xanthomonas campestris	0.0035–100	cone-and-plate	pseudoplastic; *K* increases continually; *n* levels off when xanthan concentration reaches 0.5%; cell mass (max 0.6%) has relatively little effect on viscosity	[20]
Cellulomonas uda (whole broth)	0.8–100	anchor impeller	shredded newspaper used as substrate; broth pseudoplastic with constant *n* until end of cellulose degradation; Newtonian thereafter	[11]
Nicotiana tabacum (whole broth)	not given	rotating spindle	pseudoplastic	[21]
Datura stramonium (whole broth)	0–1000	rotating spindle/ parallel plate	pseudoplastic and viscoelastic, with yield stress	[22]
Perilla frutescens (whole broth)	7.2–72	coaxial cylinder	Bingham plastic	[23]

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Factors Affecting Broth Viscosity

Rheology of fermentation broths affected by

- Cell concentration
- Cell morphology, including size, shape, mass, and vacuolation
- Flexibility and deformability of cells
- Osmotic pressure of the suspending fluid
- Concentration of polymeric substrate
- Concentration of polymeric product
- Rate of shear

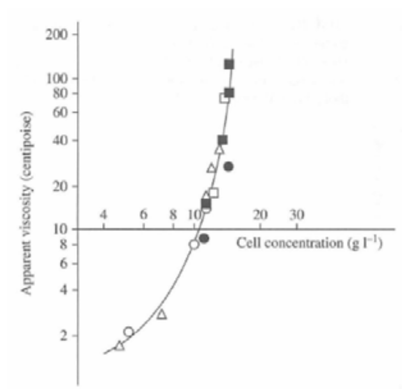
Factors Affecting Broth Viscosity – Cell Concentration

Vand equation

$$\mu = \mu_L\left(1 + 2.5\psi + 7.25\psi^2\right)$$

μ_L is the viscosity of the suspending liquid

ψ is the volume fraction of solids



Summary

Remember Bernoulli's equation for flowing fluids

Use of Reynold's number

- Make distinction between expected regions of laminar, transition, or turbulent flow
- Determine frictional losses

Newtonian vs Non-Newtonian fluids

- Expressions to describe non-Newtonian fluids