Comment on “Elastic wave propagation in a solid layer with laser-induced point defects” [J. Appl. Phys. 110, 064906 (2011)]

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The title problem concerns waves in an isotropic solid in which there are atomic point defects. The density of the defects is \( n(\mathbf{r}, t) \), where \( \mathbf{r} = (x_1, x_2, x_3) \) is a point in the solid. (Although Ref. 1 starts with two kinds of defects, most of the analysis is restricted to one type.) The constitutive relation between the stresses \( \sigma_{ij} \), \( n \), and the displacement components, \( u_i(\mathbf{r}, t) \) \((i = 1, 2, 3) \), is

\[
\sigma_{ij} = \lambda \delta_{ij} \Delta + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \vartheta_d \hbar \delta_{ij},
\]

where \( \lambda \) and \( \mu \) are Lamé moduli, \( \Delta = \frac{\partial u_i}{\partial x_i} \) is the dilatation (with the usual summation convention), and the constant \( \vartheta_d \) controls the strain-defect interaction. We note, in passing, that Eq. (1) has the same structure as the constitutive relation for thermoelasticity, with \( n \) playing the role of temperature.

The governing equations of motion are

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad i = 1, 2, 3,
\]

\[
\frac{\partial n}{\partial t} = -\frac{\partial Q_i}{\partial x_i} + g - \gamma n.
\]

Here, \( \rho \) is the mass density, and \( g \) and \( \gamma \) are nonlinear functions of the dilatation,

\[
g = \mathcal{G} \exp(\vartheta_d \Delta/k_T), \quad \gamma = \tau^{-1} \exp(\vartheta_m \Delta/k_T),
\]

where \( \mathcal{G}, \vartheta_d, k_T = k_B T, \tau \), and \( \vartheta_m \) are constants. (In another paper, Mirzade\(^4\) has considered a simpler problem, with \( g = \mathcal{G} \) and \( \gamma = \tau^{-1} \).) The defect flux has components \( Q_i \) given by

\[
Q_i = -D \frac{\partial n}{\partial x_i} + v_i n,
\]

where \( D \) is a diffusion constant and the components of the defect-drift velocity are (see above Eq. (3) in Ref. 1 or above Eq. (4) in Ref. 2)

\[
v_i = \frac{D}{k_T} F_i = -\frac{D}{k_T} \frac{\partial U_{int}}{\partial x_i} = \frac{D}{k_T} \frac{\partial \Delta}{\partial x_i}.
\]

Thus,

\[
\frac{\partial Q_i}{\partial x_i} = -D \nabla^2 n + \frac{D}{k_T} \frac{\partial}{\partial x_i} \left( n \frac{\partial \Delta}{\partial x_i} \right).
\]

To make progress, Mirzade linearizes Eq. (3). Thus, assume small strains and put \( n = n_0(x, y, z) + N_1(x, y, z, t) \) with \( |N_1|/n_0 \ll 1 \). For Eq. (2) to be satisfied at leading order, we must have \( n_0 = \mathcal{G} \). Then, from Eq. (3) at leading order, we obtain \( n_0 = \mathcal{G} \).

At next order, Eqs. (1) and (2) give

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x_i} + \mu \nabla^2 u_i - \vartheta_d \frac{\partial n_0}{\partial x_i}, \quad i = 1, 2, 3.
\]

From Eqs. (3), (4), and (6), we obtain

\[
\frac{\partial n_0}{\partial t} = g_e \Delta - \tilde{g}_d \nabla^2 \Delta + D \nabla^2 n_1 - \tau^{-1} n_1,
\]

where \( \tilde{g}_d = D n_0 \vartheta_d/k_T \) and \( g_e = \mathcal{G} (\vartheta_d - \vartheta_m)/k_T \). Equation (8) should be compared with Eq. (9) in Ref. 1. Notationally, our \( g_e \) and \( \tilde{g}_d \) are Mirzade’s \( g \) and \( d \), respectively. As \( g_e \) and \( \tilde{g}_d \) have different dimensions, for later use, we define

\[
\tilde{g}_d = \mathcal{G} \vartheta_d/k_T \quad \text{giving} \quad \tilde{g}_d = D \vartheta_d.
\]

Using our notation, Mirzade’s equation (9) has \( \gamma \) instead of the constant \( \tau^{-1} \); see Eq. (4). For a consistent linearization, the approximation \( \gamma \approx \tau^{-1} \) should be used.

**PLANE WAVES**

Mirzade\(^1\) considers waves in a layer, in a half-space, and in an unbounded space. Here, we focus on the simplest problem of determining plane waves in an unbounded space. Mirzade introduces various potentials; we bypass this step. Thus, we try \( u = \mathcal{R} \{ A \mathcal{E} \} \) and \( n_1 = \mathcal{R} \{ N \mathcal{E} \} \) with \( \mathcal{E} = \exp \{ i(\mathbf{K} \cdot \mathbf{r} - \omega t) \} \). The constant vectors \( A \) and \( \mathbf{K} \) are allowed to be complex: they are *bivectors*.\(^4\) Also, \( \mathcal{N} \) is a complex constant. We have \( \Delta = \mathcal{R} \{ i(\mathbf{A} \cdot \mathbf{K}) \mathcal{E} \} \) and \( \nabla^2 \mathcal{E} = -q^2 \mathcal{E} \), where

\[
q^2 = \mathbf{K} \cdot \mathbf{K} = K_x^2 + K_y^2 + K_z^2
\]

\[
= K^+ \cdot K^+ - K^- \cdot K^- + 2i K^+ \cdot K^-.
\]
and we have written $K = (K_1, K_2, K_3) = K^+ + iK^-$ (see p. 16 of Ref. 4). We have used the notation $q^2$ so that we can compare with Ref. 1, but we emphasise that $q^2$ is complex unless $K$ is real ($K = K^+$, $K^- = 0$). Substitution in Eqs. (7) and (8) gives
\[-\rho \omega^2 A = -(\lambda + \mu)(A \cdot K)K - \mu q^2 A - i\partial_x N K,\]
\[-i\omega N = -Dq^2 N + i(g_e + g_d q^2)(A \cdot K) - \tau^{-1} N.\]

We seek non-trivial solutions of this system. Simplify the notation by putting $X = \mu q^2 - \rho \omega^2$, $L = \lambda + \mu$, $G = g_e + g_d q^2$, and $H = i\omega - \tau^{-1} - Dq^2$. Then, the system becomes
\[L(A \cdot K)K + XA + i\partial_x N K = 0,\]
\[iG(A \cdot K) + HN = 0.\]

Write this system in matrix form as $C \mathbf{x} = 0$ with $\mathbf{x}^T = (N, A_1, A_2, A_3)$ and
\[C = \begin{pmatrix} H & iG K_1 & iG K_2 & iG K_3 \\ i\partial_x K_1 & X + L K_1^2 & L K_1 K_2 & L K_1 K_3 \\ i\partial_x K_2 & L K_1 K_2 & X + L K_2^2 & L K_2 K_3 \\ i\partial_x K_3 & L K_1 K_3 & L K_2 K_3 & X + L K_3^2 \end{pmatrix}.\]

Direct calculation gives
\[\det C = X^2 \Lambda \quad \text{with} \quad \Lambda = H(X + L q^2) + \partial_q G q^2.\]

Allowable solutions follow by setting $\det C = 0$. Thus, $X = 0$ or $\Lambda = 0$. The first of these gives $\omega^2 = c_T^2 q^2$, where $c_T^2 = \mu / \rho$ and $c_T$ is the speed of transverse (shear) waves in an isotropic elastic solid: such waves propagate independently of any atomic point defects. This result was found by Mirzade; see Eq. (25) in Ref. 1.

The second option, $\Lambda = 0$, gives
\[[(\lambda + 2\mu)q^2 - \rho \omega^2](Dq^2 + \tau^{-1} - i\omega) - \partial_q g^2 (g_e + g_d q^2) = 0.\]

We compare this with Mirzade’s equation (25b). Thus, introduce a length $\ell$ defined by $D\tau = \ell^2$ and let $c_L^2 = (\lambda + 2\mu) / \rho$ so that $c_L$ is the speed of longitudinal (compressional) waves in an isotropic elastic solid. In addition, introduce two independent dimensionless parameters, $\delta_c$ and $\delta_d$, defined by (recall Eq. (9))
\[\delta_c = \frac{\partial_d g e}{\lambda + 2\mu} \quad \text{and} \quad \delta_d = \frac{\partial_d g d}{\lambda + 2\mu}.\]

Mirzade’s $\delta$ is our $\delta_c$; see below Eq. (19) in Ref. 1. Then, Eq. (10) becomes
\[(q^2 - \omega^2 c_L^{-2})(q^2 + (1 + i\omega)\ell^{-2}) - \delta_c \ell^{-2} q^2 - \delta_d q^4 = 0.\]

This should be compared with Eq. (25b) in Ref. 1, namely,
\[(q^2 - \omega^2 c_L^{-2})(q^2 + (1 + i\omega)\ell^{-2}) - \delta_c \ell^{-2} q^2 = 0.\]

The difference between $(1 - i\omega)$ in Eq. (12) and $(1 + i\omega)$ in Eq. (13) is simply due to us assuming $e^{-i\omega t}$ and Mirzade taking $e^{i\omega t}$. However, the most striking difference is the absence of the last term in Eq. (12). This error can be traced to Eq. (13) in Ref. 1, where a term proportional to $V^2 g$ has been omitted. This omission implies that much of the analysis and computation in Ref. 1 for layers and half-spaces will require correction.

One could regard Eq. (13) as a special case of Eq. (12), obtained by putting $\delta_d = 0$. However, this case is not very interesting because it implies that $\delta_d = 0$, which means that there is no strain-defect interaction; see Eq. (7). In addition, $\delta_d = 0$ implies that $\delta_c = 0$ (see Eq. (11)), in which case Eq. (12) factors.

Mirzade also gives a perturbation analysis of Eq. (13) in which it is assumed that $\delta_c \ll 1$. One could presumably give a similar analysis of Eq. (12), but this would require both $\delta_c \ll 1$ and $\delta_d \ll 1$.

Further analysis of the dispersion relation Eq. (12) could be interesting. It can be regarded as a cubic equation for $\omega$, given $q$, or as a quadratic equation for $q^2$, given the frequency $\omega$. (Recall that $q^2$ need not be real.) It is noted that the case $\delta_d = 1$ is special because Eq. (12) contains a term $(1 - \delta_d) q^4$.

3Equation (4) in Ref. 1 and Eq. (3b) in Ref. 2 give $-c_{\rho l}$ in Eq. (5), but these are typographical errors, F. Mirzade, private communication (2011).