Multivariable Control of Aluminum Reduction Cells

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Abstract

This paper considers control of the aluminum reduction process, using a dynamic model developed from the literature and Virtpot, a model developed at Kaiser Aluminum. Analysis shows the process is controllable and observable, but not easily stabilizable using only one input, and that short-term changes in measured voltage result primarily from changes in alumina concentration rather than anode-to-cathode distance (ACD). Next, a multivariable control strategy is developed to regulate cell voltage by adjusting feed rate rather than beam movement. We introduce the idea of a feed voltage, obtained by subtracting expected voltage deviations due to ACD changes and beam moves from the filtered voltage. Feed rate is adjusted to compensate for deviations of feed voltage from its target. Simultaneously, beam movements are made to compensate for the difference in expected anode consumption and metal pad rise, based on changes in feed period. Simulations show the effectiveness of the proposed control strategy.

Introduction

The primary production of aluminum is a key process in the aluminum industry. A critical issue in this process is proper regulation of both the anode-to-cathode distance and the concentration of dissolved alumina. These quantities have a primary effect on the overall efficiency of the reduction process [1]. Unfortunately, it is difficult to directly measure either variable. Typical installations of aluminum reduction cells allow only the measurement of cell voltage, which is affected by both the anode-to-cathode distance and the dissolved alumina concentration. This fact makes the control of the aluminum reduction cell difficult [2].

In this paper we consider control of the aluminum reduction process. We take a model-based, control-theoretic approach, using a dynamic model structure to describe the process. This dynamic model is motivated by first principles arguments drawn from the literature and from results obtained from a detailed simulation model called Virtpot (Virtual Pot), which incorporates consideration of the stochiometric and thermal-physical properties of the process [3].

The paper is organized as follows. First we describe the dynamic model that is used. Then we present analysis that shows the process is controllable and observable, but not easily stabilizable using only one input, and that short-term changes in measured voltage result primarily from changes in alumina concentration rather than anode-to-cathode distance (ACD). A key aspect of this analysis is consideration of step response tests from the Virtpot simulation. Based on this analysis, we then introduce a multivariable control strategy, developed to regulate cell voltage by adjusting feed rate rather than beam movement. The strategy uses the idea of a "feed voltage," which is obtained by subtracting expected voltage deviations due to ACD changes and beam moves from measured and filtered voltage. Feed rate is then adjusted to compensate for deviations of feed voltage from its target. Simultaneously, beam movements are made to compensate for the difference in expected anode consumption and metal pad rise, based on changes in feed period. Finally, simulations are presented to show the effectiveness of the proposed control strategy.

Modelling the Process

An Initial Model

Any effective control strategy should take into account a reasonably accurate model of the process to be controlled. In initial work we have considered the aluminum reduction process first from an input/output perspective and then from a dynamic perspective [4,5]. The model that was developed at that time was a nonlinear, third-order system with appropriate mechanisms to handle saturation and the characteristic resistance versus alumina curve [1]. This model seemingly captures the essential primary effects of the aluminum reduction process. Specifically, it showed:

1. The alumina in solution is (after a short time constant) the integral of the difference between how fast alumina is added (via the feed rate) and how fast it is removed (via conversion to aluminum by the current).
2. The ACD is the integral of the beam move rate and a constant times the current. The latter term is considered to lump the effect of ACD changes due to both anode consumption and metal pad increase due to the production of aluminum.
3. Around a nominal operating point voltage is considered to be directly proportional to both alumina concentration and ACD.

Although this model presented reasonably accurate qualitative predictions of cell behavior, its analysis presented some troubling conclusions. Specifically, the model was not observable (in the sense of control theory [6]. The basic implication is that, if the result was true, the system can not be controlled using output measurements. This is a standard result from control theory and we will not discuss it further here, other than to note that (1) a system may actually be unobservable if there is not an adequate number of independent sensors available, but (2) a system may seem to be unobservable if it has not been modelled properly. This second observation led us to consider further dynamic modelling of the process, using Virtpot.

Virtpot Simulation Model

Virtpot is a computer-based dynamical model of the aluminum reduction process. It was developed at Kaiser Aluminum Company's Center for Technology (CFT) by Nobuo Urata and is built on the original modelling work of A. Wright [7]. Virtpot consists of two modules; cell model module and cell controller module. The cell model calculates ion transport and reaction equations, solves the electrical and thermal field equations and simulates the basic cell operations such as alumina feeding, bath and metal tapping, anode change-out, aluminum and sodium fluoride additions and their dissolution. In this work, the cell model simulated the cell at Mead, Washington, operating at 68 kA. The cell controller modelled in Virtpot is essentially the existing commercial controller running on the modeling computer instead of on the control hardware. In the simulations below, the control model in Virtpot, designed to be replaceable with any new control modules, is replaced with the authors' control module developed using Matlab. The cell model module, made as a dynamic link library, is called from the control codes written in the Matlab language.

Open-Loop Step Response Tests

Virtpot implements a complicated system of nonlinear differential equations. Given this, one could argue that modelling for control is unnecessary because the Urata/Wright simulation already models the process. While this is true, the Virtpot source code contains an intricate set of equations that model numerous secondary effects. As we will see, some of these secondary effects are important to our controller development process. However, many other secondary effects are not relevant to controller development, and it is advantageous to have a fundamental, lumped parameter model of the system to use for controller design. Thus, in order to extract the essential features of the system behavior, the most straightforward approach is to simulate step responses.

Figure 1 shows the input/output view of the process that we used in developing our step response tests. We view the system as having three primary inputs and one output. Also shown are the two intermediate states that combine (with the current input) to produce the output voltage.

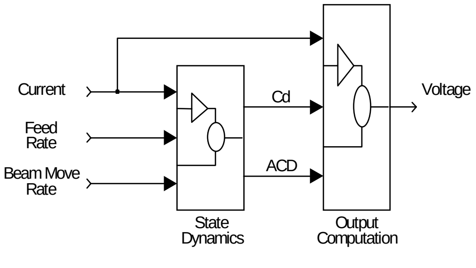

Figure 1: Input/output block diagram.

Figure 2 shows the response of Virtpot to a step change in feed rate from 140 secs/feed to 130 secs/feed occurring at seven hours. There are no beam moves in this test. It seems from the figure that there is minimal effect on ACD and that there is an integrative effect on alumina. That is, a constant level of feed rate, slightly higher than that required to counteract the effect of conversion to aluminum results in a slightly increasing alumina concentration. Then, when the feed rate is increased even higher, the slope of the alumina as a function of time also increases. Notice that relative to the time scale shown, this change in slope is essentially instantaneous. If we were to zoom in on the plot we would see that the time required for the alumina curve to reach its new slope is on the order of 3-5 minutes, which is approximately the same order of magnitude as the time between feeds in our simulation. It is also important to note that if we zoom in on the ACD plot, we in fact find that there is a small change in ACD slope due to the change in the feed rate. This effect is shown in Figure 3 and is an illuminating result that points out the error in our original assumptions that led to the conclusion that the system was unobservable. In retrospect, it does appear that the effect of feed rate on ACD is reasonable. A change in feed rate affects the alumina concentration, which affects the current efficiency, which affects the rate at which the metal pad rises, which affects the ACD. Further, detailed study of the Virtpot source code and A.Wright's dissertation [7] leads to the same conclusions.

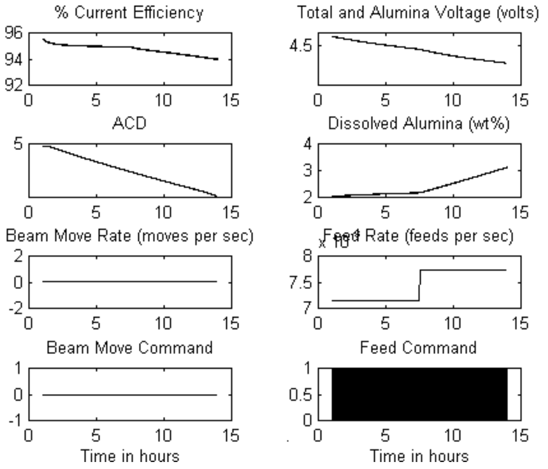

Figure 2: Open-loop step response - feed rate.

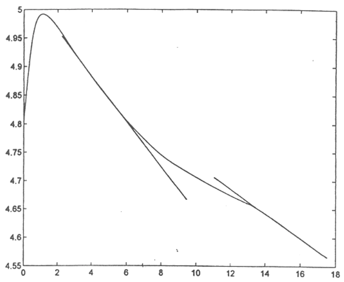

Figure 3: Zoom-in on ACD - open-loop feed rate step response.

Next, Figure 4 shows the same type of test where now the feed rate is held constant and the beam move rate (moves/second) is doubled from its initial value at seven hours (an error in the data collection resulted in the beam move and feed rates not being recorded correctly in the figure, though the actual feed and move commands are shown properly). In this figure we again see that from the perspective of primary effects, our original model is qualitatively correct. The change in beam move rate has no apparent effect on the concentration of dissolved alumina, while a change in the ACD slope is observed. Further, Figure 4 shows that the change in ACD slope is not instantaneous. That is, there seems to be a relaxation effect or time constant associated with ACD. Although this could be attributed to a type of quantization error associated with the fact that we are not moving the beam continuously, but only in discrete quantities, other plots not shown here are additionally revealing, leading to the conclusion that there is in fact a relaxation effect from beam move rate to ACD. Finally, as in the case of feed rate effect on ACD, if we zoom in on the alumina concentration following a step change in the beam move rate, we observe a small change in alumina concentration. The logic to support the effect is also similar: a change in beam move rate affects ACD, which affects current efficiency, which affects how quickly alumina is being reduced to aluminum, which thus affects the concentration of alumina in solution.

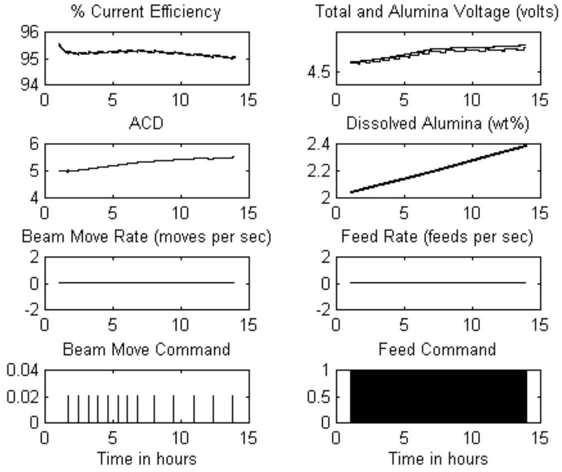

Figure 4: Open-loop test response - beam move rate.

A "Final" Model

In summary then, from the step response tests we can conclude that there are both primary and secondary effects that must be considered:

1. Feed primarily affects alumina concentration.
2. Beam movements primarily affect ACD.
3. Current affects both (a figure from this test was not shown).
4. Feed also (slightly) affects ACD.
5. Beam movements also (slightly) affect alumina.

We are thus led to argue for a second-order model structure of the form:

$$\begin{pmatrix} \dot{\mathbf{Cd}} \\ \dot{\mathbf{Acd}} \end{pmatrix} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{bmatrix} \begin{pmatrix} \mathbf{Cd} \\ \mathbf{Acd} \end{pmatrix} + \begin{bmatrix} \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix} \begin{pmatrix} \mathbf{F} \\ \mathbf{B} \end{pmatrix} + \begin{bmatrix} \mathbf{b}_{13} \\ \mathbf{b}_{23} \end{bmatrix} \mathbf{I}$$

$$\mathbf{v}(\mathbf{t}) = \begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{pmatrix} \begin{pmatrix} \mathbf{Cd} \\ \mathbf{Acd} \end{pmatrix} + \mathbf{d} \times \mathbf{I}$$

where the various variables are defined as:

Cd: concentration of alumina dissolved in solution.
Acd: anode-to-cathode distance.
F: feed rate (weight/time).
B: beam move rate (moves/time).
I: cell current.
v: cell voltage.

In this expression, the entries in the matrices, a_{ij}, b_{ij}, c_i, and d are gains to be identified from the step response tests. Some arguments can be made in this regard. There is no evidence of any relaxation effects related to alumina (we have already discounted the effect of the time constant associated with alumina going from feed to suspension to solution). Based on this, the relaxation effect related to ACD, and other reasoning from systems theory, we can assume $a_{11} = 0, a_{22} \neq 0$. The problem, however, comes in trying to properly model the off-diagonal terms. At this time the best numerical model we can give from the data, using heuristic system identification methods from the step-response simulation tests is [4]:

$$\begin{pmatrix} \dot{Cd} \\ \dot{Acd} \end{pmatrix} = \begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Cd \\ Acd \end{pmatrix} + \begin{bmatrix} 6\times10^{-2} & b_{12} \\ b_{21} & 2.717 \end{bmatrix} \begin{pmatrix} F \\ B \end{pmatrix} + \begin{bmatrix} -2\times10^{-9} \\ -1\times10^{-10} \end{bmatrix} I$$

$$v(t) = \begin{pmatrix} -0.3679 & 0.3681 \end{pmatrix} \begin{pmatrix} Cd \\ Acd \end{pmatrix} + 2\times10^{-5} I$$

So far we have not been able to produce good values for the off-diagonal terms. Likewise, a specific number for the relaxation constant a_{22} has not been determined. Nevertheless, it is knowledge of the structure of the model (i.e., where the zeros are) that is of primary importance in developing controllers, and, as we discuss in the next section, the fact that there are non-zero off-diagonal and relaxation terms in the model is very important.

Control-Motivated Analysis of the Process Model

Model Analysis

Consider again the general model suggested above, keeping in mind that $a_{11} = 0, a_{22} \neq 0$. For this model analysis shows:

1. *Stability*: If a_{22} is negative and the off-diagonal terms in the *A* matrix are non-zero, then the system is stable. Note that the relaxation effect observed in the previous section implies that a_{22} is indeed non-zero and negative. Also, process knowledge and Virtpot simulation suggests that the system is actually unstable, because we know that there are cases where we have a bounded input (i.e., the current, the feed rate, and the beam move rate are all fixed to a constant) yet the output and states begin to grow unbounded. See Figure 2 or Figure 4, for example. If the system is in fact unstable, this implies that either $a_{12} = 0$, $a_{21} = 0$, or both.
2. *State Controllability*: If both inputs are considered the system can be shown to be completely state controllable. Controllability from only feed or only beam movement depends on the specific values of the entries in the matrices *A* and *B*. So far we have been able to conclude that the diagonal elements of the *B* matrix and a_{22} are non-zero. If all other entries in the *A* and *B* matrixes are zero then the system is not controllable from only one of the inputs. If some of these entries are non-zero, the controllability of the system depends on the specific element, but in general it would appear that the system is, in principle, state controllable from either input alone. Unfortunately, in practice, the impact of feed rate on ACD and beam move rate on alumina concentration is so slight as to make the system efectively not state controllable from a single input.
3. *Observability*: As long as any of the elements a_{ij} in the *A* matrix given above are non-zero then the system is observable.
4. *Output Controllability*: There are several cases to consider. If only one output is used it can be shown that it is not possible to stabilize using constant gain feedback. Rather, a dynamical controller is required and it must be carefully designed. In particular, a proportional-integral-derivative (PID) control is not sufficient. Instead, a lead-lag or lag-lead compensator structure is necessary. On the other hand, if we use feedback from two inputs, it is possible to stabilize using both constant gain or PI control. Again, however, careful design is required.
5. *Tracking and Disturbance Rejection*: Although the open-loop system contains a free integrator, it turns out that constant gain feedback is not sufficient to ensure tracking, due to the presence of the current signal. Rather, at a minimum the controller must contain an integrator to obtain zero steady-state error. It can be shown that the system as modelled can be stabilized and gives zero steady-state error and disturbance rejection using PI controllers if feedback from both inputs is used.

Primary and Secondary Effects

Analysis of the model implies that it is probably possible to control the system from a single output. However, through a series of simulation experiments attempting to control the process using either feed only or beam movements only, we have found it difficult to control from a single input even in simulation [4]. Space limitations prohibit presentation or discussion of these results, but we can conclude that the process is not easily stabilizable from a single input (because of the slight effects of feed rate on ACD and beam move rate on alumina concentration) and we note that from an operational perspective this is not unexpected. One reason that the difficulty in controlling from a single input is not unexpected is because of an intuitive understanding of the process and how it behaves. In particular, it is clear that there are different time scales associated with primary and secondary effects. For the aluminum reduction process we can classify the primary and secondary effects based on relative gains as follows (as seen in the open-loop step response tests):

	Feed Rate	Beam Move Rate
Effect on alumina concentration	**Primary**	Secondary or none
Effect on ACD	Secondary or none	*Primary*

We can also note the time scales associated with changes in alumina concentration and ACD as follows (as seen in the open-loop step response tests):

	Alumina Conc.	ACD
Rate of change	**Fast**	*Slow*

Following the bold-face indicators, these classifications point out that **in the short term, changes in measured voltage (resistance) will be primarily due to changes in alumina concentration and not due to changes in ACD.** But, alumina concentration is primarily affected by the feed rate input. The important deduction one can draw from this chain of implications is that **cell control should focus on feed rate adjustment rather than beam movement as the primarily means of voltage control**. Of course, this does not imply that beam movement should be ignored, but simply that the primary voltage control strategy should be focused on feed rate adjustment rather than beam movement.

Implications for Control

Based on analysis of the process model, simulations, and consideration of primary and secondary effects, several alternative control strategies could be suggested:

1. Make a more concerted effort to estimate an accurate model and then attempt to control the system using just one input (beam movement or feed rate). For the reasons given in the previous subsections, we do not like this approach. In particular, this approach using beam movement as the primary control seems intuitively incorrect. It does not seem reasonable to expect to control alumina concentration by moving the beam.
2. Switching back and forth between beam and feed control, with a primary control strategy of adjusting beam movement based on voltage measurements and periodically adjusting the alumina concentration. This is typical of industrial practice for aluminum reduction.
3. Analogous to the previous suggestion, one could use a feedback loop that adjusted feed rate based on observed changes in voltage. Then one could make periodic adjustments to the beam position. The multivariable strategy presented in the next section uses this idea as a basis, but adds the concept of also adjusting the interval between beam moves.
4. Use a multivariable control scheme to adjust both the beam move rate and the feed rate simultaneously based on observed changes in cell voltage. Recall that we have noted that this type of strategy can both stabilize the system and ensure steady-state tracking and disturbance rejection using only PI controllers.

A Specific Multivariable Control Scheme

In this section we present a multivariable, feed-based approach to control of the aluminum reduction cell. In our controller we introduce the idea of a "feed voltage," which is determined by subtracting expected voltage deviations due to ACD changes and beam moves from a filtered voltage. Thus, the controller has essentially acted as an estimator, by estimating the contribution of the voltage that is due to alumina concentration. Feed rate is adjusted to compensate for changes in this feed voltage signal from a desired setpoint and beam movements are based on a beam move rate designed to compensate for the difference between anode consumption and metal pad movement. The beam move rate is adjusted to compensate for changes in feed period. The proposed control strategy is motivated by two key observations from the previous sections:

1. The process is both controllable and observable, but it is not easily stabilizable from a single input. This implies the **need for a multivariable control strategy** that makes decisions about beam move rate and feed rate simultaneously, based on measurements of cell voltage.
2. In the short term, changes in measured voltage will be primarily due to changes in alumina concentration and not due to changes in ACD. This implies that cell **control should focus on feed rate adjustment rather than beam movement**.

Multivariable Feedback Controller

Figure 5 shows a conceptual block diagram of the proposed multivariable control scheme. The essential feature of the control strategy is the computation of what we call a "feed voltage," denoted as *Vfeed* in the figure. This voltage is determined by discounting the effect of both beam moves and expected ACD variation from the measured voltage. As shown in the block diagram, the computation of *Vfeed* is implemented in three stages:

1. First, the measured voltage is filtered. It should also be noted at this point that in an actual implementation of the strategy one would expect to work with cell pseudo-resistance and not directly with voltage. In the simulations shown here, however, we simply use voltage because Virtpot has a

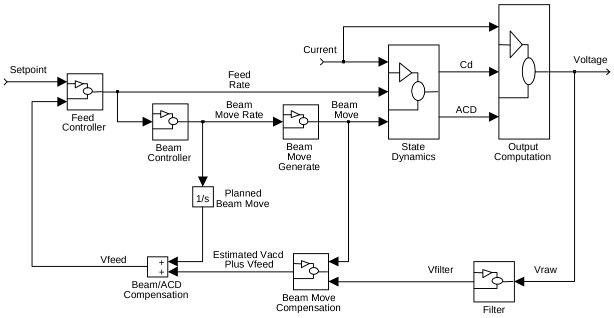

Figure 5: Conceptual view of the proposed controller.

constant current with no noise. Thus there is no reason not to just use voltage.

2. Next, the voltage change resulting from any beam moves are subtracted from the filtered voltage.
3. Third, *Vfeed* is computed by continuously adjusting the filtered, beam move- compensated voltage by the integral of the beam move rate. This action compensates for expected ACD variation during the time between beam moves and is necessary because we can not move the beam continuously. We must wait until there has been a sufficient variation in ACD to warrant a beam change. However, there is no reason to delay accounting for the expected ACD variation.

Descriptions of key processing logic blocks in Figure 5 are as follows (note that the blocks labeled “State Dynamics” and “Output Computation” are simply our models of the process used in the simulation and would be replaced by the real cell in an implementation):

Feed Controller: The feed controller adjusts the feed rate in order to force the feed voltage, *Vfeed*, to track a desired setpoint. In our simulations we used a proportional-integral (PI) controller to compute necessary changes in the feed rate.

Beam Controller: The beam controller acts to force changes in the feed rate to be zero. The logic motivating this idea is quite intuitive. Suppose the beam move rate is too low, so that the ACD is decreasing. The feed controller will interpret the voltage decrease as an overfeeding situation and will thus decrease the feed rate. This decrease in feed rate will be observed by the beam controller, which will increase the beam move rate. This will then cause the voltage variation due to ACD to reduce, which will then cause the feed rate to stop decreasing. Theoretical analysis of the model shows that if all the controller gains are chosen so that the system is stable, the beam move rate and the feed rate will come to a “happy medium” in which all signals are bounded.

Filter: The filter used for the measured voltage (resistance) should low-pass filter out variations due to the feeding process. In our simulations the steady-state feed period was about 140 secs/feed. Thus the filter should have a bandwidth of less than 1/140 Hz so the controllers won’t see the up and down voltage variations due to feeding at discrete intervals.

Beam Move Compensate: This block has a simple logic associated with it. Whenever a beam move is commanded it functions to eliminate the effect of that beam move on the feed voltage. The following pseudo-code indicates the logic to be used:

```
If  BeamMove is commanded
        Set VfilterPrevious = Vfilter
        Start counting
End
If Count  >  filter settling time
        DeltaBeam = DeltaBeam + (Vfilter - VfilterPrevious)
End
EstimatedVacd+Vfeed  =  Vfilter -  DeltaBeam
```

Beam/ACD Compensation: This block simply adds the integral of the beam rate to the output from the Beam Move Compensate block. Any type of integration routine will probably be suitable.

Beam Move Generate: This block produces an output whenever the estimated ACD variation is greater than a specified threshold. In the Matlab/Simulink environment this is implemented via a reset integrator that integrates the beam move rate starting from zero, resets to zero when the threshold is reached, and then begins integrating again.

Feed Controller: The feed controller implements a simple PI algorithm which has the setpoint error as its input and produces a correction to the feed period. This correction is added to the nominal feed period. There should also be appropriate limits placed on these values during normal operation. For instance, in our simulations we placed limits to make sure we never fed more often than every 100 seconds. This is necessary because of the nonlinearities of the process.

Beam Controller: The beam controller has been implemented in a number of ways in the various experiments we have considered. In the simulations presented below the strategy shown in Figure 6 has been used. The input to the controller is the feed period (or, feed rate can be used). A derivative is used to find the change in the feed rate. This derivative is filtered and then a saturation limiter is used. This prevents the beam move rate from changing due to rapid changes in the feed period. Such rapid changes can occur when there are disruptions to the alumina feeding, such as a clogged feeder or changes in the alumina properties, for example. We want to ensure that these effects are handled by the feed controller and not by the beam controller.

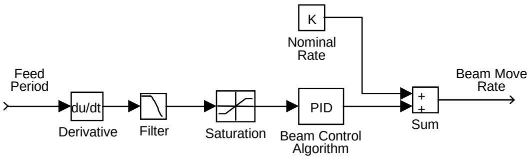

Figure 6: Beam controller.

Noise-Free Simulations

Figure 7 shows a representative simulation of the multivariable control strategy described in the previous subsection. The feed voltage setpoint was set at 4.55 volts. The feed controller used a PI algorithm with $P = -0.3179$ and $I = -0.000266$. The beam controller used only a P-type algorithm with $P = 7.14 \times 10^{-10}$. Note that the signs of these gains reflect the fact that we wrote our simulation to deal with feed rate for feed control, but feed period for beam control. Thus, if the voltage is too low, the feed controller sees a positive error, which is multiplied by a negative gain to give rise to a smaller feed rate, which in turn results in a higher feed period, which leads to a higher voltage. Likewise, when the beam controller sees an increase in feed period, it assumes this to be due to the voltage being too low (thus period is increasing) and it compensates by multiplying this positive change in feed period by a positive gain, thus increasing the beam move rate so as to increase the voltage. It can be seen from the figure that the beam rate and feed rate both adjust to a constant and the ACD and alumina concentration move to constants.

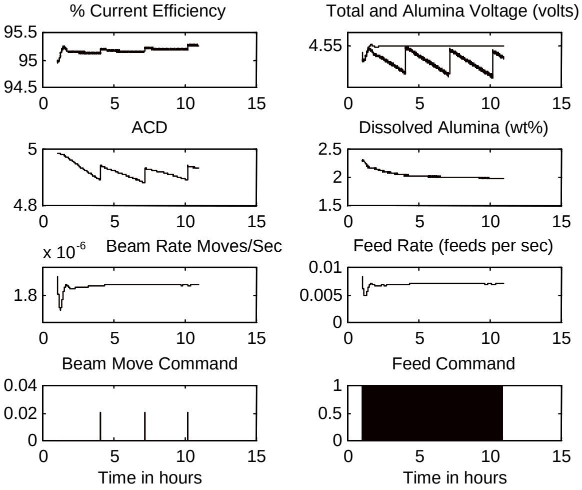

Figure 7: Representative simulation - no noise.

Disturbances, Noise, Set, and Tap

We have also developed methods for dealing with cell control to handle the effects of disturbances, noise, set, and tap and we have simulated the effectiveness of these methods using Virtpot. Space limitation prohibit including these results here, but we briefly note that disturbances and noise effects are readily handled by the multivariable control scheme we have presented. However, it should be stressed that it is necessary to distinguish between normal conditions and exception conditions (such as anode effect, set, and tap) and it is necessary to properly switch between these two types of conditions. The overriding concept is that when returning from an exception to a normal condition it is imperative that all the controller inputs, outputs, errors, and gains be reset or returned to the condition they were in before the exception occurred.

Concluding Comments

In this paper we have presented some conclusions from our study of the aluminum reduction process and its control. First, an appropriate model of the process was identified, taken primarily from the literature and from simulations using Virtpot, the dynamic model developed at Kaiser Aluminum's CFT. Second, the process and its model were analyzed using step response tests with Virtpot. Key observations were that (1) the process is both controllable and observable, but it is not easily stabilizable from a single input, which implies the need for a multivariable control strategy that makes decisions about beam move rate and feed rate simultaneously, based on measurements of cell voltage; and (2) in the short term, changes in measured voltage will be primarily due to changes in alumina concentration and not due to changes in ACD, which implies that cell control should focus on feed rate adjustment rather than beam movement. Third, we developed the conceptual design of a specific control strategy for the aluminum reduction process that uses a multivariable, feed-based approach in which feed rate is adjusted to compensate for changes in a feed voltage signal from a desired setpoint using a proportional-integral controller. In addition, beam movements are based on a beam move rate designed to compensate for the difference in anode consumption and metal pad movement. The beam move rate is adjusted to compensate for changes in feed period. These adjustments are also made with a proportional integral controller. Future work will focus on improving and validating the results presented here. Also, the work here does not consider the effect of changes current efficiency resulting from current variation or changes in voltage setpoint. This is also a subject for further research.

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