

A framework for inferring field-scale rock physics relationships through numerical simulation

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[1] Rock physics attempts to relate the geophysical response of a rock to geologic properties of interest, such as porosity, lithology, and fluid content. The geophysical properties estimated by field-scale surveys, however, are impacted by additional factors, such as complex averaging of heterogeneity at the scale of the survey and artifacts introduced through data inversion, that are not addressed by traditional approaches to rock physics. We account for these field-scale factors by creating numerical analogs to geophysical surveys via Monte Carlo simulation. The analogs are used to develop field-scale rock physics relationships that are appropriate for transforming the geophysical properties estimated from a survey into geologic properties. We demonstrate the technique using a synthetic example where radar tomography is used to estimate water content. **Citation:** Moysey, S., K. Singha, and R. Knight (2005), A framework for inferring field-scale rock physics relationships through numerical simulation, *Geophys. Res. Lett.*, 32, L08304, doi:10.1029/2004GL022152.

1. Introduction

[2] Geophysical methods are important tools for characterizing the properties of the subsurface, particularly when direct data are sparse or unavailable. In these investigations, rock physics is a key step to obtaining quantitative estimates of geologic properties of interest, such as porosity, water content or contaminant concentration, by establishing a relationship between geologic and geophysical properties. While rock physics relationships can often be determined in the laboratory, the challenge is determining valid relationships to use at the field-scale.

[3] In Figure 1 we define the core scale and the field scale as two distinct scales of interest in determining rock physics relationships; the example shown is for the relationship between water content and dielectric constant. At the core scale we assume that we can establish a relationship between geologic and geophysical properties, typically through laboratory studies. The field scale is defined by the volume over which geophysical properties are obtained from a field survey. This is the scale where an appropriate rock physics relationship must be found to obtain accurate estimates of geologic properties from the geophysical data. Determining the valid field-scale relationship is difficult

because it can be both scale-dependent and vary spatially throughout a study area.

[4] *Chan and Knight* [1999, 2001] and *Moysey and Knight* [2004] explored the scale dependence of the relationship between water content and dielectric constant for layered and spatially correlated media, respectively. The authors used ray theory and effective medium theory to demonstrate that spatial heterogeneity below the scale of an individual geophysical measurement has a significant effect on rock physics relationships. Because these effects depend on the details of the heterogeneity, this could cause the relationship to vary at a given location as the scale of the measurement increased from core scale to field scale, or to vary spatially as regions with different forms of heterogeneity are sampled.

[5] In a true field experiment, the geophysical properties determined at the field scale involve much more than is captured in the physics of upscaling by ray or effective medium theory. Many aspects of a field experiment and the way in which the geophysical data are processed and inverted affect the values of the geophysical properties determined. As a result, these factors also affect the relationship between geophysical and geologic properties. *Day-Lewis and Lane* [2004] and F. D. Day-Lewis et al. (On the limitations of applying petrophysical models to tomograms: A comparison of correlation loss for cross-hole electrical-resistivity and radar tomography, submitted to *Journal of Geophysical Research*, 2005) illustrated this problem for tomographic surveys using random-field averaging. In their work, these authors analytically showed how the spatial resolution of a tomography experiment affects the correlation between geologic and geophysical variables. *Moysey and Knight* [2003] and *Singha and Moysey* [2004] tackled this problem by taking a numerical approach that allowed them to show how a number of factors, including geologic heterogeneity, survey design, and the inversion of the geophysical data, can all impact rock physics relationships. Both the analytical and numerical studies indicated that field-scale rock relationships could vary with spatial location.

[6] In this paper, we build on these previous studies to develop a methodology for determining field scale rock physics relationships that could account for all factors influencing the link between the field-scale geologic property of interest and the field-scale geophysical property measured. Although we will describe this methodology within the context of GPR tomography, we emphasize to the reader that the approach is general and could be applied to many different geophysical problems.

2. Method for Building Field-Scale Rock Physics Relationships

[7] Our technique for determining field-scale rock physics relationships uses Monte Carlo simulations to generate a set

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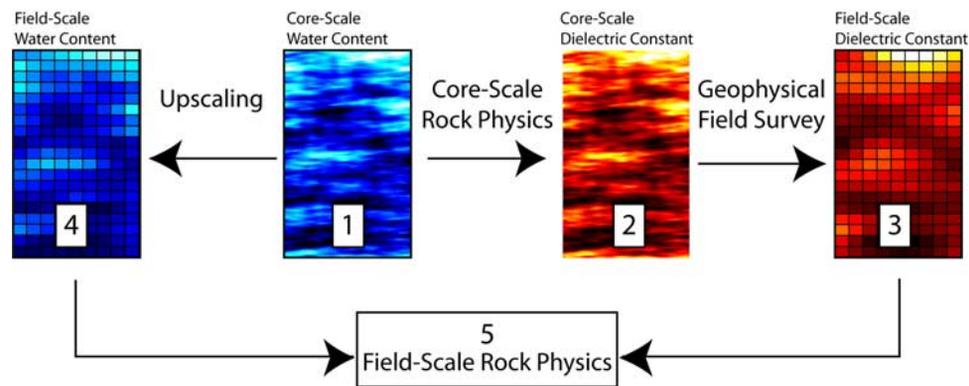


Figure 1. Conceptual schematic showing the relation between geophysical properties (dielectric constant) and geologic properties (water content) at the field and core scale. Numbers refer to the steps taken to generate a field-scale rock physics relationship as described in the text.

of site-specific numerical analogs of the geophysical survey. The analogs account for core-scale rock properties, the sampling physics of the geophysical measurements, survey design, and choices related to the inversion of the survey data. The procedure we use for creating the analogs can be described in five steps, each of which is also shown for a single analog model in Figure 1.

[8] 1) *Core-Scale Property Simulation*: A set of realizations of the geologic property of interest is created at the core scale using a technique that honors both the available data and conceptual model for the field site, e.g., geostatistical simulation.

[9] 2) *Core-Scale Rock Physics and Geophysical Properties*: Site-specific laboratory measurements and/or rock physics theories are used to determine relationships between the geologic and geophysical properties under investigation. Geophysical property realizations can then be obtained from the geologic property realizations generated in step 1 using the core-scale rock physics relationship.

[10] 3) *Generation of Field-Scale Geophysical Analogs*: Creating field-scale analogs of the geophysical properties from the core-scale simulations follows a two-part process – forward and inverse modeling – that emulates the field experiment and recovery of the geophysical property values.

[11] a. *Forward Modeling* – A numerical analog to the experiment executed in the field is performed on each geophysical property realization. The numerical experiment should parallel as closely as possible the real field experiment in both experimental design (e.g. survey geometry, data acquisition parameters) and representation of the relevant physical processes.

[12] b. *Inverse Modeling* – The synthetic observations obtained through forward modeling in the previous step are inverted. The inversion of the synthetic data should mimic the inversion of the real field data (acquired in the real field experiment), including the parameterization (e.g., model grid) and selection of regularization criteria.

[13] 4) *Generation of Field-Scale Geologic Analogs*: Each geologic property realization is upscaled from the core scale to the field scale using an appropriate spatial weighting function (Figure 1). This step can be carried out using volumetric averaging if the geologic properties of interest are volumetric properties, e.g., water content.

[14] 5) *Field-Scale Rock Physics*: The sets of field-scale analogs are used to infer a relationship between the geophysical and geologic properties at each model cell. These relationships can then be used to post-process the real-world geophysical properties to obtain an estimate of the geologic properties for the actual field site.

[15] Both error and uncertainty can be addressed within each step of our Monte Carlo framework. For example, the impact of alternative conceptual models of spatial heterogeneity (Step 1), uncertainty in the core-scale rock physics (Step 2), systematic or random errors in antennae position (Step 3), or simplifying physical assumptions made in the inversion (e.g., straight ray paths) could all be investigated. Additionally, the field-scale rock physics relationship obtained in Step 5 could be modeled either deterministically or stochastically, e.g., as a joint probability density function. The reader should recognize, however, that if a large degree of error or uncertainty exists in any of the steps, e.g., if the appropriate model for spatial heterogeneity were unknown or chosen incorrectly, then the resulting field-scale rock physics relationships could likewise have large uncertainties associated with them or could potentially yield misleading results.

[16] We note that our technique can be integrated with work being pursued in statistical rock physics [Mukerji *et al.*, 2001], which is also aimed at improving field-scale estimates of rock physics relationships, but does not address field-scale issues such as sampling physics or inversion artifacts.

3. Radar Tomography Example

[17] Although the concepts discussed in this paper are applicable to general geophysical estimation problems, we demonstrate our calibration technique for a specific synthetic example where radar tomography is used to estimate a spatially heterogeneous distribution of water content.

3.1. Reference Problem Description

[18] The reference water content distribution, which represents the ‘real’ field site in this example, is shown in Figure 2a. This model was generated using SGSim [Deutsch and Journel, 1998] as an unconditional realization of a

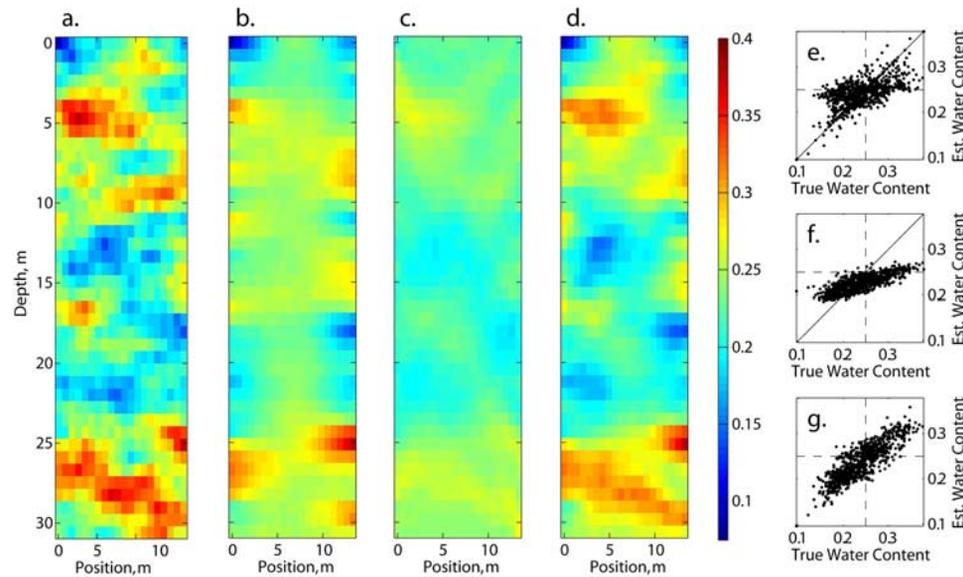


Figure 2. (a) True field-scale water content. Estimated water content distributions obtained using: (b) the Topp equation, (c) numerical analogs, (d) average of water content realizations. Plots of true vs. estimated water content illustrate the accuracy of the estimates obtained using: (e) the Topp equation, (f) numerical analogs, (g) average of water content realizations; accurate estimates fall along the 1:1 line (solid). The dashed lines indicate a water content of 0.25 (vol/vol), which is the mean of the reference field.

Gaussian random field with mean water content of 0.25 (vol./vol.), standard deviation of 0.05 (vol./vol.), and a spherical variogram (horizontal range = 8.4 m, vertical range = 2.8 m). The size of the model domain is 14 m \times 31.4 m with a 48 cm cell spacing (i.e., 29×65 cells). Dielectric constants, κ , for the reference case were calculated from the water content, θ , at each cell using the Topp equation [Topp *et al.*, 1980]. Electromagnetic wave velocity, v , was then determined from the dielectric constants (i.e., $v = c_0/(\kappa)^{1/2}$, $c_0 = 3 \times 10^8$ m/s).

[19] The radar tomography survey was simulated for this velocity model by calculating the fastest possible travel time between source and receiver positions in boreholes located on either side of the model domain ($x = 0$ and 14 m); travel time calculations were performed using a numerical solution of the eikonal equation based on the work of Sethian and Popovici [1999]. Source and receiver locations were positioned at 1 m intervals in the boreholes, resulting in a total of 961 travel time measurements.

[20] The travel time data were inverted assuming straight-rays between source and receiver locations. The inversion was performed using damped-least squares [e.g., Menke, 1989] with a regularization constraint that penalizes deviations of the estimated velocities away from the mean slowness ($1/v$). To simulate the differences in scale that occur in the field, we chose to perform the inversion using a coarser grid (20×40 cells) than the reference model (29×65 cells).

3.2. Estimating Water Content

[21] Geostatistical interpolation (e.g., kriging) can be used to estimate water content values between the well locations without using the geophysical data. We approximated the kriging estimate by averaging 50 conditional water content realizations (Figure 2b). Because the realiza-

tions are conditioned to data at the well locations (i.e., $x = 0$ and 14 m) the accuracy of the water content estimates is best there and decreases toward the center of the model, where they revert to the a priori mean value for the domain. As a result, the large-scale patterns of heterogeneity across the model are not captured by kriging.

[22] Radar tomography does a much better job of capturing the continuity of the large-scale heterogeneities. However, the high and low values of the field-scale water content estimated using the Topp equation (Figure 2c) are damped compared to the true values (Figure 2a), even though the Topp equation was the core-scale rock physics relationship used to create the reference case. This occurs because the inversion was designed to penalize large variations from the mean of the estimated slowness. In addition, a bias is introduced toward higher velocities, i.e., lower water contents, because the travel times we record represent the fastest path through the medium. Although this bias is relatively small in Figure 2f (the mean water content is underestimated by 0.01 (vol./vol.)), in experiments where we increased the variance of the water content for the reference field we have found that the bias becomes significant; the mean water content was underestimated by 0.1 (vol./vol.) when the variance of the water content field was increased to 0.15 (vol./vol.). Because effects like preferential sampling along ray paths and inversion artifacts are a consequence of the global properties of the experiment, they cannot be addressed by traditional rock physics approaches, which focus on the local properties of a medium.

[23] Figure 2d shows the water contents estimated with field-scale rock physics relationships determined using the analog approach described earlier. Specifically, the analog models in this example were created by generating 50 realizations of core-scale water content conditional to known values at the well locations (i.e., $x = 0$ and 14 m). The

realizations were simulated using the same model of spatial variability, i.e., variogram, as in the reference case. The water content values in these realizations were transformed to dielectric constants using the Topp equation. Each dielectric constant realization was then used to forward model the geophysical survey, as described for the reference case. The data for each realization were inverted to obtain apparent dielectric constants on the same coarse grid used for the inversion of the reference case, while water contents were upscaled to this grid by arithmetic averaging. At every stage in the generation of the analogs, we used the same parameters and methods to perform the simulations as for the reference case. Finally, to determine the field-scale rock physics relationships from the analogs we fit a linear relationship to each set of 50 dielectric constant and water content pairs, one pair from each realization, at every spatial location in the domain. At each point in space, the resulting relation for that location was then used to transform the dielectric constant to water content for the reference case, i.e., the 'real' field data, the results of which are shown in Figure 2d. The field-scale rock physics relationships yield an estimate that better reproduces both the large-scale patterns of heterogeneity and local magnitudes of water content than either of the other two methods (i.e., the Topp equation or kriging). The local accuracy of the water content estimates can be quantified by the root mean square error (RMSE). The RMSE of the water content estimates obtained using the field-scale approach (0.025 vol./vol.) is significantly lower than that obtained with either of the other two approaches, both of which have an RMSE of 0.039 (vol./vol.).

4. Discussion and Conclusions

[24] We have outlined an approach that uses numerical analogs to derive rock physics relationships accounting for both rock properties and additional factors influencing field-scale geophysical estimates. A major advantage of the technique is that it takes into account the spatial variability of these factors by performing calibrations as a function of spatial location. Additionally, the approach is very flexible, as it is based on Monte Carlo simulation, and can be used to investigate field-scale rock physics relationships for a wide array of problems. Overall, our approach is similar to traditional rock physics in that it attempts to locally relate geophysical and geologic properties, but differs since our approach also depends on global properties of the field experiment.

[25] There are some similarities between the mechanics of our approach and Monte Carlo based inversion. However, the goal of inversion, to select realizations with predictions that best match the true field data, is different from our goal, which is to produce field-scale relationships between geophysical properties and other properties of interest. In both cases, the ability to account for complex scenarios and non-linear behaviors are significant strengths over comparable methods. However, because Monte Carlo inversion techniques determine the goodness of each realization using a global measure, i.e., data misfit, a disadvantage of these inverse methods is that they can potentially require thousands of realizations to obtain a satisfactory result [Peck *et al.*, 1988]. In contrast, the local, spatially

variable relationships produced using the field-scale rock physics approach described here can capture the local impact of global processes in a relatively low number of realizations. As a result, our approach to field-scale rock physics, coupled with an appropriate inversion scheme, makes it possible to draw upon the strengths of a Monte Carlo inversion while reducing the computational effort.

[26] We have presented an example where using field-scale rock physics relationships lowered the RMSE of the water contents estimated by a radar tomography survey by 35% compared to the use of traditional rock physics or geostatistics (i.e., kriging). Our intention in this paper, however, is not to suggest that the proposed field-scale rock physics technique will always improve estimation accuracy. Instead, we wish to illustrate the principle that numerical analogs can be useful tools for investigating relationships between field-scale variables. This calibration technique is not a replacement for inversion, but rather a supplement that improves the local accuracy of geologic property estimates obtained using geophysical methods.

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