Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles

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CORRECTIONS AND ADDITIONS

June 13, 2024 `msbook-add.tex`

Corrections

p. 34. Example 2.5. Insert $(-1)^{n-1}$ on right-hand side, giving

$$\left(\frac{\partial}{\partial x} \pm \mathrm{i}\frac{\partial}{\partial y}\right)^n \log r = (-2)^{n-1}(n-1)!\,\frac{\mathrm{e}^{\pm \mathrm{i}n\theta}}{r^n}, \quad n = 1, 2, \ldots.$$

Error found by A. J. Yuffa.

p. 80. There is a short paragraph about the axisymmetric addition theorem:

> The special case of Theorem 3.15 in which $z_0 = j_0$ was published by Clebsch in 1863; see [M1298, p. 363]. According to Watson, the special case with $z_0 = y_0$ is due to Gegenbauer: it is formula (3) on p. 365 of [M1298] with $\nu = \frac{1}{2}$.

This is not quite right. Replace by:

> Theorem 3.15 was proved by Clebsch [106, p. 227] in 1863.

Gegenbauer gave a more general result. Email exchanges with A. F. Rawle encouraged me to look at Clebsch's paper.

p. 81. Missing $(-1)^l$ in (3.67). It should read

$$B_l^{(n)} \equiv B_l^{n,0} = \frac{(-1)^l\,(2n-2l)!}{2^n l!\,(n-l)!\,(n-2l)!}. \tag{3.67}$$

This agrees with $B_l^{n,m}$ defined by (3.27).

p. 133. The formula (4.45) is correct but the derivation is not. For a proof, see [113, Theorem 2.16]. The same criticism applies to formula (4.10) on p. 124.

p. 209. Below (6.91), the specification "where H has the following properties ... for some $Q \in B_\mathrm{e}$" is inadequate. Change to:

> where sufficient conditions on H are: for every $q \in S$, $H(P;q)$ is an outgoing wavefunction for all $P \in B_\mathrm{e}$ whereas $H(P;q)$ must have singularities at some points $P \in B$.

p. 213. Remove overbars in (6.108), giving

$$[U, V] = \int_{\Omega_a} \left(U\frac{\partial V}{\partial r} - V\frac{\partial U}{\partial r}\right) \mathrm{d}s = -[V, U]. \tag{6.108}$$

Error found by M. Ganesh.

p. 269. Theorem 7.4. The second property, (7.68), holds for lossless scatterers. The proof sketched assumes that

$$\int_S \left(u\frac{\partial \overline{\tilde{u}}}{\partial n} - \overline{\tilde{u}}\,\frac{\partial u}{\partial n}\right) \mathrm{d}s = 0.$$

p. 275. Remove hat from $\hat{\psi}_\nu^\mu$ in last displayed equation at bottom of page, giving

$$\psi_n^m(\boldsymbol{r}_j^P) = \sum_{\nu,\mu} \widehat{S}_{n\nu}^{m\mu}(-\boldsymbol{b}_j)\psi_\nu^\mu(\boldsymbol{r}_P), \quad j = 1, 2.$$

Error found by K. Spalding.

p. 303. At the end of the displayed equation below (8.72), replace $h_2(kR_{rj})$ by $h_2(kR_{nj})$.

p. 324. Two lines below (A.8), replace "positive" by "non-negative".

p. 362. Reference 72. It should be F.G. Bass, not F. Bass. ([71] is correct.)

p. 363. Reference 103. Remove space: should be unidirectional, not uni directional.

p. 368. Reference 212. It should be pages T14–T25, not T11–T25.

p. 378. Reference 436. Change **[241]** to [241].

p. 379. Reference 459. It should be volume **14**, not **18**.

p. 398. Reference 896. Remove space: should be electromagnetic, not electro magnetic.

p. 406. Reference 1067. It should be C: p. 202, not C: pp. 202, 202.

p. 407. Reference 1089. Hyphen missing: should be A.-R. Sebak.

p. 411. Reference 1176. Remove one *PCPS*.

Citation index. Achenbach [2]–[7] (not [3]–[7]); Banerjee [60] (omitted); Gumerov [447]–[450] (not [447]–[449]); Hasheminejad [484]–[486] (not [484, 485, 486]); Rodríguez-Zúñiga (not Rodrí guez-Zúñiga).

Several extraneous square brackets, such as Achenbach 623], [1275, which should be 623, 1275. Similarly: Barber 685, 1074; Mackowski 867, 868; Mal 768, 1351; Martin 730, 750; Pao 1251, 1260; Sabina 1042, 1120; Truell 1185, 1297.

1.1. What is 'multiple scattering'?

[Just above §1.1.1, following on from "process."] Stokes gave an earlier description, in a letter to J. Herschel in 1856, and quoted by James [224, p. 90]:

> *Suppose a number of ships moored together, or simply placed near each other, on a perfectly still ocean, the assemblage extending in two directions, and not being merely a line. Imagine a series of waves now to be propagated along the surface of the ocean. These would set the ships swinging in an irregular manner, and would then pass on. The ships would of course act as a sort of breakwater, so that as the series passed on the water would be comparatively calm behind them. The ships, however, having been set swinging, would in turn become centres of disturbance, and waves would spread from them in all directions.*

1.1.1.2. Independent scattering

Bottom of p. 2. There have been further attempts to quantify when single scattering and independent scattering are reasonable assumptions. See, for example, [370, 367].

1.1.3. Self-consistent methods

Second paragraph below (1.7), after "see [M687, §III]", insert: See also [M416, eqn (253)].

1.2. Narrowing the scope

[Beginning of §1.2, p.10]:

> *The phenomenon of multiple scattering is frequently encountered and is conventionally treated in a great variety of ways.* (Goldberger & Watson [M416, p. 750])

[End of first paragraph]: Introductory overviews include [350, 351]. For electromagnetic scattering, see [368].

There is a 1968 book by Ivanov, with a 557-page English translation in 1970 [219]; the Russian author is Evgeniĭ Ivanov, cited as both E. A. Ivanov and Ye. A. Ivanov. The book describes methods based on separation of variables and addition theorems (see Chapter 4) for two scatterers: circles, ellipses, spheres, spheroids and circular discs.

1.3.1. The Sommerfeld radiation condition

The two-dimensional condition, (1.25), was used by Kupradze [271, §3] to establish uniqueness; he called it the 'emission principle'.

1.3.3. Boundary conditions

[Top of p. 14, last sentence in §1.3.2, amended]:
For more information, including uniqueness theorems, see [M657] and [523].

1.5.1. The Kupradze radiation conditions

First sentence, insert word giving 'Sommerfeld radiation condition to'

1.5.3. [Elastic] transmission problems

"For more information on [fluid-solid interactions]...", add [202, 214]

1.5.7. Flexural waves in plates [New section]

For thin elastic plates, the simplest approximation is known as Kirchhoff theory, in which flexural waves are modelled using $(\nabla^4 - k^4)w = 0$, where $w(x, y)$ is the out-of-plane displacement of the plate, ∇^2 is the two-dimensional Laplacian and k is a wavenumber. For background, see Kitahara's book [M619]. Scattering problems can be formulated: the scatterers could be holes or inclusions of various kinds. There are more refined theories (such as Mindlin plates) but most work on scattering problems has used Kirchhoff theory.

2.4. Wavefunctions

[p. 35, below (2.11)]: For an application of $\mathcal{T}_m$, see [118, eqn (3.10)].

2.9. Elliptical wavefunctions

[End of §2.9, p. 49, just before §2.9.1]: See also [59].

2.9.1. Addition theorem [for elliptical wavefunctions]

[p. 53, just before §2.10]: See also [219, Chapter 4, §5].

2.11. Multipoles for water waves

[p. 55, below (2.81)]: "For more information on multipole potentials ..." Add [142].

3.1. Introduction [to 3D addition theorems]

For an overview, covering scalar and vector additions theorems, see [194].

3.3. Legendre's addition theorem

[p. 66, end of §3.3]: Putting $\hat{\boldsymbol{r}}_1 = \hat{\boldsymbol{r}}_2 = \hat{\boldsymbol{r}}$, say, in (3.20), and using $P_n(1) = 1$, we obtain

$$\sum_{m=-n}^{n} |Y_n^m(\hat{\boldsymbol{r}})|^2 = \frac{2n+1}{4\pi}.$$

The fact that the left-hand side of this formula is independent of $\hat{\boldsymbol{r}}$ is sometimes known as Unsöld's theorem [506, p. 379]; see also [404, eqn (21–16)].

3.5. Hobson's theorem

[p. 68, below (3.29)]: For a lengthy proof of (3.29), see [554, Appendix A].

3.6. Wavefunctions and the operator $\mathcal{Y}_n^m$

[p. 71, 5 lines from bottom, after "$\ldots \mathcal{P}_n^m u = \mathrm{i}^n \mathcal{Y}_n^m u$."] In detail, using (A.2),

$$\mathcal{P}_n^0 = P_n(\mathrm{i}\mathcal{D}_1^0) = P_n\left(\frac{1}{\mathrm{i}k}\frac{\partial}{\partial z}\right),$$

whence $\mathcal{P}_n^0 z_0(kr) = \mathrm{i}^n\, z_n(kr) P_n(\cos\theta)$. In fact, this formula is stated (without proof) in a 1912 paper by Sudhansukumar (S.K.) Banerji [35, p. 8].

3.11. Addition theorem for $j_n Y_n^m$

[End of part (v), amend last sentence]: For direct proofs, see [M318, Theorem 3] and [476, §IV].

3.12. Addition theorem for $h_n^{(1)} Y_n^m$

[p. 90, 5 lines below (3.87), after "§1.2."]: See also [219, Chapter 3, §7].

[Top of p. 91, line 5, after "For direct proofs ..."]: add [412].

In [139], the authors find $\widehat{S}_{n\nu}^{m\mu}$ by letting $r_1 \to \infty$ in (3.85), using (3.90), $h_n^{(1)}(w) \sim \mathrm{i}^{-n-1} w^{-1} \mathrm{e}^{\mathrm{i}w}$ as $w \to \infty$. This gives

$$\sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \widehat{S}_{n\nu}^{m\mu}(\boldsymbol{b})\, \mathrm{i}^{-\nu} Y_\nu^\mu(\hat{\boldsymbol{r}}_1) = \lim_{r_1 \to \infty} \left(\mathrm{i}^{-n} \frac{r_1}{r_2}\, \mathrm{e}^{\mathrm{i}k(r_2 - r_1)} Y_n^m(\hat{\boldsymbol{r}}_2) \right).$$

But $r_2 = |\boldsymbol{r}_1 + \boldsymbol{b}| \sim r_1 + \boldsymbol{b} \cdot \hat{\boldsymbol{r}}_1$ and $\hat{\boldsymbol{r}}_2 = \boldsymbol{r}_2 / r_2 \sim \hat{\boldsymbol{r}}_1$ as $r_1 \to \infty$ so that the limit is $\mathrm{i}^{-n} \mathrm{e}^{\mathrm{i}k\boldsymbol{b}\cdot\hat{\boldsymbol{r}}_1} Y_n^m(\hat{\boldsymbol{r}}_1)$. Use of orthogonality, (3.9), then gives (3.79).

3.13.1. Computation of the separation matrices

Bottom of p. 93, "See, for example": add [452, 548]. For a short review, see [463].

Top of p. 94: For more on recursive methods, see [250].

3.14. Two-centre expansions

p. 96. Theorem 3.30 can be found in [544, eqn (2)], together with related expansions. For an earlier study, see [M919, §5].

3.17. Vector spherical wavefunctions

Bottom of p. 107, "For alternative proofs...", add: [497], [189, Appendix A3], [M196, Appendix D], [51, §1.8], [100, 139, 563, 330], [268, §7.4.2]. Wittmann [M1332] has defined and exploited a vector analogue of the operator $\mathcal{Y}_n^m$ (Definition 3.9).

3.18. Multipoles for water waves

[p. 110, above Theorem 3.40]: "For more information on three-dimensional multipole ..." Add [142].

Chapter 4

> *For* [single] *spheres and circular cylinders the solutions are well known, and even the scattering problem has answers which must suffer the contempt bred by familiarity.*
>
> (Morse & Feshbach [M884, p. 1513])

4.5.1. Literature [on acoustic Záviška]

For two cylinders..., add [216, 140, 372] and [219, Chapter 6]

For more than two cylinders ... from the 1970s: [200]; ... from the 1980s: [32, 47]; ... from the 1990s: [145, 335]; ... from the 2000s: [528, 526, 505, 443, 31, 327]; ... from the 2010s: [387, 24, 450, 510, 128, 366, 453, 386]; ... since 2020: [114, 566, 565].

For circles centred at the vertices of concentric regular polygons, see [225].

Chamberlain [81] has combined Záviška's method with a mild-slope equation inside each cylinder so as to model the scattering of water waves by various circular structures. Random water waves are considered in [230].

Záviška's method has been used to study the interaction of SH waves with tunnels. The earliest paper in this context is by Lee & Trifunac from 1979 [284]. They consider one circular tunnel near the flat boundary of a half-plane, which is equivalent to two circles in a full space. For more tunnels (three in a full space and two in a half-plane), see [559]. Extension to lined tunnels has been made. Each tunnel is circular with a concentric lining in which a different Helmholtz equation holds. There is a Neumann condition on the inner boundary and transmission conditions at the outer boundary. For two lined tunnels in a half-plane, see [34].

[Top of p. 130, discussion of Grote & Kirsch [M444]]. For applications, numerical analysis and generalizations, see [4, 5, 108, 265, 357].

Circular scatterers in a layered medium have been considered in many papers, mainly in the context of electromagnetic waves. A typical problem is two half-spaces with waves in both but circular cylinders in one. For example, see [52, 283, 130, 397, 163, 406, 280, 281, 282]. For cylinders in both half-spaces, see [407]. There are analogous treatments for problems involving a slab sandwiched between two half-spaces. For example, see [278, 161, 162, 164, 275]. The paper by Lai *et al.* [275] also gives results for non-circular cylinders using a T-matrix method.

[End of §4.5.1, p. 130, on orders-of scattering]. For other applications, see [144, 316, 486, 550].

4.5.3. Numerical solution of the system (4.27)

[Top of p. 132, end of first paragraph]. For another investigation into the choice of M, see [405]. For errors incurred by truncating the infinite series in Graf's addition theorem, see [362] and [69, Lemma 9].

Zhang & Li [567] combined (4.27), Theorem 2.7 and a two-dimensional form of the Fast Multipole Method (Section 6.14). Results for up to 2200 cylinders are given. Amirkulova & Norris [15] have exploited the algebraic structure of (4.27) when the cylinders are identical and arranged in a rectangular array, leading to a fast recursive algorithm. Results for up to 1000 cylinders are given when the distance between the centres of adjacent circular cross-sections is $2.02a$.

For additional papers dedicated to numerical aspects, see [22, 490, 432, 193]. For a detailed numerical analysis, see [159].

4.5.4. Three-dimensional problems [New section]

Suppose we have a configuration of parallel cylinders, with generators parallel to the z-axis. Thus far, we have considered incident fields that do not depend on z, but three-dimensional incident fields can be of interest.

The simplest situations occur when we have a plane wave at oblique incidence,

$$u_{\rm inc}(x,y,z) = \exp\{{\rm i}(\ell_1 x + \ell_2 y + \ell_3 z)\} \quad \text{with} \quad k^2 = \ell_1^2 + \ell_2^2 + \ell_3^2.$$

Then, assuming the boundary conditions do not depend on z, we can write

$$u_{\rm sc}(x,y,z) = v(x,y)\,{\rm e}^{{\rm i}\ell_3 z},$$

where v solves the two-dimensional Helmholtz equation in a cross-sectional plane, $(\nabla^2 + \ell^2)v = 0$, with $\ell^2 = \ell_1^2 + \ell_2^2$, together with boundary and radiation conditions. For circular cylinders, the two-dimensional boundary-value problem for v can be solved by multipole methods. (For non-circular cylinders, v can be calculated using methods described later in the book.) Some of the papers cited in Section 4.5.1 use Záviška's method to find v for plane waves at oblique incidence; see, for example, [M1367] and [M700]. See also [279].

Similar methods can be developed for cylinders in a waveguide, with infinite plane walls at $z = \pm h$, say. See [501, 63, 394] and references therein. For other axisymmetric structures, see [220]. There are also related water-wave problems for vertical cylinders and water of constant finite depth (Section 4.11). For example, in [41], the authors develop a hybrid method, combining an indirect integral equation method (Section 5.9) with modal expansions.

When the incident wave is not plane, the natural approach is to use a Fourier transform in the z-direction. This decomposes the problem into a superposition of cross-sectional problems. For incident waves generated by a point source, we can effect the decomposition using Weyrich's formula ([M278], §2.9.4). See also [M133], eqn (2.9). For a point source and one circular cylinder, see the 1955 paper by Wait [524]. Wait has discussed analogous electromagnetic problems in his book [525]. For acoustic problems with many circular cylinders perpendicular to an impenetrable plane, see [511]. For an acoustic point source and an elastic cylinder, see [306]. For an analogous problem with elastic waves generated by a point force near a cylindrical cavity, see [53]. For two cylinders, see [M38] and [M1164].

For a point source in a waveguide, with infinite plane walls at $z = \pm h$, modal decompositions can be used, with propagating and evanescent modes. (See Section 4.11.1 for such a decomposition in the context of water waves.) Results for acoustic scattering by four penetrable cylinders are given in [63].

4.6.2 Literature [on scattering by one sphere]

[p. 134, end of first paragraph]: Rayleigh's 1872 paper on scattering by a sphere [480] treats several problems: scattering of a plane wave by a rigid sphere, by a fluid-filled sphere (transmission problem) and by a movable sphere. He also indicates how to proceed when the incident waves are generated by a point source.

[Top of p. 135]: "For further results, see [M561]": add [157, 448]

[p. 135, end of §4.6.2]: For scattering by one sphere in a tube, see [346] and references therein. For two spheres in a tube, see [269].

4.7.1. Literature [on multipole method for two spheres]

[p. 137, first paragraph]: In his 1912 paper, Banerji [35] attempted an orders-of-scattering calculation for a vibrating sphere in the presence of a fixed rigid sphere. Sircar extended this approach in 1929, replacing the rigid sphere by a rigid spheroid [470].

[p. 137, 2nd paragraph, with discussion of Marnevskaya]: At about the same time, Ivanov [219, Chapter 7, §3] made an extensive study of two-sphere problems.

[p. 137, 2nd paragraph, line 8]: "method of images ... M391]" add [468].

In [435], the authors consider scattering by two spheres when one of the spheres is small ($ka \ll 1$). In [422], the authors consider a radiation problem for two spheres with emphasis on computing the (nonlinear) radiation forces. For acoustic scattering by two spheres, with many numerical results and associated software, see [434, Chapter 4].

[The last paragraph, from "In [880]" to "for 162 cylinders are given" has been absorbed into the (new) Section 4.13; see below.]

4.8.3. Literature [on multipole method for N spheres]

p. 139. For scattering by a few hard spheres, see [13]. For computational aspects, with results for 100 spheres and $ka = \frac{1}{10}\pi$, see [251].

4.9. Electromagnetic waves [by separation of variables]

[End of first paragraph on p. 140, after "by Mie [M849, M717]."]: See also the book edited by Hergert & Wriedt [197].

[3rd paragraph on p. 140, after "see also [M117, §4.2]."]: add [51, §5.2]. [End of 3rd paragraph, add] For a very detailed study of scattering by a layered sphere, see [462]. For anisotropic spheres, see [268, Chapter 8].

"Many papers on electromagnetic scattering by clusters of spheres ..." (p. 141): add [182, 21, 188, 437, 137, 369, 55, 308, 58, 307, 40],

Also, add: For a good overview, including computational aspects, see [330]. For a 'tutorial', see [160]. For scattering of waveguide modes by several spheres inside a tube (or optical fibre), see [103].

[New paragraph, end of §4.9]:

Electromagnetic scattering by spheres near the boundary of a (penetrable) dielectric half-space has been studied by many authors, perhaps starting with the paper by Bobbert & Vlieger [46] on one sphere. For multiple spheres, see [51, Chapter 6], [332]. For scattering by a sphere near a cylinder, using an orders-of-scattering approach, see [445].

4.10. Elastic waves [by separation of variables]

[First sentence]: add reference to Clebsch [106].

[p. 141, 4th paragraph]: "Elastic waves exterior to single circular cylinders...": add [319], [509, §2.2]. [End of paragraph]: "... Veksler [M1261]": add [509, §2.1]. For acoustic scattering by a radially inhomogeneous elastic sphere, see [493]. For layered spheres, see [515, 516].

Bottom of p. 141, "The analogous problems for incident shear waves": add [222], [509, §2.3].

Top of p. 142, "Movable rigid spheres are considered in [M1333, M943]": add [221]. Same paragraph, add: For a review, see [30].

3rd paragraph on p. 142, after "Petrashen and his students": add [263].

[After "see also [M421].", middle of 4th paragraph on p. 142]: Avilés & Sánchez-Sesma [32] gave results for scattering of P-waves and S-waves by a row of eight rigid movable cylinders.

[End of 5th paragraph]: "later review [M459]." Add: see also [98].

Moore & Guan [374] have considered elastodynamic problems for two lined tunnels in an unbounded elastic space. They treated the problem using an orders-of-scattering approach. This approach was used earlier for two circular cavities by Sancar & Pao [441], with extensive numerical results and comparisons

with experiments in a companion paper by Sancar & Sachse [442]. The twin-lined-tunnel problem with three-dimensional loadings has also been studied: Kuo *et al.* [270] use orders-of-scattering whereas Yuan *et al.* [555, 556, 557, 558] give a complete analysis. Transient plane-strain problems with two identical circular cavities have been solved by Chen & Zhang [93] and by Itou [218]. They combine symmetry, separated solutions and Fourier integrals (addition theorems are not used); the resulting method is complicated and special. The method of [M103] has been extended to plane-strain problems [481], to flexural waves in plates [534] and to Mindlin plates [253].

For scattering by hexagonal arrangements of circular inclusions with imperfect interfaces, see [533]. For two circular defects where surface elasticity is included, see [529, 396].

[p. 143, end of second paragraph]: Doyle has developed [M292] into a full paper [136] on scattering of elastic waves by spheres.

[p. 143, last paragraph of §4.10]: "For scattering by one sphere ..." add [80]. "For axisymmetric ... see [M484]." For axisymmetric scattering by two spheres (representing bubbles), see [508]. "For scattering by one poro-elastic sphere, see [M93, M1380]": add [317]. For Záviška-like studies of circular cavities in an unbounded poroelastic space, see [192, 66].

For two elliptical cavities in an unbounded poroelastic space, there are two papers [532, 578] in which circular-cylindrical wavefunctions are used. This is expected to be valid for low eccentricities, otherwise the Rayleigh hypothesis will be relevant.

4.11. Water waves [by multipoles]

[p. 146, end of 2nd paragraph]: Multipole methods for oblique incidence on several horizontal parallel circular cylinders, submerged in water of finite depth, have been used by Shen *et al.* [458]; similar methods have been used for two-layer fluids [156].

[p. 146, end of 3rd paragraph]: See also [86, §5.9]. For N movable spheres in water of constant finite depth, see [298]. For several porous hemispheres (known as 'reef balls') on the bottom of finite-depth water, see [299].

4.11.2. Matched eigenfunction expansions

[First paragraph, after "another expansion of some kind."] In [1], the authors try to represent the enclosed bodies using the 'method of fundamental solutions', as described in Section 7.3.2.

[Bottom of p. 147 and first paragraph at top of p. 148, with insertions]: "McIver [M828] ... pair of immersed rectangular cylinders"; see also [552]. "For a pair of thin vertical barriers, see [M982]." See also [461]; this paper includes comparisons with experiments. "For a vertical stack of aligned horizontal plates, see [M1278]." For a pair of truncated vertical circular cylinders, see [576]. There is an earlier paper by Silva *et al.* [467] in which evanescent modes are neglected. For several truncated vertical circular cylinders, see [464, 101, 541, 560, 87, 88, 570] and [86, §2.9]. For several *submerged* truncated vertical circular cylinders, see [353]. For a pair of truncated vertical elliptical cylinders, using Mathieu functions (defined in Section 4.12.1), see [85]. For several floating elastic plates, see [261, 260]. For floating elastic discs, see [373, 571]. For several thin vertical porous barriers, see [244, 245]. For horizontal porous barriers, see [102].

[New paragraph, remainder of first paragraph on p. 148]: "In [M927, M815] ... orders-of-scattering ... bodies ... are given." The combination of matched eigenfunction expansions with an orders-of-scattering approach has been revived and applied in the context of arrays of wave-energy devices [262, 564].

[End of §4.11.2, p. 148]: after "scattering by N rectangular grooves." Interest in problems of this kind has increased; see [530, 33, 304, 305, 546]. See also [149, Chapter 1]. For scattering by N holes drilled through a thick screen, see [400].

[End of §4.11.2, p. 148, new paragraph]: In [315], the authors use (4.67) for a vertical cylinder of non-circular cross-section. Their direct application of the boundary condition presupposes that the Rayleigh hypothesis is valid; see p. 249 and [342]. This limitation on the geometry is not noted. Nevertheless, extension to multiple vertical truncated cylinders has been made [573, 572].

4.12.1. Elliptic cylinders

[Middle of p. 149]: after "and [M1139]." In his book on Mathieu functions, McLachlan [355, p. 3] pointed out that Sieger had 'published an important paper on the diffraction of electromagnetic waves by an elliptical cylinder' in 1908 [465]; see also [271, Chapter II] and [355, §XIX].

Then, insert, giving "see [M1321, M1375], [86, Chapter 4] and references therein."

[Bottom of p. 149]: Scattering by two elliptic cylinders was first analysed by Ivanov [219, Chapter 8, §3].

Nigsch [389] has given results for time-dependent scattering by three elliptical cylinders. Results for time-harmonic scattering by a pair of identical elliptical cylinders were given by Chatjigeorgiou & Mavrakos [90] and by Kleshchev & Kuznetsova [257]. For arrays of ellipses, see [89] and [86, §4.6]. For a combination of circular and elliptical cylinders, see [84].

4.12.2. Spheroids

[End of 2nd paragraph] ... and [M1, Chapter 21]. See also [219, Chapter 5] and [301, Chapter 2].

[3rd paragraph] See also [258, 436, 6]. For scattering by a penetrable spheroid, see [551, 264, 229]. For acoustic scattering by elastic spheroids, see [466]. For interaction of water waves with one spheroid, see [91] and [86, §§5.7 & 5.8].

[4th paragraph] ... see also [M710], [M870, §6.1] and [301, 215]. [Then, at end] For scattering of elastic waves by a spheroidal inclusion, see [233].

[5th paragraph, with insertions] "Addition theorems for spheroidal wavefunctions are available ..." Also [219, Chapter 5, §5]. Ivanov [219, Chapter 9] has given results for two co-axial spheroids including the special case of two circular discs.

[Last paragraph, with insertions] "Addition theorems for vector spheroidal wavefunctions are also available [M1106, M253]." Also: [121, 208]. "They have been used extensively for scattering by two perfectly conducting spheroids [M1107, M252], by two dielectric parallel spheroids [M234] and by spheroids that are oriented arbitrarily [M232, M233, M215]." Also: [117, 385]. "All of these results are reviewed in Chapter 4 of [M866]."

4.12.3. Ellipsoids

[Top of p. 151] "Low-frequency scattering ... [M262, Chapter 8]" Add "and in [123, Chapter 14]." For the scattering of water waves by one submerged ellipsoid, see [86, §6.16].

4.13. Expansion-collocation methods [New section]

The scattered field exterior to N circular cylinders can be represented as (4.24). Truncating this expansion gives the approximation

$$u_{\rm sc}(\boldsymbol{r}) \simeq \sum_{j=1}^{N} \sum_{m=-M}^{M} c_m^j \psi_m(\boldsymbol{r}_j)$$

containing $Q = (2M+1)N$ coefficients. To determine these, we may apply the boundary condition at a total of P points on the cylinders, with $P \geq Q$; when $P > Q$, the overdetermined system may be solved in a least-squares sense. This basic method is usually known as 'point-matching'; there is an extensive literature on its implementation and limitations [M76, M625, M213]. Note that for multiple-scattering problems, point-matching yields a numerical method that avoids addition theorems but one whose conditioning should be monitored carefully. For some generalisations of this method, see Section 7.3.2.

The basic method, as described above, was used in [M103], where results for 162 cylinders are given. See also [290, 293, 477].

For scattering by two spheres, using a truncated form of (4.51), see [M880]. For application to a few hard spheres, see [286].

For elliptic cylinders, outgoing elliptical wavefunctions $\psi_{\sigma n}$ (see Definition 2.15) can be used. Lee [285] gave numerical results for scattering by two and three ellipses. For the degenerate case of thin strips, see [110]. For several spheroids, using outgoing spheroidal wavefunctions, see [291, 292].

5.4.4. Spectral properties [New section]

When S is a single circle of radius a, we can take $x(t) = a\cos 2\pi t$ and $y(t) = a\sin 2\pi t$. Then, all the boundary integral operators defined above have eigenfunctions $\mathrm{e}^{2\pi \mathrm{i} n t}$ and their eigenvalues can be calculated explicitly in terms of Bessel and Hankel functions. Of course, this follows because separation of variables is available. These spectral properties can be used to estimate condition numbers, for example. For detailed calculations, see Kitahara's book [M619, Chapter 4], [M659], [266], [M23] and [517]; these four papers also give analogous results for one sphere. For one ellipse, using Mathieu functions, see [44].

Antoine & Thierry [25, 489] have studied the spectral properties of the single-layer operator when S consists of N circles.

5.9.1. Literature [on indirect method]

[Bottom of p. 171]: Prior to Kupradze, in 1910, Poincaré used (5.50) and derived (5.51); see [414, p. 181 and §4]. He was interested in electromagnetic scattering by a sphere, with the incident field generated by a point dipole, this being a contemporary problem of direct relevance to the understanding of long-distance radio wave propagation. For an analysis of Poincaré's long paper [414], see [415]. Kupradze's work is also available in the translation [271, §3].

[End of 2nd paragraph on p. 172, "the indirect method has also been used in"]: add [365, 364, 39]. Domínguez *et al.* [135] have developed an effective method for solving the two-dimensional form of (5.54); they give numerical results for a pair of ellipses. For scattering by thousands of three-dimensional sound-hard obstacles, using (5.51), see [170].

[End of 3rd paragraph on p. 172]: Single-layer potentials have also been used for two-dimensional mixture problems, with a combination of soft, hard and penetrable cylinders [427].

5.10.4. Literature [on direct method]

[First paragraph on p. 178, line 9, insert giving]: "Linton & Evans [M726]; see also [178]."

[End of first paragraph on p. 178]: For mixture problems, see [523].

6.2. Transmission problems [by integral equations]

[Top of p. 180]: "Examples of other direct methods ... M470]" add [95, 138]. For an application to a transmission problem where the interior Helmholtz equation is replaced by Poisson's equation, see [375].

The computational performance of several formulations, both direct and indirect, has been evaluated [513], with detailed results for scattering by four spheres.

[New paragraphs, end of §6.2, p. 180]: When an obstacle has internal interfaces, essentially two situations can arise. The simpler is when the interfaces are disjoint as with an onion: for concentric circles or spheres, separated solutions can be written down for each layer, and then patched together using the transmission conditions across each interface. When the geometry is more complicated, more-or-less standard boundary integral formulations can be developed. For scattering by several layered ellipses, see [455].

Suppose now that interfaces meet. For example, consider a circular scatterer with a single internal interface along a diameter; call this a 'Janus circle' [527]. The two points at the ends of the diameter (known as 'triple points' or 'junctions') can be reached from three different subdomains (the two halves of the scatterer and the exterior) and this makes it more difficult to construct well-conditioned systems of boundary integral equations; see [523, 421, 68, 146, 184, 104, 105, 227, 196] and references therein. In particular, numerical results for scattering by a Janus circle are available [227, 196]. The difficulties at junction points have been avoided [410] by using simple boundary element methods (with piecewise-constant

approximations and collocation at element midpoints). In fact, there are several earlier papers by Ström & Zheng [479, 574, 575], [M1155, M1376] on electromagnetic scattering by objects with interior interfaces (such as a ball composed of two dissimilar halves); they use null-field methods (see Chapter 7). For acoustic scattering by Janus spheres, see [434, Chapter 3].

6.3.2. An integral representation and an integro-differential eqn

[p. 185, 3rd paragraph]: "In fact ... variational principles." A similar coupled system is derived in Kanaun's book [240, eqns (3.242) & (3.243)].

6.3.3. The Lippmann–Schwinger equation

[New paragraph, end of §6.3.3, p. 186]: The Lippmann–Schwinger equation (6.26) was derived by a direct method; the unknown w is a physical quantity, defined above (6.26). There is an indirect form, derived in [14, eqn (1.7)]. Start by writing

$$U(P) = \int_B G(P,Q)\,\varphi(Q)\,\mathrm{d}V_Q$$

where U is defined by (6.6) and φ is to be found. This gives a representation for $p_{\mathrm{sc}}(P)$ as a volume potential when $P \in B_{\mathrm{e}}$. When $P \in B$, $U = p_0 - p_{\mathrm{inc}}$ and we require $(\nabla^2 + k_0^2)p_0 = 0$ in B. Imposing this gives

$$\begin{aligned} 0 &= (\nabla^2 + k_0^2)p_{\mathrm{inc}} + (\nabla^2 + k_0^2)\int_B G(P,Q)\,\varphi(Q)\,\mathrm{d}V_Q \\ &= (k_0^2 - k^2)p_{\mathrm{inc}} + 2\varphi(P) + (k_0^2 - k^2)\int_B G(P,Q)\,\varphi(Q)\,\mathrm{d}V_Q \end{aligned}$$

using (6.8) and $(\nabla^2 + k^2)p_{\mathrm{inc}} = 0$ in B. Rearranging gives

$$\varphi(P) - \frac{1}{2}k^2\{1 - n(P)\}\int_B G(P,Q)\,\varphi(Q)\,\mathrm{d}V_Q = k^2\{1 - n(P)\}p_{\mathrm{inc}}(P).$$

This should be compared with (6.26), where $\{1 - n(Q)\}$ appears under the integral sign.

6.3.4. An alternative equation [for inhomogeneous obstacles]

Equations similar to (6.28), involving both volume and surface integrals, were derived in two dimensions by Jin *et al.* [231]; see also [522, §3.2.5]. For three-dimensional electromagnetic problems, see [521] and [522, §3.1.7].

6.4.6. Electromagnetic waves [by integral equations]

[p. 195, end of 4th paragraph, on books]: add [522].

For applications of the EFIE and the MFIE to scattering by several polygonal cylinders, see [326].

For applications of (6.34) to three-dimensional multiple scattering problems, see [168].

For applications of the EFIE to three-dimensional multiple scattering problems, see [172].

[6th paragraph, on transmission problems]: add [439, 82] to "see also [M617, M471]". For more complicated configurations of scatterers, see [553, 256].

For scattering by a dielectric sphere near a thin square plate, see [28].

For applications of volume integral equations to scattering by many spheroids, see [503, 38]. For a $20 \times 20 \times 20$ array of spheres, see [340]. For other shapes, see [94]. For reviews of the work done by Tsang and his collaborators, see [502, 500].

For inhomogeneous scatterers, boundary integral equations for the exterior region can be coupled to finite-element discretisations inside the scatterers [119].

6.5.5. Elastic waves [by integral equations]

[p. 201, after the Kupradze quotation]: For two-dimensional (plane-strain) problems, Kupradze noted much earlier (1935) that 'the integral equations have singular kernels, and the applicability of the Fredholm theory requires an additional preliminary analysis, which reduces to the regularization of the singular system of equations obtained' [271, p. 117].

[p. 201, end of 2nd paragraph]: The idea of using two scalar potentials for plane-strain elastodynamic problems has been revived by Lai & Li [276]. For example, they give results for scattering by 1000 fixed rigid obstacles.

[p. 201, end of 3rd paragraph on indirect methods]: Cavity problems have been treated by solving the hypersingular equation $\mathbf{N}\boldsymbol{f} = \boldsymbol{g}$ [36].

[p. 202, 2nd paragraph]: Numerical results for two-dimensional problems have been presented in [403]. [End of paragraph, on 3D transmission problems]: add [62].

[p. 202, end of 3rd paragraph]: "Orthotropic cylinders are considered in [M698]" and [277].

[p. 202, 4th paragraph on half-space problems]: for the indirect method [using $\mathbf{G}$], see [M1052, M752]: add [431]; for the direct method, see [M1018, M593]: add [402].

For several rigid discs attached to a half-space, see [498].

Sheng *et al.* [460] have used $\mathbf{G}$ for tunnels in a half-space. They used a hybrid method, coupling finite elements around the tunnels with a boundary integral representation for the surrounding exterior domain.

Chaillat & Bonnet [72] have shown how the computation of $\mathbf{G}^{\mathrm{H}}$ can be improved so that a version of the Fast Multipole Method is effective.

[p. 202, 5th paragraph on anisotropy]: For inhomogeneous scatterers, using volume integral equations, see [48, 49].

For collections of elastodynamic fundamental solutions, see [129, 398] and Kausel's book [246].

[Top of p. 203]: After "several reviews": add [54, 336]

For flexural waves in a thin elastic plate, we may use Kirchhoff theory. The governing partial differential equation is $(\nabla^4 - k^4)w = 0$, where $w(x, y)$ is the out-of-plane displacement and ∇^4 is the two-dimensional biharmonic operator. The relevant fundamental solution can be taken as

$$G(P, Q) = H_0^{(1)}(kR) - H_0^{(1)}(\mathrm{i}kR) = 1 + O(R^2 \log R) \quad \text{as } R \to 0.$$

The fact that G is finite at $R = 0$ can be exploited to good effect. For G itself, see [M619, p. 211], [234, eqn (7.66)], [391, eqn (28)], [151, eqn (2.3)] and [344, eqn (11)], for example. For boundary integral equations, see Kitahara's book [M619]. For scattering of flexural waves by a hole of arbitrary shape in an infinite plate, see [471]. For several circular holes and inclusions, see [287, 288, 289].

In [420], the authors attach the ends of N parallel Kirchhoff plates to the boundary of an elastic half-space; plane-strain waves interact with the plates.

6.6.3. Water waves [by integral equations]

[End of first paragraph on p. 205]: For two flat *elastic* strips, see [198].

[End of 4th paragraph on p. 205]: For applications of (6.81) to several wave-energy devices, see [150].

[Near the end of 5th paragraph on p. 205]: After "floating buoys", add: For applications of *WAMIT* to arrays of floating axisymmetric wave-energy devices, see [147, 324]. Then "For other applications of the direct method to multibody problems, see [M303, M510]": add [579, 483, 185, 568, 115].

[After citation of Chakrabarti [M181], bottom of p. 205]: Chen & Mahrenholtz [97] solved (6.83) and related integral equations for two floating cylinders in finite-depth water. [End of same paragraph] ... between two ships, see [M837, M591]: add [297, 577].

Porter [416] started with the hypersingular integral equation obtained from the normal derivative of (6.82), for two-dimensional problems and submerged cylinders. He regularised the equation by using the Cauchy–Riemann equations, leading to a Fredholm integral equation of the first kind with a symmetric weakly-singular kernel. He applied this method in a search for trapped modes above a pair of elliptical cylinders [417].

[End of last-but-one paragraph in §6.6.3, p. 206, after citations of [M506, M527, M28]]: Cho *et al.* [102] used integral equations based on a simple fundamental solution in a finite region containing two submerged inclined plates, and then matched these with eigenfunction expansions (as described in Section 4.11.2). For another application, see [499].

[End of last paragraph in §6.6.3, p. 206, after citation of [M304]]: Leonard *et al.* [294] used finite elements, truncating the computational domain with simple outgoing-wave conditions, and gave results for two half-immersed circular cylinders. See also [444, 384] for similar approaches. More recently, several authors [474, 302, 303] have used the 'scaled boundary finite element method' for two or three vertical cylinders in water of finite depth.

For a review of integral-equation methods with a focus on the simulation of wave-energy devices, see [399].

6.7. Cracks and other thin scatterers

[p. 207. End of paragraph about use of Fourier transforms]: For scattering by parallel finite cracks in a thin elastic plate over water, see [418].

6.7.1. Two-dimensional [crack] problems

[Top of p. 208] Revised paragraph, with insertions:

If there are N screens, one obtains a system of N coupled hypersingular integral equations; the hypersingularity can be treated using Chebyshev expansions. This and other methods have been used for acoustic scattering by several straight rigid strips [217], [M443, M1124, M1364], [186, 393]. Murai *et al.* [379] gave results for scattering by up to 72 identical parallel rigid strips using a piecewise-constant approximation for $u(t)$ on each strip. For two coplanar sound-soft strips, see [454]. Chebyshev expansions have been used for up to 28 curved sound-soft strips [228]. Hewett *et al.* [199] have developed a hybrid analytical-numerical method for scattering by N coplanar sound-soft strips. For sound-hard strips, see [111].

For scattering of elastic waves by several straight cracks, see [M545, M519], [562], [M1373, M1056, M548, M259]. For several straight coplanar strips, see [M1364]. For acoustic scattering by several elastic strips, see [109]. "For the interaction of water waves with thin plates ... M397]": add [175, 438, 538, 209, 210].

Analogous problems arise when water waves are scattered by thin vertical rectangular barriers in a three-dimensional ocean of constant finite depth; this is a special case of the formulation described in §1.6.4, where each vertical cylinder has no interior. Such problems have been solved using hypersingular integral equations and Chebyshev expansions in the context of wave-energy devices; see [429] for one device and [428, 447, 446] for several. The last of these discusses how machine learning may be used to optimise the layout of arrays of devices; see also [564].

For two scatterers, one can contemplate using an orders-of-scattering approach. This has been pursued by Meguid & Wang for two straight strips [358] and for two straight cracks [359].

Acoustic scattering by strips on which the boundary condition involves a fractional derivative (thus interpolating between sound-soft and sound-hard conditions) has been studied [514, 484]. A numerical method can be developed in which Gegenbauer polynomials are used instead of Chebyshev polynomials; this method has been used for scattering by two parallel strips [484].

6.7.2. Three-dimensional [crack] problems

[Top of p. 209]. For two coplanar rectangular cracks, see [314]. For scattering by a penny-shaped crack near a rigid movable disc or a spherical inclusion, see [383, 380]. Earlier, Mykhas'kiv & Khai had derived the governing integral equations for N plane cracks and then given results for two or three coplanar penny-shaped cracks [381, 382].

For scattering by two coplanar discs, using special integral-equation methods, see [112, 491, 339].

6.8. Modified integral equations: general remarks

"There is an extensive literature ... for reviews, see ... [M959, Chapter 6]": add [338, 131]. Note that [131] includes numerical results for scattering by four sound-hard circles.

6.9. Modified fundamental solutions

[p. 210, new paragraph]: The analysis in this section makes use of the bilinear expansion for $G(\boldsymbol{r}_P, \boldsymbol{r}_Q)$, as given in (6.102); this is an infinite series. It is natural to ask if G can be approximated in the separated form

$$G(\boldsymbol{r}_P, \boldsymbol{r}_Q) \simeq \sum_{m=0}^{M} f_m(\boldsymbol{r}_P)\, g_m(\boldsymbol{r}_Q),$$

where M and the functions f_m and g_m are to be chosen. For investigations in this direction, see [148].

6.10.1. A combined-layer method

Numerical solutions of (6.121) and (6.122) for two three-dimensional obstacles have been presented by Ganesh & Hawkins [166]. Subsequently, they [169], Bremer *et al.* [57] and Hao *et al.* [190] solved (6.122) for many soft obstacles.

6.10.2. The method of Burton and Miller

p. 218. "Buffa & Hiptmair [M158] first multiply (5.59) by a regularising operator ... Dirichlet problem." This idea has been developed in several subsequent papers; see, for example, [23, 61].

[Top of p. 219] "However, we are not aware of any published numerical results for multiple-scattering problems, based on (6.124)." In 2007, Shen & Liu [456] combined (6.124) with a fast multipole method (see §6.14) for scattering by many objects; see also [543] for scattering by a cube containing 27 spherical inclusions.

"The choice of η has been investigated in [M23]." For a review, see [337].

[End] "in particular, [M404] contains results for scattering by two circular cylinders." The method used in [M404] is designed for high-frequency problems and it uses an iterative orders-of-scattering approach. It has been analysed in detail [143] and it has been extended to three dimensions [20] with results given for scattering by two soft ellipsoids. Gibbs *et al.* [177] consider scattering by soft cylinders, one of which is a large convex polygon; they also start from (6.125), and they give results for one small triangle near a large triangle.

6.11. Augmentation methods

[p. 221 "... has been implemented numerically [M1158]." In [255], the authors add one null-field equation (that with $n = m = 0$ in the unnumbered equation at the top of p. 221) to (5.58) (see [255, eqn (19)]) and give some numerical results.

[p. 221 "We know of two ... [M287, M1353]." For an application to three two-dimensional scatterers, see [408].

Woodworth & Yaghjian [542] augmented using (6.127) evaluated for $P \in S'$, with S' being inside but close to S, giving what they call 'dual-surface integral equations'.

6.11.1. Extensions [Augmentation methods]

[There is an unnumbered equation on p. 221]:

$$u(P) = \int_{S \cup S'} \mu(q)\, G(P, q)\, \mathrm{d}s_q, \quad P \in B_\mathrm{e}.$$

This representation has been used by Bremer [56] for two-dimensional problems involving many non-smooth star-like scatterers.

[Bottom of p. 221]: "For more details ... see [M694]" add [239].

6.12.1. The exact Green's function

[Bottom of p. 223, after "Boley [M112]."]: For numerical solutions of (6.135), see [485, eqn (21)] and [73, eqn (13)] (where G^{E} is called a 'tailored Green's function').

6.12.2. Multiple scattering by two obstacles

[p. 224, just before §6.12.2.1] Maystre [352, eqn (44)] characterises each scatterer using an operator S_j leading to a formulation that has similarities with others discussed in §6.12.

6.12.2.1. Partitioning

Multiplying (6.137) by A_{11}^{-1} and (6.138) by A_{22}^{-1} gives an equivalent pair of equations. Thierry [488] has investigated the numerical consequences of doing this for N scatterers: one can say that the system has been *preconditioned*. He gives some numerical results for $N = 30$.

6.12.3. Generalised Born series for two scatterers

[Bottom of p. 227]: "In [M290] ... elastic half-space." For an application to scattering by two coplanar penny-shaped cracks, see [254].

"Geuzaine *et al.* [M404] ... $ka = 1000$." For two ellipsoids, see [20].

[End of §6.12.3, p. 228]: Balabane's general approach, combining N single-scatterer problems with some kind of iterative process, has been developed by several groups [176, 519, 531, 547, 518].

There are alternative interpretations. For the exterior Neumann problem with two obstacles, we can write $u(P) = u_1(P) + u_2(P)$, where

$$2u_j(P) = \int_{S_j} \Big(f_j(q) G(P,q) - u(q) \, \frac{\partial}{\partial n_q} G(P,q) \Big) \, \mathrm{d}s_q,$$

valid for all points P outside S_j (including inside the other obstacle). Thus u_j is a radiating wavefunction outside S_j. To use this formula, we need to find, or approximate, $u(q)$ for $q \in S_j$. For work in this direction, using variants of the 'on-surface radiation condition' of Kriegsmann *et al.* [267], see [2, 12, 3, 248].

Another way to break a multiple scattering problem into pieces is to use a *domain decomposition method* (DDM) [134]. In some variants, the domains are the scatterers themselves [569, 119]. For a DDM that Schwarz might recognise, see [409], where a two-dimensional cluster is enclosed by a square which is itself broken into smaller square subdomains, each containing a number of scatterers. Numerical results for up to 40,960 scatterers are given.

6.13.5. Extensions [of Twersky's method]

Kyurkchan has developed a variant of Twersky's method in several papers, some with collaborators; see, for example, [273, 272] and [274, §3.5]. Results are given for scattering by two obstacles.

6.13.6. Weyl's integral representation [new section]

Friedman & Russek [M366, eqn (11)] used a three-dimensional analogue of Sommerfeld's integral representation due to Weyl [537], namely

$$h_0(kr) = \frac{\mathrm{e}^{\mathrm{i}kr}}{\mathrm{i}kr} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{0}^{\pi/2-\mathrm{i}\infty} \mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\hat{\boldsymbol{s}}}\,\sin\alpha\,\mathrm{d}\alpha\,\mathrm{d}\beta; \tag{1}$$

see also [M1149, p. 578, eqn (26)], [M1199, eqn (8)] and [433, eqn (3.54)]. In (1), the complex vector $\hat{\boldsymbol{s}} = (\sin\alpha\cos\beta,\, \sin\alpha\sin\beta,\, \cos\alpha)$; although $\hat{\boldsymbol{s}}$ is not a unit vector in general, we do have $\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}} = 1$. Although not stated in [M366], (1) is valid for $z > 0$. To check this, we compare with a formula derived by Clemmow [107, eqn (2.120)],

$$\frac{\mathrm{i}\,\mathrm{e}^{-\mathrm{i}kr}}{kr} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}k(\ell x+my\mp z\sqrt{1-\ell^2-m^2})}\frac{\mathrm{d}\ell\,\mathrm{d}m}{\sqrt{1-\ell^2-m^2}}$$

where the upper (lower) sign is taken for $z > 0$ ($z < 0$). Noting that Clemmow uses $\mathrm{e}^{+\mathrm{i}\omega t}$, we obtain the two-dimensional Fourier integral,

$$\frac{\mathrm{e}^{\mathrm{i}kr}}{\mathrm{i}kr} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}k(\ell x+my+z\sqrt{1-\ell^2-m^2})}\frac{\mathrm{d}\ell\,\mathrm{d}m}{\sqrt{1-\ell^2-m^2}}, \quad z > 0;$$

see also [M177, pp. 338–339], [M1332, eqn (18)] and [433, eqn (3.51)]. Use plane polar coordinates for the double integral, putting $\ell = \tau\cos\beta$ and $m = \tau\sin\beta$. This gives

$$\frac{\mathrm{e}^{\mathrm{i}kr}}{\mathrm{i}kr} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{0}^{\infty} \mathrm{e}^{\mathrm{i}k(\ell x+my+z\sqrt{1-\tau^2})}\frac{\tau\,\mathrm{d}\tau\,\mathrm{d}\beta}{\sqrt{1-\tau^2}}, \quad z > 0,$$

with $\sqrt{1-\tau^2} = \mathrm{i}\sqrt{\tau^2-1}$ when $\tau > 1$, giving exponential decay with increasing z. Next, put $\tau = \sin\alpha$. This gives $\ell x + my + z\sqrt{1-\tau^2} = \boldsymbol{r}\cdot\hat{\boldsymbol{s}}$, as in (1). The interval $\tau = 0$ to $\tau = 1$ becomes $\alpha = 0$ to $\alpha = \pi/2$. For $\tau > 1$, put $\alpha = A + \mathrm{i}B$. Then $\mathrm{i}\sqrt{1-\tau^2} = \mathrm{i}\cos\alpha = \mathrm{i}\cos A\cosh B + \sin A\sinh B$. Hence, for exponential decay with z, we can take $A = \pi/2$ with B going to $-\infty$. This leads precisely to (1), with a continuous contour going from $\alpha = 0$ to $\alpha = \pi/2 - \mathrm{i}\infty$, valid for $z > 0$; see also [537, Fig. 1].

Let us apply the operator $\mathcal{Y}_n^m$ to (1). From Theorems 3.10 and 3.11, we have $\mathcal{Y}_n^m\{h_0\} = h_nY_n^m$ and $\mathcal{Y}_n^m\{\mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\hat{\boldsymbol{s}}}\} = (-\mathrm{i})^n\mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\hat{\boldsymbol{s}}}Y_n^m(\hat{\boldsymbol{s}})$. Hence

$$\psi_n^m(\boldsymbol{r}) = h_n(kr)Y_n^m(\hat{\boldsymbol{r}}) = \frac{(-\mathrm{i})^n}{2\pi}\int_{-\pi}^{\pi}\int_{0}^{\pi/2-\mathrm{i}\infty} Y_n^m(\hat{\boldsymbol{s}})\,\mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\hat{\boldsymbol{s}}}\,\sin\alpha\,\mathrm{d}\alpha\,\mathrm{d}\beta, \quad z > 0, \tag{2}$$

which agrees with [M366, eqn (15)]; see also [M1332, eqn (19)].

For the addition theorem, we assume that $(\boldsymbol{r}+\boldsymbol{b})\cdot\hat{\mathbf{z}} > 0$. Thus

$$\psi_n^m(\boldsymbol{r}+\boldsymbol{b}) = \frac{(-\mathrm{i})^n}{2\pi}\int_{-\pi}^{\pi}\int_{0}^{\pi/2-\mathrm{i}\infty} Y_n^m(\hat{\boldsymbol{s}})\,\mathrm{e}^{\mathrm{i}k(\boldsymbol{r}+\boldsymbol{b})\cdot\hat{\boldsymbol{s}}}\,\sin\alpha\,\mathrm{d}\alpha\,\mathrm{d}\beta. \tag{3}$$

Using the plane-wave expansion $\mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\hat{\boldsymbol{s}}} = 4\pi\sum_{\nu,\mu}\mathrm{i}^\nu\hat{\psi}_\nu^\mu(\boldsymbol{r})\overline{Y_\nu^\mu(\hat{\boldsymbol{s}})}$, we obtain (3.84), in which

$$S_{n\nu}^{m\mu}(\boldsymbol{b}) = 2(-1)^\mu\mathrm{i}^{\nu-n}\int_{-\pi}^{\pi}\int_{0}^{\pi/2-\mathrm{i}\infty} \mathrm{e}^{\mathrm{i}k\boldsymbol{b}\cdot\hat{\boldsymbol{s}}}\,Y_n^m(\hat{\boldsymbol{s}})Y_\nu^{-\mu}(\hat{\boldsymbol{s}})\,\sin\alpha\,\mathrm{d}\alpha\,\mathrm{d}\beta, \quad \boldsymbol{b}\cdot\hat{\mathbf{z}} > 0;$$

here, we have used (3.7). This formula for $S_{n\nu}^{m\mu}$ reduces to (3.87) after use of Theorem 3.20 and (2).

6.14. Fast multipole methods

[Bottom of p. 235]: Chapter 6 of Liu's book [318] is concerned with acoustic scattering.

6.14.2. Literature [on FMM]

[Bottom of p. 240, after "... M255, M256"]: In particular, the electromagnetic version of (6.195) can be found in [M1130, eqn (10)] (in discrete form); for an application to scattering by 56 unmanned aerial vehicles, see [545].

[Bottom of p. 241]: For applications of FMM to scattering of two-dimensional elastic waves by many circular inclusions, see [440]. For three-dimensional elastic waves, see the review [71]. For applications to multiple scattering of water waves, see [50]. For acoustic scattering by finite periodic arrays, see [226].

7.1. Introduction [to null-field and T-matrix methods]

[End of first paragraph, p. 242]: After "For electromagnetic problems, see ... [M873]." add: See also [132] and [482, Chapter 3].

7.3.1. Literature [on Kupradze's method]

[End of section, p. 244]. For a later review, see [274].

7.3.2. Generalized multipole techniques

MFS has been used for multiple scattering problems arising in computer graphics, where the aim is to give realistic sounds in virtual environments [223, 360]. For an application of MFS to scattering by a cluster of sound-hard spheres, see [475].

[Top of p. 246] For a combination of GMT with a fast multipole method (FMM, see §6.14), with applications to electromagnetic problems, see [45].

[Line 5, amend:] ... rigid wall); for comparisons with Foldy-type approximations (Section 8.3), see [247].

[New paragraph, end of §7.3.2, p. 246]: Another related numerical technique is known as the *Trefftz method* [99]. In two dimensions, surround the scatterers by a circle, C. Outside C, approximate $u_{\rm sc}$ using a (finite) series of outgoing cylindrical wavefunctions, ψ_n. Break the region between C and S into a finite number of simply-connected subregions, B_m, $m = 1, 2, \ldots, M$. In each B_m, approximate $u_{\rm sc}$ using a (finite) series of wavefunctions; these could be plane waves in various directions or regular cylindrical wavefunctions Finally, determine the coefficients in each expansion by imposing continuity conditions across each interface and the boundary condition on S. Variants of this approach have been used for multiple scattering problems. For four squares, see [478]. For five circles, see [512]. Trefftz methods have also been coupled with boundary element methods [29].

7.8.1. Regular wavefunctions

Waterman [535] has refined his approach for spheroids and other quadric surfaces. For electromagnetic scattering by spheroids, see [473]. For fluid-solid problems, see [312], where Waterman's ideas are combined with representations used by Doicu *et al.* [M285].

7.8.4. Numerical experience and convergence

[p. 265, before §7.8.4.1]: For more examples of poor conditioning when computing the Q-matrix using regular wavefunctions, see [472]. For static problems, see [155].

7.8.6. The scattered field

[Bottom of p. 267]: Harness & Ditkowski [191] have given an iterative method for computing the coefficients c_n^m without a direct solution of the null-field equations.

7.9.4. Using a BIE to calculate the T-matrix

[p. 272] "... this connection between boundary integral equations and T-matrix methods has not been exploited." In 2013, Gimbutas & Greengard [180] did this in the context of electromagnetic scattering (although they use the terminology 'scattering matrix' for the T-matrix), and then gave results for scattering by 200 ellipsoids. See also [252].

Other authors have constructed the T-matrix in a similar way (using regular spherical wavefunctions as incident fields) except they used other methods to solve the associated boundary-value problems [331, 388, 457, 125]. Ganesh & Hawkins [167, 171] went one step further and computed the corresponding far-field patterns, f_N^M, say. Then using $d_n^m = \delta_{nN}\delta_{mM}$ (see above (7.74)) in (4.46) gives

$$f_N^M(\hat{\boldsymbol{r}}) = \sum_{n,m} (-\mathrm{i})^n Y_n^m(\hat{\boldsymbol{r}})\, T_{nN}^{mM} \quad \text{and} \quad T_{nN}^{mM} = \mathrm{i}^n \int_\Omega f_N^M\, \overline{Y_n^m}\, \mathrm{d}\Omega.$$

In [165], the authors construct approximations to the T-matrix using many plane waves as incident fields. This idea had been used earlier in [356] for analogous hydrodynamic problems. For electromagnetic problems, see [208].

7.9.5. The far field

[End of section, p. 273]: The elastodynamic analogues of (7.77) and (7.80) are in [122] and [96], respectively.

7.9.6. Literature [on T-matrix for one obstacle]

The T-matrix for a perturbed circular cylinder was constructed by Givoli [181].

Methods for computing the T-matrix recursively have been developed, using invariant imbedding and matrix Riccati equations; for a review, see [133].

Hu *et al.* [206] consider electromagnetic scattering by axisymmetric objects. They build matrices akin to T-matrices by scattering plane waves in various directions, and then use them for scattering by a few objects. Similar methods, where 'characteristic basis functions' are constructed at the discrete level, were developed by Mittra and his co-workers for large-scale computational electromagnetics [325], and they have been used for various multiple scattering problems [207, 120].

[p. 274, end of line 2]: For a careful discussion of averaging over random orientations, see [371].

[p. 274, paragraph starting "For an elastic half-space"]: See also [M132] and [311].

The T-matrix for one defect in a thin plate (using Kirchhoff theory) has been studied [349, 401, 64, 65, 494].

7.10.3. Discussion [of T-matrix for two obstacles]

The key equations are (7.102) and (7.103). It turns out that they are in a 1972 paper by Wilton & Mittra [539, eqn (28)], where they are derived in the form

$$\beta_I a_I + H_{II} a_{II} = -E_I, \quad H_I a_I + \beta_{II} a_{II} = -E_{II}.$$

Here, a_I and a_{II} are vectors of coefficients (equivalent to $\mathbf{c}_1$ and $\mathbf{c}_2$), E_I and E_{II} are analogous vectors for the incident field, the matrices H_I and H_{II} are recognised as separation matrices and the inverses of the matrices β_I and β_{II} can be recognised as individual T-matrices. As the authors note, their equations 'include all the interactions between the two scatterers and [are] therefore exact' [539, p. 315]. Numerical results for scattering by a pair of square cylinders are given.

The formalism in section 7.10 uses two T-matrices, whereas the formalism in section 7.10 uses partitioned boundary integral equations. A hybrid approach can be developed, using a T-matrix for one scatterer and an integral equation for the other. This development is given for electromagnetic problems in [323].

7.10.4. Literature [on T-matrix for two obstacles]

[End of first paragraph]: For scattering by two spheroids, see [M1309], [549].

[Top of p. 281, after first paragraph]: The T-matrix is not very convenient if one wants to compute the near field. Methods for doing this, using various re-expansions, have been developed [487]. For an extreme case, Martin [347] has shown how to compute the electromagnetic field between a pair of coaxial discs, with a small gap between the discs.

7.11.1. T-matrix for N obstacles: discussion and literature

[p. 282, paragraph starting "The two-dimensional version"]: For applications in the context of homogenization theory, see [495, 496].

[p. 282, paragraph starting "The electromagnetic version"]: For a good overview, see [268, Chapter 9]. For applications to scattering by an array of thin-wire structures, see [561]. For scattering by closely spaced spheroids, see [487, 451]. For scattering by inhomogeneous objects, see [341]. The method used in [9] is essentially a T-matrix method.

[After citations of Cai & Williams [M170, M171], bottom of p. 282]: Liu & Cai [320] have extended the method of [M169] to scattering of elastic waves by spherical inclusions. For defects in a thin plate, see [64, 494].

[Top of p. 283, end of first paragraph]: For a review and comparison of several recursive algorithms, including those in [M203, M1048], see [361]. For applications to clusters of non-circular cylinders, see [124].

[End of next paragraph]: After "see, for example, [M607,..., M575]", add [238, 356, 152, 153, 492]. The method of Kagemoto & Yue [M576] has been extended to deep water [411]. Their notion of a T-matrix has also been used in conjunction with the mild-slope equation so as to calculate how water waves over varying bathymetry interact with several rigid obstacles [363].

8.2.2. Three dimensions: sound-hard obstacles

[p. 288, end of paragraph containing (8.25)]: See also [201, §2.8.3].

[Below (8.26), correct "polarisability" to "polarizability". Then, end of paragraph, add]: It is also known that $\mathbf{X}(1)$ vanishes (see [M262], eqn (5.27)) so that the scattering is isotropic when there is no density mismatch across S; this fact was known to Rayleigh [M1010, §335]. In this very special case, we can use the Lippmann–Schwinger equation (§6.3.3), and Foldy's method (§8.6.3) is available. We observe that the transmission problem with $\gamma = 1$ (see (1.26)) is atypical in that the far-field scattering is isotropic (as for sound-soft scatterers) but, in the far field, $|u_{\mathrm{sc}}|^2$ is 'proportional to ω^4 in the low-frequency limit [as for sound-hard scatterers]. This dependence is well known as the signature of *Rayleigh scattering*' [459, p. 57] but the reader should keep in mind the underlying assumption that $\gamma = 1$.

8.2.7. Literature [on small scatterers]

[Bottom of p. 296]: After "[M884, p. 1298]". For another approach, see [334]. Bispherical coordinates have also been used to study sub-wavelength resonances of two close-to-touching spheres; this is relevant to a pair of bubbles (transmission problem) [16]. Extensions to other geometries have been made [300].

[p. 297, first paragraph]: "Standard low-frequency theory gives an approximation for a fixed geometry (a/b fixed) and long waves ($kb \ll 1$)." Garnaud & Mei [173] have developed a multiple-scales analysis for such problems in the context of water waves.

[End of first paragraph]: The 'small-body' approximation ($a \to 0$ with k and minimum spacing held fixed) has been developed in a long series of papers by Ammari and his collaborators. For example, Ammari *et al.* [17, eqn (23)] give the following generalisation of (8.26) for N small scatterers:

$$f(\hat{\boldsymbol{r}};\hat{\boldsymbol{\alpha}}) \sim -\frac{\mathrm{i}k^3}{4\pi}\sum_{j=1}^{N}\left\{\hat{\boldsymbol{r}}\cdot\mathbf{X}_j(\gamma)\cdot\hat{\boldsymbol{\alpha}} + [1-\gamma(k_0/k)^2]|B_j|\right\}\exp\left(\mathrm{i}k[\hat{\boldsymbol{\alpha}}-\hat{\boldsymbol{r}}]\cdot\boldsymbol{b}_j\right).$$

Here $|B_j|$ is the volume of the scatterer B_j located at $\boldsymbol{b}_j$, $j = 1, 2, \ldots, N$, and $\mathbf{X}_j$ is the polarizability tensor for B_j *in isolation*. This formula for f says that the far-field contributions from each scatterer are summed with a suitable phase correction to take account of a change of origin, as given by (4.47): at leading order, there is no multiple scattering. For related acoustic results, see [26, 42]. For electromagnetic scattering, see [520, 18].

[End of section, p. 297]: Alker [10] has used matched asymptotic expansions for two-dimensional water-wave problems, with results for two docks and two immersed cylinders.

8.3. Foldy's method

[Beginning of section 8.3, p. 297]:

> *The point interaction approximation is a natural tool to analyze a variety of interesting problems in the continuum or homogenization limit. In the physical literature it goes back to Foldy's paper* [M354] *on sound propagation in a bubbly liquid and perhaps earlier. In almost all papers that followed Foldy's, the point interaction approximation is not treated as an important approximation in itself and averaging is carried out over the* [scatterer] *center locations. The closure problem that arises is then treated in a variety of ways depending on other parameters in the problem.* [See Section 8.6.] *. . . But averaging is not necessary. The continuum limit holds for deterministic sequences* [of scatterer locations] *subject to some conditions that hold for "most" realizations in the random case. The closure difficulties are thus avoided for many problems.*
>
> (Figari *et al.* [158, p. 47])

[Top of p. 299, after first paragraph]: The Monte Carlo approach of Fikioris [M345] was implemented by Hahn [187]. An earlier study was made by Bruno & Novarini in 1982 [60]. They solved the Foldy system (8.49) [60, eqn (20)] (although Foldy is not cited) for scattering by a row of N bubbles, with random spacings between adjacent bubbles.

The idea of approximating small spherical bubbles by point sources is natural; for historical remarks, see [343]. For two-dimensional analogues, see [536].

The problem of scattering by N isotropic point scatterers in the vicinity of a large scatterer has been treated by combining Foldy's method with a boundary integral equation [211, 213, 212, 203, 204].

For other applications and developments of Foldy's method, see [504, 430, 127, 507, 37, 259, 309, 70, 295, 310, 449, 174], [321, §4] and [195, Chapter 26]. An iterative variant of Foldy's method has been analysed [69].

Bendali *et al.* [43] have given a detailed analysis using matched asymptotic expansions; the small parameter is ka, with N and the geometrical configuration of the scatterers held fixed. They recover the Foldy scheme at leading order and then go further.

Foldy-type methods can be developed for flexural waves in plates. In the simplest situations, the plate is 'pinned' at a configuration of points, and incident waves are scattered by these points. As the basic fundamental solution, $G(P, Q) = H_0^{(1)}(kR) - H_0^{(1)}(\mathrm{i}kR)$, is finite at $R = 0$, collocation at the pinned points is permitted, leading to a simple numerical scheme. This approach was introduced by Evans & Porter in 2007 [151]. Since then, many other investigations have appeared [419, 395, 83, 377, 344, 376, 348].

8.3.1. The scattering coefficient

For a formula similar to (8.62), see [235, Proposition 2.1].

8.3.2. Ramm's method

[End of section, p. 302.] Ramm has revived his method in numerous papers; see, for example, [424]. He has also extended it to electromagnetic problems [423, 425, 426]. Similar methods, starting from (8.63), have been developed by Argatov & Sabina [26, 27]. For a more detailed analysis of Ramm's method, see

[76, 7, 78] with extensions to elastodynamic problems [77, 11]. We also mention an earlier paper [378] in which scattering by N small obstacles is reduced to solving an $N \times N$ linear system.

Ammari & Zhang [19] have used a variant of Ramm's method for bubbly liquids. They start with the single-layer representation for $u_{\rm sc}$ and then use it for the transmission problem with N small spherical bubbles. Under certain circumstances, the limit $N \to \infty$ can be taken; bubbles can resonate so that the limiting process is not straightforward.

8.4. Point scatterers

[Bottom of p. 305, after "... the approximation (8.79)."] Challa & Sini [75] have used (8.82) for direct and inverse problems. They have made similar applications to elastodynamic and electromagnetic problems; see also [205, 74].

The standard book on point scatterers is [8]. Its bibliography contains 747 items. Apart from a few books such as Abramowitz & Stegun [M1] and Gradshteyn & Ryzhik [M427], the only items in common are three of Foldy's papers, [M354, M460]. Foldy's seminal paper is described in one sentence [8, p. 353]: 'Multiple scattering of waves by randomly distributed point scatterers has been discussed in [M354]'.

8.5.2. Two-dimensional water waves [wide-spacing approximations]

[p. 308, end of first paragraph, after "For detailed derivations... [M731, §6.3]."] For N bodies, see [M329] and [244, 245]. Kagemoto [236] has given an extended theory for two bodies in which evanescent modes are taken into account; see also [237].

[End of second paragraph]: McIver [354] has extended the wide-spacing approximation within a formal procedure of matched asymptotic expansions.

8.5.4. Three-dimensional water waves [wide-spacing approx]

p. 312, after "It was exploited ... used in [M827]." Add: [469].

8.6.3. Foldy's approximation [random problems]

[p. 316, second paragraph, after citing M492, M493]: add [322]

8.6.6. Finite-size effects [random problems]

[p. 319, end of first paragraph, after citing [M492, M493, M586]: A two-dimensional version of (8.117) was obtained by Noskov [392, eqn (32)].

[p. 319, end of second paragraph]: For a review of some applications of (8.118) with $Q = 0$, see [333].

[End of §8.6.6]: The literature on formulas such as (8.118), (8.121) and (8.122), including generalisations and applications, has grown. For a new proof of the Lloyd–Berry formula (8.121), see [313]. For further results on scattering by random arrangements of spheres, see [79, 328, 67, 413, 329], [509, §2.4].

For two-dimensional acoustic problems, see [126, 345, 249, 390].

For plane-strain elastodynamics, see [154, 116, 92]. For waves in plates, see [401]. For elastodynamic problems involving spheres, see [328] and [141]; the second of these includes comparisons with experiments.

Gower *et al.* [183] have obtained generalisations for media comprising random arrangements of spheres of two (or more) sizes.

8.6.7. Effective field methods and effective medium methods

The book by Kanaun & Levin [243] reviews effective field methods and effective medium methods, with an emphasis on elastodynamic problems. For example, [243, Chapter 7] considers random configurations of spherical inclusions; see also [M583] and [241, 242].

Appendix B. Integrating a product of three spherical harmonics

[p. 326, end of 2nd paragraph]: For numerical computation of 3-j symbols, see [232] and references therein.

Appendix C. Rotation matrices

[p. 330, end of paragraph starting "An explicit formula", add]: For additional general references on rotations, see [540] and [476, §III].

[At the end, p. 333]: Further computational algorithms have been developed [189, Appendix A2], [179, 296].

References

[1] M.S. Abou-Dina & A.F. Ghaleb, Multiple wave scattering by submerged obstacles in an infinite channel of finite depth. I. Streamlines. *European Journal of Mechanics B/Fluids* **59** (2016) 37–51.

[2] S. Acosta, On-surface radiation condition for multiple scattering of waves. *Comput. Methods Appl. Mech. Engrg.* **283** (2015) 1296–1309.

[3] S. Acosta, Local on-surface radiation condition for multiple scattering of waves from convex obstacles. *Comput. Methods Appl. Mech. Engrg.* **378** (2021) 113697 (17 pages).

[4] S. Acosta & V. Villamizar, Coupling Dirichlet-to-Neumann boundary condition and finite difference methods in curvilinear coordinates for multiple scattering. *J. Comp. Phys.* **229** (2010) 5498–5517.

[5] S. Acosta, V. Villamizar & B. Malone, The DtN nonreflecting boundary condition for multiple scattering problems in the half-plane. *Comput. Methods Appl. Mech. Engrg.* **217–220** (2012) 1–11.

[6] R. Adelman, N.A. Gumerov & R. Duraiswami, Semi-analytical computation of acoustic scattering by spheroids and disks. *J. Acoust. Soc. Amer.* **136** (2014) EL405–EL410.

[7] B. Ahmad, D.P. Challa, M. Kirane & M. Sini, The equivalent refraction index for the acoustic scattering by many small obstacles: with error estimates. *J. Math. Anal. & Appl.* **424** (2015) 563–583.

[8] S. Albeverio, F. Gesztesy, R. Høegh-Krohn & H. Holden, with an appendix by P. Exner, *Solvable Models in Quantum Mechanics*, second edition. Providence: Amer. Math. Soc., 2005.

[9] M. Alian & H. Oraizi, Electromagnetic multiple PEC object scattering using equivalence principle and addition theorem for spherical wave harmonics. *IEEE Trans. Antennas & Propag.* **66** (2018) 6233–6243.

[10] G. Alker, The radiation and scattering of short surface waves by a number of cylinders. *Quart. J Mech. Appl. Math.* **31** (1978) 391–408.

[11] F. Al-Musallam, D.P. Challa & M. Sini, The equivalent medium for the elastic scattering by many small rigid bodies and applications. *IMA J. Appl. Math.* **81** (2016) 1020–1050.

[12] H. Alzubaidi, X. Antoine & C. Chniti, Formulation and accuracy of On-Surface Radiation Conditions for acoustic multiple scattering problems, *Appl. Math. & Computation* **277** (2016) 82–100.

[13] M.L. Amamou, A theoretical and numerical resolution of an acoustic multiple scattering problem in three-dimensional case. *Acoustical Physics* **62** (2016) 280–291.

[14] S. Ambikasaran, C. Borges, L.-M. Imbert-Gerard and L. Greengard, Fast, adaptive, high-order accurate discretization of the Lippmann–Schwinger equation in two dimensions. *SIAM J. Sci. Computing* **38** (2016) A1770–A1787.

[15] F.A. Amirkulova & A.N. Norris, Acoustic multiple scattering using recursive algorithms. *J. Comp. Phys.* **299** (2015) 787–803.

[16] H. Ammari, B. Davies & S. Yu, Close-to-touching acoustic subwavelength resonators: eigenfrequency separation and gradient blow-up. *Multiscale Modeling & Simulation* **18** (2020) 1299–1317.

[17] H. Ammari, E. Iakovleva & S. Moskow, Recovery of small inhomogeneities from the scattering amplitude at a fixed frequency. *SIAM J. Math. Anal.* **34** (2003) 882–900.
[18] H. Ammari & D. Volkov, The leading-order term in the asymptotic expansion of the scattering amplitude of a collection of finite number of dielectric inhomogeneities of small diameter. *Int. J. for Multiscale Computational Eng.* **3** (2005) 149–160.
[19] H. Ammari & H. Zhang, Effective medium theory for acoustic waves in bubbly fluids near Minnaert resonant frequency. *SIAM J. Math. Anal.* **49** (2017) 3252–3276.
[20] A. Anand, Y. Boubendir, F. Ecevit & F. Reitich, Analysis of multiple scattering iterations for high-frequency scattering problems. II: the three-dimensional scalar case. *Numer. Math.* **114** (2010) 373–427.
[21] A.C. Andersen, J.A. Sotelo, V.N. Pustovit & G.A. Niklasson, Extinction calculations of multi-sphere polycrystalline graphitic clusters. A comparison with the 2175 Å peak and between a rigorous solution and discrete-dipole approximations. *Astronomy & Astrophysics* **386** (2002) 296–307. Erratum: **411** (2003) 481–482.
[22] X. Antoine, C. Chniti & K. Ramdani, On the numerical approximation of high-frequency acoustic multiple scattering problems by circular cylinders. *J. Comp. Phys.* **227** (2008) 1754–1771.
[23] X. Antoine, C. Geuzaine & K. Ramdani, Computational methods for multiple scattering at high frequency with applications to periodic structure calculations. In: *Wave Propagation in Periodic Media* (ed. M. Ehrhardt) pp. 73–107. Bentham Science Publishers, 2010.
[24] X. Antoine, K. Ramdani & B. Thierry, Wide frequency band numerical approaches for multiple scattering problems by disks. *J. Algorithms & Computational Technology* **6** (2012) 241–259.
[25] X. Antoine & B. Thierry, Spectral and condition number estimates of the acoustic single-layer operator for low-frequency multiple scattering in dilute media. *Comput. Methods Appl. Mech. Engrg.* **265** (2013) 242–256.
[26] I.I. Argatov & F.J. Sabina, Acoustic diffraction by a finite number of small soft bodies. *Wave Motion* **45** (2008) 238–253.
[27] I.I. Argatov & F.J. Sabina, Acoustic scattering by a cluster of small sound-soft obstacles. *Wave Motion* **47** (2010) 537–551.
[28] E. Arvas, A. Rahhal-Arabi, A. Sadigh & S.M. Rao, Scattering from multiple conducting and dielectric bodies of arbitrary shape. *IEEE Antennas & Propag. Magazine* **33**, 2 (1991) 29–36.
[29] O. Atak, B. Bergen, D. Huybrechs, B. Pluymers & W. Desmet, Coupling of boundary element and wave based methods for the efficient solution of complex multiple scattering problems. *J. Comp. Phys.* **258** (2014) 165–184.
[30] R. Ávila-Carrera & F.J. Sánchez-Sesma, Scattering and diffraction of elastic P- and S-waves by a spherical obstacle: a review of the classical solution. *Geofísica Internacional* **45** (2006) 3–21.
[31] R. Ávila-Carrera, F.J. Sánchez-Sesma & J. Avilés, Transient response and multiple scattering of elastic waves by a linear array of regularly distributed cylindrical obstacles: anti-plane S-wave analytical solution. *Geofísica Internacional* **47** (2008) 115–126.
[32] J. Avilés & F.J. Sánchez-Sesma, Piles as barriers for elastic waves. *J. Geotechnical Engineering* **109** (1983) 1133–1145.
[33] J.-F. Babadjian, E. Bonnetier & F. Triki, Enhancement of electromagnetic fields caused by interacting subwavelength cavities. *Multiscale Modeling & Simulation* **8** (2010) 1383–1418.
[34] T. Balendra, D.P. Thambiratnam, C.G. Koh & S.-L. Lee, Dynamic response of twin circular tunnels due to incident SH-waves. *Earthquake Eng. & Structural Dynamics* **12** (1984) 181–201.
[35] S. Banerji, On sound-waves due to prescribed vibration on a spherical surface in the presence of a rigid and fixed spherical obstacle. *Bulletin of the Calcutta Mathematical Society* **4** (1912–1913) 5–14.
[36] G. Bao, L. Xu & T. Yin, Boundary integral equation methods for the elastic and thermoelastic waves in three dimensions. *Comput. Methods Appl. Mech. Engrg.* **354** (2019) 464–486.
[37] M.C. Barr, M.P. Zaletel & E.J. Heller, Quantum corral resonances: lossy scattering as acoustics. *Nano Letters* **10** (2010) 3253–3260.
[38] B.E. Barrowes, C.O. Ao, F.L. Teixeira & J.A. Kong, Sparse matrix/canonical grid method applied to 3-D dense medium simulations. *IEEE Trans. Antennas & Propag.* **51** (2003) 48–58.
[39] H. Barucq, J. Chabassier, H. Pham & S. Tordeux, Numerical robustness of single-layer method with Fourier basis for multiple obstacle acoustic scattering in homogeneous media. *Wave Motion* **77** (2018) 40–63.

[40] S. Batool, F. Frezza, F. Mangini & Y.-L. Xu, Scattering from multiple PEC sphere using translation addition theorems for spherical vector wave function. *J. Quant. Spectroscopy & Radiative Transfer* **248** (2020) 106905 (6 pages).
[41] K.A. Belibassakis, G. Arnaud, V. Rey & J. Touboul, Propagation and scattering of waves by dense arrays of impenetrable cylinders in a waveguide. *Wave Motion* **80** (2018) 1–19.
[42] A. Bendali, P.-H. Cocquet & S. Tordeux, Scattering of a scalar time-harmonic wave by N small spheres by the method of matched asymptotic expansions. *Numerical Analysis & Applications* **5** (2012) 116–123.
[43] A. Bendali, P.-H. Cocquet & S. Tordeux, Approximation by multipoles of the multiple acoustic scattering by small obstacles in three dimensions and application to the Foldy theory of isotropic scattering. *Arch. Rational Mech. Anal.* **219** (2016) 1017–1059.
[44] T. Betcke, J. Phillips & E.A. Spence, Spectral decompositions and nonnormality of boundary integral operators in acoustic scattering. *IMA J. Numer. Anal.* **34** (2014) 700–731.
[45] B. Blankrot & Y. Leviatan, FMM-accelerated source-model technique for many-scatterer problems. *IEEE Trans. Antennas & Propag.* **65** (2017) 4379–4384.
[46] P.A. Bobbert & J. Vlieger, Light scattering by a sphere on a substrate. *Physica A* **137** (1986) 209–242.
[47] T.J. Bogdan & F. Cattaneo, The normal modes of a resonant cavity containing discrete inhomogeneities: The influence of fibril magnetic fields on the solar acoustic oscillations. *Astrophysical J.* **342** (1989) 545–557.
[48] M. Bonnet, Solvability of a volume integral equation formulation for anisotropic elastodynamic scattering. *J. Integral Equations & Appl.* **28** (2016) 169–203.
[49] M. Bonnet, A modified volume integral equation for anisotropic elastic or conducting inhomogeneities: Unconditional solvability by Neumann series. *J. Integral Equations & Appl.* **29** (2017) 271–295.
[50] B. Borgarino, A. Babarit & P. Ferrant, An implementation of the fast multipole algorithm for wave interaction problems on sparse arrays of floating bodies. *J. Eng. Math.* **77** (2012) 51–68.
[51] F. Borghese, P. Denti & R. Saija, *Scattering from Model Nonspherical Particles*, second edition. Berlin: Springer, 2007. The first edition is [M117].
[52] R. Borghi, F. Gori, M. Santarsiero, F. Frezza & G. Schettini, Plane-wave scattering by a set of perfectly conducting circular cylinders in the presence of a plane surface. *J. Opt. Soc. Amer.* A **13** (1996) 2441–2452.
[53] A. Boström & A. Burden, Propagation of elastic surface waves along a cylindrical cavity and their excitation by a point force. *J. Acoust. Soc. Amer.* **72** (1982) 998–1004.
[54] M. Bouchon & F.J. Sánchez-Sesma, Boundary integral equations and boundary elements methods in elastodynamics. *Advances in Geophysics* **48** (2007) 157–189
[55] L. Boyde, K.J. Chalut & J. Guck, Near- and far-field scattering from arbitrary three-dimensional aggregates of coated spheres using parallel computing. *Phys. Rev.* E **83** (2011) 026701 (14 pages).
[56] J. Bremer, A fast direct solver for the integral equations of scattering theory on planar curves with corners. *J. Comp. Phys.* **231** (2012) 1879–1899.
[57] J. Bremer, A. Gillman & P.-G. Martinsson, A high-order accurate accelerated direct solver for acoustic scattering from surfaces. *BIT* **55** (2015) 367–397.
[58] P. Briard, J.J. Wang & Y.P. Han, Shaped beam scattering by an aggregate of particles using generalized Lorenz–Mie theory, *Optics Comm.* **365** (2016) 186–193.
[59] C. Brimacombe, R.M. Corless & M. Zamir, Computation and applications of Mathieu functions: a historical perspective. *SIAM Rev.* **63** (2021) 653–720.
[60] D.R. Bruno & J.C. Novarini, Coherence and multiple scattering effects on acoustic backscattering from linear arrays of gas-filled bubbles. *J. Acoust. Soc. Amer.* **71** (1982) 1359–1367.
[61] O. Bruno, T. Elling & C. Turc, Regularized integral equations and fast high-order solvers for sound-hard acoustic scattering problems. *Int. J. Numer. Meth. Eng.* **91** (2012) 1045–1072.
[62] F. Bu, J. Lin & F. Reitich, A fast and high-order method for the three-dimensional elastic wave scattering problem. *J. Comp. Phys.* **258** (2014) 856–870.
[63] L.-W. Cai, D.K. Dacol, D.C. Calvo & G.J. Orris, Acoustical scattering by arrays of cylinders in waveguides. *J. Acoust. Soc. Amer.* **122** (2007) 1340–1351.
[64] L.-W. Cai & S.A. Hambric, Multiple scattering of flexural waves on thin plates. *J. Vibration & Acoustics* **138** (2016) 011009 (10 pages)

[65] L.-W. Cai & S.A. Hambric, Movable rigid scatterer model for flexural wave scattering on thin plates. *J. Vibration & Acoustics* **138** (2016) 031016 (10 pages)
[66] Y.-Q. Cai, G.-Y. Ding & C.-J. Xu, Amplitude reduction of elastic waves by a row of piles in a poroelastic soil. *Computers & Geotechnics* **36** (2009) 463–473.
[67] M. Caleap, B.W. Drinkwater & P.D. Wilcox, Coherent acoustic wave propagation in media with pair-correlated spheres. *J. Acoust. Soc. Amer.* **131** (2012) 2036–2047.
[68] M. Carr, E. Topsakal & J.L. Volakis, A procedure for modeling material junctions in 3-D surface integral equation approaches. *IEEE Trans. Antennas & Propag.* **52** (2004) 1374–1379.
[69] M. Cassier & C. Hazard, Multiple scattering of acoustic waves by small sound-soft obstacles in two dimensions: mathematical justification of the Foldy–Lax model. *Wave Motion* **50** (2013) 18–28.
[70] A. Chai, M. Moscoso & G. Papanicolaou, Imaging strong localized scatterers with sparsity promoting optimization. *SIAM J. Imaging Sciences* **7** (2014) 1358–1387.
[71] S. Chaillat & M. Bonnet, Recent advances on the fast multipole accelerated boundary element method for 3D time-harmonic elastodynamics. *Wave Motion* **50** (2013) 1090–1104.
[72] S. Chaillat & M. Bonnet, A new fast multipole formulation for the elastodynamic half-space Green's tensor. *J. Comp. Phys.* **258** (2014) 787–808.
[73] S. Chaillat, B. Cotté, J.-F. Mercier, G. Serre & N. Trafny, Efficient evaluation of three-dimensional Helmholtz Green's functions tailored to arbitrary rigid geometries for flow noise simulations. *J. Comp. Phys.* **452** (2022) 110915 (21 pages).
[74] D.P. Challa, G. Hu & M. Sini, Multiple scattering of electromagnetic waves by finitely many point-like obstacles. *Math. Models & Methods in Appl. Sciences* **24** (2014) 863–899.
[75] D.P. Challa & M. Sini, Inverse scattering by point-like scatterers in the Foldy regime. *Inverse Prob.* **28** (2012) 125006 (39 pages).
[76] D.P. Challa & M. Sini, On the justification of the Foldy–Lax approximation for the acoustic scattering by small rigid bodies of arbitrary shapes. *Multiscale Modeling & Simulation* **12** (2014) 55–108. Corrigendum: **20** (2022) 882–892.
[77] D.P. Challa & M. Sini, The Foldy–Lax approximation of the scattered waves by many small bodies for the Lamé system. *Mathematische Nachrichten* **288** (2015) 1834–1872.
[78] D.P. Challa & M. Sini, Multiscale analysis of the acoustic scattering by many scatterers of impedance type. *Zeitschrift für angewandte Mathematik und Physik* **67** (2016) article 58 (33 pages).
[79] R.E. Challis, M.J.W. Povey, M.L. Mather & A.K. Holmes, Ultrasound techniques for characterizing colloidal dispersions. *Reports on Progress in Physics* **68** (2005) 1541–1637.
[80] R.E. Challis, J.S. Tebbutt & A. K. Holmes, Equivalence between three scattering formulations for ultrasonic wave propagation in particulate mixtures. *J. Phys. D: Appl. Phys.* **31** (1998) 3481–3497.
[81] P.G. Chamberlain, Water wave scattering by finite arrays of circular structures. *IMA J. Appl. Math.* **72** (2007) 52–66.
[82] J.C. Chao, Y.J. Liu, F.J. Rizzo, P.A. Martin & L. Udpa, Regularized integral equations and curvilinear boundary elements for electromagnetic wave scattering in three dimensions. *IEEE Trans. Antennas & Propag.* **43** (1995) 1416–1422.
[83] G.J. Chaplain, M.P. Makwana & R.V. Craster, Rayleigh–Bloch, topological edge and interface waves for structured elastic plates. *Wave Motion* **86** (2019) 162–174.
[84] I.K. Chatjigeorgiou, Three dimensional wave scattering by arrays of elliptical and circular cylinders. *Ocean Eng.* **38** (2011) 1480–1494.
[85] I.K. Chatjigeorgiou, The hydrodynamics of arrays of truncated elliptical cylinders. *European Journal of Mechanics B/Fluids* **37** (2013) 153–164.
[86] I.K. Chatjigeorgiou, *Analytical Methods in Marine Hydrodynamics.* Cambridge: Cambridge University Press, 2018.
[87] I.K. Chatjigeorgiou, Water wave trapping in a long array of bottomless circular cylinders. *Wave Motion* **83** (2018) 25–48.
[88] I.K. Chatjigeorgiou, K. Chatziioannou & T. Mazarakos, Near trapped modes in long array of truncated circular cylinders. *J. Waterway, Port, Coastal, & Ocean Engineering* **145** (2019) 04018035 (18 pages).
[89] I.K. Chatjigeorgiou & V. Katsardi, Hydrodynamics and near trapping effects in arrays of multiple elliptical cylinders in waves. *Ocean Eng.* **157** (2018) 121–139.
[90] I.K. Chatjigeorgiou & S.A. Mavrakos, An analytical approach for the solution of the hydrodynamic

diffraction by arrays of elliptical cylinders. *Appl. Ocean Res.* **32** (2010) 242–251.

[91] I.K. Chatjigeorgiou & T. Miloh, Wave scattering of spheroidal bodies below a free surface. *J. Ship Res.* **57** (2013) 141–154.

[92] M. Chekroun, L. Le Marrec, B. Lombard & J. Piraux, Time-domain numerical simulations of multiple scattering to extract elastic effective wavenumbers. *Waves in Random & Complex Media* **22** (2012) 398–422.

[93] B. Chen & X. Zhang, Plane dynamic response of two circular holes to transient elastic waves. *Archive of Applied Mechanics* **61** (1991) 567–577.

[94] D. Chen, W. Cai, B. Zinser & M.H. Cho, Accurate and efficient Nyström volume integral equation method for the Maxwell equations for multiple 3-D scatterers. *J. Comp. Phys.* **321** (2016) 303–320.

[95] J.-T. Chen , P.-Y. Chen & C.-T. Chen, Surface motion of multiple alluvial valleys for incident plane SH-waves by using a semi-analytical approach. *Soil Dynamics & Earthquake Engineering* **28** (2008) 58–72.

[96] K.-C. Chen & C.-L. Lin, An expansion theorem for two-dimensional elastic waves and its application. *Math. Meth. Appl. Sci.* **29** (2006) 1849–1860.

[97] S. Chen & O. Mahrenholtz, Interaction of water waves and floating twin cylinders in beam waves. *Appl. Ocean Res.* **14** (1992) 371–379.

[98] M.A. Cherevko, Diffraction of plane flexural waves in an infinite multiply-connected plate. *Soviet Applied Mechanics* **10** (1974) 620–625.

[99] Y.K. Cheung, W.G. Jin & O.C. Zienkiewicz, Solution of Helmholtz equation by Trefftz method. *Int. J. Numer. Meth. Eng.* **32** (1991) 63–78.

[100] W.C. Chew, Vector addition theorem and its diagonalization. *Comm. Comput. Phys.* **3** (2008) 330–341.

[101] B.F.M. Child & V. Venugopal, Optimal configurations of wave energy device arrays. *Ocean Eng.* **37** (2010) 1402–1417.

[102] I.H. Cho, H.J. Koh, J.R. Kim & M.H. Kim, Wave scattering by dual submerged horizontal porous plates. *Ocean Eng.* **73** (2013) 149–158.

[103] I.D. Chremmos & N.K. Uzunoglu, Analysis of scattering by a linear chain of spherical inclusions in an optical fiber. *J. Opt. Soc. Amer.* A **23** (2006) 3054–3062.

[104] X. Claeys & R. Hiptmair, Multi-trace boundary integral formulation for acoustic scattering by composite structures. *Comm. Pure Appl. Math.* **66** (2013) 1163–1201.

[105] X. Claeys & R. Hiptmair, Integral equations for acoustic scattering by partially impenetrable composite objects. *Integral Equations & Operator Theory* **81** (2015) 151–189.

[106] A. Clebsch, Ueber die Reflexion an einer Kugelfläche. *Journal für die reine und angewandte Mathematik* **61** (1863) 195–262.

[107] P.C. Clemmow, *The Plane Wave Spectrum Representation of Electromagnetic Fields*. Oxford: Oxford University Press and IEEE Press, 1996.

[108] J. Coatléven & P. Joly, Operator factorization for multiple-scattering problems and an application to periodic media. *Comm. Comput. Phys.* **11** (2012) 303–318.

[109] M.J. Colbrook & L.J. Ayton, A spectral collocation method for acoustic scattering by multiple elastic plates. *J. Sound Vib.* **461** (2019) 114904 (15 pages).

[110] M.J. Colbrook & A.V. Kisil, A Mathieu function boundary spectral method for scattering by multiple variable poro-elastic plates, with applications to metamaterials and acoustics. *Proc. Roy. Soc.* A **476** (2020) 20200184 (21 pages).

[111] M.J. Colbrook & M.J. Priddin, Fast and spectrally accurate numerical methods for perforated screens (with applications to Robin boundary conditions). *IMA J. Appl. Math.* **85** (2020) 790–821.

[112] W.D. Collins, On the solution of some axisymmetric boundary value problems by means of integral equations V. Some scalar diffraction problems for circular disks. *Quart. J. Mech. Appl. Math.* **14** (1961) 101–117.

[113] D. Colton & R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, third edition. New York: Springer, 2013. The second edition is [M225].

[114] P. Cong, L. Chen & Y. Gou, Hydrodynamic interaction among multiple columns in front of a vertical wall. *Ocean Eng.* **197** (2020) 106877 (19 pages).

[115] P. Cong, M. Zhao, B. Teng, Y. Gou & Y. Liu, Second-order near trapping of water waves by a square array of vertical columns in bi-chromatic seas. *J. Fluids & Struct.* **121** (2023) 103916 (22 pages).

[116] J.-M. Conoir & A.N. Norris, Effective wavenumbers and reflection coefficients for an elastic medium containing random configurations of cylindrical scatterers. *Wave Motion* **47** (2010) 183–197.
[117] M.F.R. Cooray & I.R. Ciric, Scattering by systems of spheroids in arbitrary configurations. *Computer Physics Comm.* **68** (1991) 279–305.
[118] R.V. Craster & A.V. Shanin, Embedding formulae for diffraction by rational wedge and angular geometries. *Proc. Roy. Soc.* A **461** (2005) 2227–2242.
[119] Z. Cui, Y. Han, X. Ai & W. Zhao, A domain decomposition of the finite element–boundary integral method for scattering by multiple objects. *Electromagnetics* **31** (2011) 469–482.
[120] Z. Cui, Y. Han & Q. Xu, Numerical simulation of multiple scattering by random discrete particles illuminated by Gaussian beams. *J. Opt. Soc. Amer.* A **28** (2011) 2200–2208.
[121] J. Dalmas & R. Deleuil, Translational addition theorems for prolate spheroidal vector wave functions $\mathbf{M}^r$ and $\mathbf{N}^r$. *Quart. Appl. Math.* **44** (1986) 213–222.
[122] G. Dassios, The Atkinson–Wilcox expansion theorem for elastic waves. *Quart. Appl. Math.* **46** (1988) 285–299.
[123] G. Dassios, *Ellipsoidal Harmonics*. Cambridge: Cambridge University Press, 2012.
[124] M. Degen, V. Jandieri, B. Sievert, J.T. Svejda, A. Rennings & D. Erni, An efficient analysis of scattering from randomly distributed obstacles using an accurate recursive aggregated centered T-matrix algorithm. *IEEE Trans. Antennas & Propag.* **71** (2023) 9849–9862.
[125] G. Démesy, J.-C. Auger & B. Stout, Scattering matrix of arbitrarily shaped objects: combining finite elements and vector partial waves. *J. Opt. Soc. Amer. A* **35** (2018) 1401–1409.
[126] A. Derode, V. Mamou & A. Tourin, Influence of correlations between scatterers on the attenuation of the coherent wave in a random medium. *Phys. Rev.* E **74** (2006) 036606 (9 pages).
[127] A.J. Devaney, E.A. Marengo & F.K. Gruber, Time-reversal-based imaging and inverse scattering of multiply scattering point targets. *J. Acoust. Soc. Amer.* **118** (2005) 3129–3138.
[128] F. Dikmen, E. Sever, S. Vatansever & Y.A. Tuchkin, Well-conditioned algorithm for scattering by a few eccentrically multilayered dielectric circular cylinders. *Radio Sci.* **50** (2015) 99–110.
[129] P.S. Dineva, G.D. Manolis & F. Wuttke, Fundamental solutions in 3D elastodynamics for the BEM: a review. *Engineering Analysis with Boundary Elements* **105** (2019) 47–69.
[130] M. Di Vico, F. Frezza, L. Pajewski & G. Schettini, Scattering by a finite set of perfectly conducting cylinders buried in a dielectric half-space: a spectral-domain solution. *IEEE Trans. Antennas & Propag.* **53** (2005) 719–727.
[131] G.C. Diwan, J. Trevelyan & G. Coates, A comparison of techniques for overcoming non-uniqueness of boundary integral equations for the collocation partition of unity method in two-dimensional acoustic scattering. *Int. J. Numer. Meth. Eng.* **96** (2013) 645–664.
[132] A. Doicu, T. Wriedt & Y.A. Eremin, *Light Scattering by Systems of Particles. Null-Field Method with Discrete Sources: Theory and Programs*. Berlin: Springer, 2006.
[133] A. Doicu, T. Wriedt & N. Khebbache, An overview of the methods for deriving recurrence relations for T-matrix calculation. *J. Quant. Spectroscopy & Radiative Transfer* **224** (2019) 289–302.
[134] V. Dolean, P. Jolivet & F. Nataf, *An Introduction to Domain Decomposition Methods: Algorithms, Theory, and Parallel Implementation*. Philadelphia: SIAM, 2015.
[135] V. Domínguez, S.L. Lu & F.-J. Sayas, A Nyström method for the two dimensional Helmholtz hypersingular equation. *Adv. Comput. Math.* **40** (2014) 1121–1157.
[136] T.E. Doyle, Iterative simulation of elastic wave scattering in arbitrary dispersions of spherical particles. *J. Acoust. Soc. Amer.* **119** (2006) 2599–2610.
[137] T.E. Doyle, D.A. Robinson, S.B. Jones, K.H. Warnick & B.L. Carruth, Modeling the permittivity of two-phase media containing monodisperse spheres: effects of microstructure and multiple scattering. *Phys. Rev.* B **76** (2007) 054203 (14 pages).
[138] M. Dravinski & M.C. Yu, Scattering of plane harmonic *SH* waves by multiple inclusions. *Geophys. J. Int.* **186** (2011) 1331–1346.
[139] T.J. Dufva, J. Sarvas & J.C.-E. Sten, Unified derivation of the translational addition theorems for the spherical scalar and vector wave functions. *Progress in Electromagnetic Research* B **4** (2008) 79–99.
[140] T.M. Dunster, Electromagnetic wave scattering by two parallel infinite dielectric cylinders. *Studies in Appl. Math.* **131** (2013) 302–316.
[141] M. Duranteau, T. Valier-Brasier, J.-M. Conoir & R. Wunenburger, Random acoustic metamaterial

with a subwavelength dipolar resonance. *J. Acoust. Soc. Amer.* **139** (2016) 3341–3352.
[142] R. Eatock Taylor & C.S. Hu, Multipole expansions for wave diffraction and radiation in deep water. *Ocean Eng.* **18** (1991) 191–224.
[143] F. Ecevit & F. Reitich, Analysis of multiple scattering iterations for high-frequency scattering problems. I: the two-dimensional case. *Numer. Math.* **114** (2009) 271–354.
[144] A.Z. Elsherbeni, M. Hamid & G. Tian, Iterative scattering of a Gaussian beam by an array of circular conducting and dielectric cylinders. *J. Electromagnetic Waves & Appl.* **7** (1993) 1323–1342.
[145] A.Z. Elsherbeni & A.A. Kishk, Modeling of cylindrical objects by circular dielectric and conducting cylinders. *IEEE Trans. Antennas & Propag.* **40** (1992) 96–99.
[146] P. Emmanouil, The direct scattering problem: uniqueness and existence for anisotropic media. *Quart. Appl. Math.* **69** (2011) 747–758.
[147] J. Engström, M. Eriksson, M. Göteman, J. Isberg & M. Leijon, Performance of large arrays of point absorbing direct-driven wave energy converters. *J. Appl. Phys.* **114** (2013) 204502 (6 pages).
[148] B. Engquist & H. Zhao, Approximate separability of the Green's function of the Helmholtz equation in the high-frequency limit. *Comm. Pure Appl. Math.* **71** (2018) 2220–2274.
[149] H.J. Eom, *Wave Scattering Theory*. Berlin: Springer, 2001.
[150] İ.Ö. Erselcan & A. Kükner, A numerical analysis of several wave energy converter arrays deployed in the Black Sea. *Ocean Eng.* **131** (2017) 68–79.
[151] D.V. Evans & R. Porter, Penetration of flexural waves through a periodically constrained thin elastic plate in *vacuo* and floating on water. *J. Eng. Math.* 58 (2007) 317–337.
[152] F. Fàbregas Flavià & A.H. Clément, Extension of Haskind's relations to cylindrical wave fields in the context of an interaction theory. *Appl. Ocean Res.* **66** (2017) 1–12.
[153] F. Fàbregas Flavià, C. McNatt, F. Rongère, A. Babarit & A.H. Clément, A numerical tool for the frequency domain simulation of large arrays of identical floating bodies in waves. *Ocean Eng.* **148** (2018) 299–311.
[154] X.-Q. Fang, D.-B. Wang & J.-X. Liu, Multiple scattering of elastic waves in metal-matrix composite materials with high volume concentration of particles. *European Journal of Mechanics A/Solids* **28** (2009) 377–386.
[155] V. Farafonov, V. Il'in, V. Ustimov & M. Prokopjeva, On the analysis of Waterman's approach in the electrostatic case. *J. Quant. Spectroscopy & Radiative Transfer* **178** (2016) 176–191.
[156] B. Feng, Y.H. Zheng, Y.G. You, B.J. Wu & S.W. Sheng, Interaction of linear waves with an array of infinitely long horizontal circular cylinders in a two-layer fluid of infinite depth. *Ocean Eng.* **35** (2008) 963–982.
[157] C. Feuillade & C.S. Clay, Anderson (1950) revisited. *J. Acoust. Soc. Amer.* **106** (1999) 553–564.
[158] R. Figari, G. Papanicolaou & J. Rubinstein, Remarks on the point interaction approximation. In: *Hydrodynamic Behavior and Interacting Particle Systems* (ed. G. Papanicolaou) pp. 45–55. New York: Springer, 1987.
[159] B. Fitzpatrick, E. De Sena & T. van Waterschoot, On the convergence of the multipole expansion method. *SIAM J. Numer. Anal.* **59** (2021) 2473–2499.
[160] F. Frezza, F. Mangini & N. Tedeschi, Introduction to electromagnetic scattering, part II: tutorial. *J. Opt. Soc. Amer.* A **37** (2020) 1300–1315.
[161] F. Frezza, L. Pajewski, C. Ponti & G. Schettini, Scattering by perfectly conducting circular cylinders buried in a dielectric slab through the cylindrical wave approach. *IEEE Trans. Antennas & Propag.* **57** (2009) 1208–1217.
[162] F. Frezza, L. Pajewski, C. Ponti & G. Schettini, Scattering by dielectric circular cylinders in a dielectric slab. *J. Opt. Soc. Amer.* A **27** (2010) 687–695.
[163] F. Frezza, L. Pajewski, C. Ponti & G. Schettini, Accurate wire-grid modelling of buried conducting cylindrical scatterers. *Nondestructive Testing & Evaluation* **27** (2012) 199–207.
[164] F. Frezza, L. Pajewski, C. Ponti & G. Schettini, Through-wall electromagnetic scattering by N conducting cylinders. *J. Opt. Soc. Amer.* A **30** (2013) 1632–1639.
[165] M. Fruhnert, I. Fernandez-Corbaton, V. Yannopapas & C. Rockstuhl, Computing the T-matrix of a scattering object with multiple plane wave illuminations. *Beilstein J. Nanotechnology* **8** (2017) 614–626. Correction: **9** (2018) 953.
[166] M. Ganesh & S.C. Hawkins, Simulation of acoustic scattering by multiple obstacles in three dimensions.

ANZIAM J. **50** (2008) C31–C45.
[167] M. Ganesh & S.C. Hawkins, A far-field based T-matrix method for three dimensional acoustic scattering. *ANZIAM J.* **50** (2008) C121–C136.
[168] M. Ganesh & S.C. Hawkins, A high-order algorithm for multiple electromagnetic scattering in three dimensions. *Numerical Algorithms* **50** (2009) 469–510.
[169] M. Ganesh & S.C. Hawkins, An efficient $\mathcal{O}(N)$ algorithm for computing $\mathcal{O}(N^2)$ acoustic wave interactions in large N-obstacle three dimensional configurations. *BIT* **55** (2015) 117–139.
[170] M. Ganesh & S.C. Hawkins, A fast high order algorithm for multiple scattering from large sound-hard three dimensional configurations. *J. Computational & Appl. Math.* **362** (2019) 324–340.
[171] M. Ganesh & S.C. Hawkins, A numerically stable T-matrix method for acoustic scattering by non-spherical particles with large aspect ratios and size parameters. *J. Acoust. Soc. Amer.* **151** (2022) 1978–1988.
[172] M. Ganesh, J.S. Hesthaven & B. Stamm, A reduced basis method for electromagnetic scattering by multiple particles in three dimensions. *J. Comp. Phys.* **231** (2012) 7756–7779.
[173] X. Garnaud & C.C. Mei, Wave-power extraction by a compact array of buoys. *J. Fluid Mech.* **635** (2009) 389–413.
[174] D. Gaspard & J.-M. Sparenberg, Multiple scattering model of the quantum random Lorentz gas. *Phys. Rev. A* **105** (2022) 042204 (17 pages).
[175] R. Gayen & A. Mondal, Water wave interaction with two symmetric inclined permeable plates. *Ocean Eng.* **124** (2016) 180–191.
[176] C. Geuzaine, A. Vion, R. Gaignaire, P. Dular & R.V. Sabariego, An amplitude finite element formulation for multiple-scattering by a collection of convex obstacles. *IEEE Trans. Magnetics* **46** (2010) 2963–2966.
[177] A. Gibbs, S.N. Chandler-Wilde, S. Langdon & A. Moiola, A high-frequency boundary element method for scattering by a class of multiple obstacles. *IMA J. Numer. Anal.* **41** (2021) 1197–1239.
[178] B. Gilvey & J. Trevelyan, A comparison of high-order and plane wave enriched boundary element basis functions for Helmholtz problems. *Engineering Analysis with Boundary Elements* **122** (2021) 190–201.
[179] Z. Gimbutas & L. Greengard, A fast and stable method for rotating spherical harmonic expansions. *J. Comp. Phys.* **228** (2009) 5621–5627.
[180] Z. Gimbutas & L. Greengard, Fast multi-particle scattering: a hybrid solver for the Maxwell equations in microstructured materials. *J. Comp. Phys.* **232** (2013) 22–32.
[181] D. Givoli, Scattering matrix of a nearly circular cylinder. *Wave Motion* **30** (1999) 239–251.
[182] G. Gouesbet & G. Grehan, Generalized Lorenz–Mie theory for assemblies of spheres and aggregates. *Journal of Optics A: Pure and Applied Optics* **1** (1999) 706–712.
[183] A.L. Gower, M.J.A. Smith, W.J. Parnell & I.D. Abrahams, Reflection from a multi-species material and its transmitted effective wavenumber. *Proc. Roy. Soc.* A **474** (2018) 20170864 (27 pages).
[184] L. Greengard & J.-Y. Lee, Stable and accurate integral equation methods for scattering problems with multiple material interfaces in two dimensions. *J. Comp. Phys.* **231** (2012) 2389–2395.
[185] J.R. Grice, P.H. Taylor & R. Eatock Taylor, Near-trapping effects for multi-column structures in deterministic and random waves. *Ocean Eng.* **58** (2013) 60–77.
[186] B. Guizal & D. Felbacq, Electromagnetic beam diffraction by a finite strip grating. *Optics Comm.* **165** (1999) 1–6.
[187] T.R. Hahn, Low frequency sound scattering from spherical assemblages of bubbles using effective medium theory. *J. Acoust. Soc. Amer.* **122** (2007) 3252–3267.
[188] A.-K. Hamid, M.I. Hussein & M. Hamid, Electromagnetic scattering by a system of dielectric spheres coated with a dielectric shell. *ACES J.* **18** (2003) 91–96.
[189] J.E. Hansen (ed.), *Spherical Near-Field Antenna Measurements*. London: Peter Peregrinus Ltd., 1988.
[190] S. Hao, P.G. Martinsson & P. Young, An efficient and highly accurate solver for multi-body acoustic scattering problems involving rotationally symmetric scatterers. *Computers & Math. with Applications* **69** (2015) 304–318.
[191] Y. Harness & A. Ditkowski, The null-field method: a reconstruction kernel approach. *J. Comp. Phys.* **248** (2013) 127–146.
[192] S.M. Hasheminejad & R. Avazmohammadi, Harmonic wave diffraction by two circular cavities in a poroelastic formation. *Soil Dynamics & Earthquake Engineering* **27** (2007) 29–41.

[193] S.C. Hawkins, Algorithm 1009: MieSolver–an object-oriented Mie series software for wave scattering by cylinders. *ACM Trans. Math. Software* **46** (2020), article no. 19 (28 pages).
[194] B. He & W.C. Chew, Addition theorem. In: *Modeling and Computations in Electromagnetics* (ed. H. Ammari) pp. 203–226. Berlin: Springer, 2008.
[195] E.J. Heller, *The Semiclassical Way to Dynamics and Spectroscopy*. Princeton: Princeton University Press, 2018.
[196] J. Helsing & A. Karlsson, Physical-density integral equation methods for scattering from multi-dielectric cylinders. *J. Comp. Phys.* **387** (2019) 14–29.
[197] W. Hergert & T. Wriedt (eds.), *The Mie Theory: Basics and Applications*. Heidelberg: Springer, 2012.
[198] A.J. Hermans, Interaction of free-surface waves with floating flexible strips. *J. Eng. Math.* **49** (2004) 133–147.
[199] D.P. Hewett, S. Langdon & S.N. Chandler-Wilde, A frequency-independent boundary element method for scattering by two-dimensional screens and apertures. *IMA J. Numerical Analysis* **35** (2015) 1698–1728.
[200] B.A. Howarth & T.J.F. Pavlasek, Multipole induction: a novel formulation of multiple scattering of scalar waves. *J. Appl. Phys.* **44** (1973) 1162–1167. Erratum: **44** (1973) 4255.
[201] M. Howe, *Acoustics and Aerodynamic Sound*. Cambridge: Cambridge University Press, 2015.
[202] G.C. Hsiao, R.E. Kleinman & G.F. Roach, Weak solutions of fluid-solid interaction problems. *Mathematische Nachrichten* **218** (2000) 139–163.
[203] G. Hu, A. Mantile & M. Sini, Direct and inverse acoustic scattering by a collection of extended and point-like scatterers. *Multiscale Modeling & Simulation* **12** (2014) 996–1027.
[204] G. Hu, A. Mantile, M. Sini & T. Yin, Direct and inverse time-harmonic elastic scattering from point-like and extended obstacles. *Inverse Problems & Imaging* **14** (2020) 1025–1056.
[205] G. Hu & M. Sini, Elastic scattering by finitely many point-like obstacles. *J. Math. Phys.* **54** (2013) 042901 (16 pages).
[206] J. Hu, Y.-K. Li, Z. Nie & H. Zhao, Modal characteristic basis function method for solving scattering from multiple conducting bodies of revolution. *IEEE Trans. Antennas & Propag.* **62** (2014) 870–877.
[207] L. Hu, L.-W. Li & R. Mittra, Electromagnetic scattering by finite periodic arrays using the characteristic basis function and adaptive integral methods. *IEEE Trans. Antennas & Propag.* **58** (2010) 3086–3090.
[208] H. Huang, L. Tsang, A. Colliander, R. Shah, X. Xu & S. Yueh, Multiple scattering of waves by complex objects using hybrid method of T-matrix and Foldy–Lax equations using vector spherical waves and vector spheroidal waves. *Progress In Electromagnetics Research* **168** (2020) 87–111.
[209] J. Huang & R. Porter, Water wave propagation through arrays of closely spaced surface-piercing vertical barriers. *J. Fluid Mech.* **960** (2023) A20 (33 pages).
[210] J. Huang & R. Porter, Scattering of water waves by multiple rows of vertical thin barriers. *Wave Motion* **125** (2024) 103258 (25 pages).
[211] K. Huang & P. Li, A two-scale multiple scattering problem. *Multiscale Modeling & Simulation* **8** (2010) 1511–1534.
[212] K. Huang, P. Li & H. Zhao, An efficient algorithm for the generalized Foldy–Lax formulation. *J. Comp. Phys.* **234** (2013) 376–398.
[213] K. Huang, K. Solna & H. Zhao, Generalized Foldy–Lax formulation. *J. Comp. Phys.* **229** (2010) 4544–4553.
[214] T. Huttunen, J.P. Kaipio & P. Monk, An ultra-weak method for acoustic fluid-solid interaction. *J. Computational & Appl. Math.* **213** (2008) 166–185.
[215] V.B. Il'in, D.G. Turichina, V.G. Farafonov, S.I. Laznevoi, G.A. Gontcharov, A.A. Marchuk, A.V. Mosenkov, D.M. Poliakov, S.S. Savchenko, A.A. Smirnov & M.S. Prokopjeva, A new practical approach to light scattering by spheroids with the use of spheroidal and spherical function bases. *J. Quant. Spectroscopy & Radiative Transfer* **311** (2023) 108759 (19 pages).
[216] W.A. Imbriale & R. Mittra, The two-dimensional inverse scattering problem. *IEEE Trans. Antennas & Propag.* **AP-18** (1970) 633–642.
[217] S. Itou, Diffraction of an antiplane shear wave by two coplanar Griffith cracks in an infinite elastic medium. *Int. J. Solids Struct.* **16** (1980) 1147–1153.
[218] S. Itou, Transient dynamic stresses around two equal circular cavities in an infinite elastic strip. *Archive*

of Applied Mechanics **64** (1994) 192–205.
[219] Ye.A. Ivanov, *Diffraction of Electromagnetic Waves on Two Bodies*. NASA Technical Translation F-597, Washington, D.C., 1970.
[220] S.M. Ivansson, Multiple underwater sound scattering by cylindrically symmetric anomalies. *J. Acoust. Soc. Amer.* **147** (2020) 1429–1440.
[221] Y. Iwashimizu, Scattering of elastic waves by a movable rigid sphere embedded in an infinite elastic solid. *J. Sound Vib.* **21** (1972) 463–469. Errata **33** (1974) 374–375.
[222] Y. Iwashimizu, Scattering of shear waves by an elastic sphere embedded in an infinite elastic solid. *J. Sound Vib.* **40** (1975) 267–271.
[223] D.L. James, J. Barbič & D.K. Pai, Precomputed acoustic transfer: output-sensitive, accurate sound generation for geometrically complex vibration sources. *ACM Transactions on Graphics* **25** (2006) 987–995.
[224] F.A.J.L. James, The conservation of energy, theories of absorption and resonating molecules, 1851–1854: G. G. Stokes, A. J. Ångström and W. Thomson. *Notes & Records Roy. Soc.* **38** (1983) 79–107.
[225] V. Jandieri & K. Yasumoto, Electromagnetic scattering by layered cylindrical arrays of circular rods. *IEEE Trans. Antennas & Propag.* **59** (2011) 2437–2441.
[226] C. Jelich, W. Zhao, S. Chen & S. Marburg, Fast multipole boundary element method for the acoustic analysis of finite periodic structures. *Comput. Methods Appl. Mech. Engrg.* **391** (2022) 114528 (23 pages).
[227] C. Jerez-Hanckes, C. Pérez-Arancibia & C. Turc, Multitrace/singletrace formulations and domain decomposition methods for the solution of Helmholtz transmission problems for bounded composite scatterers. *J. Comp. Phys.* **350** (2017) 343–360.
[228] C. Jerez-Hanckes & J. Pinto, High-order Galerkin method for Helmholtz and Laplace problems on multiple open arcs. *ESAIM: Mathematical Modelling & Numerical Analysis* **54** (2020) 1975–2009.
[229] T.S. Jerome, Y.A. Ilinskii, E.A. Zabolotskaya & M.F. Hamilton, Acoustic radiation force on a compressible spheroid. *J. Acoust. Soc. Amer.* **148** (2020) 2403–2415.
[230] X. Ji, S. Liu, H.B. Bingham & J. Li, Multi-directional random wave interaction with an array of cylinders. *Ocean Eng.* **110** (2015) 62–77.
[231] J.M. Jin, V.V. Liepa & C.T. Tai, A volume-surface integral equation for electromagnetic scattering by inhomogeneous cylinders. *J. Electromagnetic Waves & Applications* **2** (1988) 573–588.
[232] H.T. Johansson & C. Forssén, Fast and accurate evaluation of Wigner $3j$, $6j$, and $9j$ symbols using prime factorization and multiword integer arithmetic. *SIAM J. Sci. Computing* **38** (2016) A376–A384.
[233] L.R. Johnson, Scattering of elastic waves by a spheroidal inclusion. *Geophys. J. Int.* **212** (2018) 1829–1858.
[234] M.C. Junger & D. Feit, *Sound, Structures, and their Interaction*, second edition. New York: Acoustical Society of America, 1993.
[235] M. Kachanovska, A new class of uniformly stable time-domain Foldy–Lax models for scattering by small particles. Acoustic sound-soft scattering by circles. *Multiscale Modeling & Simulation* **22** (2024) 1–38.
[236] H. Kagemoto, Revisiting the complete wave transmission and reflection due to an array of 2-D surface-piercing truncated vertical plates fixed in regular incident waves. *Ocean Eng.* **38** (2011) 976–982.
[237] H. Kagemoto, Revisiting the complete wave transmission and reflection due to an array of 2-D surface-piercing truncated vertical plates fixed in regular incident waves (part 2). *Ocean Eng.* **108** (2015) 173–180.
[238] H. Kagemoto & D.K.P. Yue, Hydrodynamic interaction analyses of very large floating structures. *Marine Structures* **6** (1993) 295–322.
[239] I. Kahraman, E. Tunca & B. Uğurlu, Eliminating the irregular frequencies in wave-body interaction: an investigation for accuracy assessment with higher-order elements. *J. Fluids & Struct.* **116** (2023) 103807 (25 pages).
[240] S. Kanaun, *Heterogeneous Media: Local Fields, Effective Properties, and Wave Propagation*. Amsterdam: Elsevier, 2021.
[241] S.K. Kanaun & V.M. Levin, Propagation of shear elastic waves in composites with a random set of spherical inclusions (effective field approach). *Int. J. Solids Struct.* **42** (2005) 3971–3997.
[242] S.K. Kanaun & V.M. Levin, Propagation of longitudinal elastic waves in composites with a random

set of spherical inclusions (effective field approach). *Archive of Applied Mechanics* **77** (2007) 627–651.
[243] S.K. Kanaun & V.M. Levin, *Self-Consistent Methods for Composites*, vol. 2: *Wave Propagation in Heterogeneous Materials*. Dordrecht: Springer, 2008.
[244] D. Karmakar, J. Bhattacharjee & C. Guedes Soares, Scattering of gravity waves by multiple surface-piercing floating membrane. *Appl. Ocean Res.* **39** (2012) 40–52.
[245] D. Karmakar & C. Guedes Soares, Wave transformation due to multiple bottom-standing porous barriers. *Ocean Eng.* **80** (2014) 50–63.
[246] E. Kausel, *Fundamental Solutions in Elastodynamics*. New York: Cambridge University Press, 2006.
[247] J. Kawahara, T. Ohno & K. Yomogida, Attenuation and dispersion of antiplane shear waves due to scattering by many two-dimensional cavities. *J. Acoust. Soc. Amer.* **125** (2009) 3589–3596.
[248] T. Khajah, Iterative on surface radiation conditions for fast and reliable single and multiple scattering analyses of arbitrarily shaped obstacles. *Comput. Methods Appl. Mech. Engrg.* **420** (2024) 116715 (17 pages).
[249] J.-Y. Kim, Models for wave propagation in two-dimensional random composites: a comparative study. *J. Acoust. Soc. Amer.* **127** (2010) 2201–2209.
[250] K.T. Kim, Efficient recursive generation of the scalar spherical multipole translation matrix. *IEEE Trans. Antennas & Propag.* **55** (2007) 3484–3494.
[251] K.T. Kim, Comparison and improvement of the computational efficiencies of two FFT-based iterative solution methods for the scalar multiple-scattering equation. *Comm. Comput. Phys.* **5** (2009) 108–125.
[252] K.T. Kim & B.A. Kramer, Direct determination of the T-matrix from a MoM impedance matrix computed using the Rao–Wilton–Glisson basis function. *IEEE Trans. Antennas & Propag.* **61** (2013) 5324–5327.
[253] Y. Kinoshita, S. Biwa & Z. Wang, Multiple scattering of flexural waves on Mindlin plates with circular scatterers. *Z. Angew. Math. Mech.* **101** (2021) e202000221 (15 pages).
[254] M. Kitahara & K. Nakagawa, Multiple scattering calculations by Born series integral equations. In: *Review of Progress in Quantitative Nondestructive Evaluation*, vol. 17 (eds. D.O. Thompson & D.E. Chimenti), pp. 33–40. New York: Plenum Press, 1998.
[255] E. Klaseboer, F.D.E. Charlet, B.-C. Khoo, Q. Sun & D.Y.C. Chan, Eliminating the fictitious frequency problem in BEM solutions of the external Helmholtz equation. *Engineering Analysis with Boundary Elements* **109** (2019) 106–116.
[256] A. Kleanthous, T. Betcke, D.P. Hewett, M.W. Scroggs & A.J. Baran, Calderón preconditioning of PMCHWT boundary integral equations for scattering by multiple absorbing dielectric particles. *J. Quant. Spectroscopy & Radiative Transfer* **224** (2019) 383–395.
[257] A.A. Kleshchev & E.I. Kuznetsova, Interaction of acoustic scatterers. *Acoustical Physics* **57** (2011) 505–510.
[258] A.A. Klëshchev & L.S. Sheiba, Scattering of a sound wave by ideal prolate spheroids. *Soviet Physics – Acoustics* **16** (1970) 219–222.
[259] Y.A. Kobelev, Multiple monopole scattering of sound waves from spherical particles in liquid and elastic media. *Acoustical Physics* **57** (2011) 731–740.
[260] A.L. Kohout & M.H. Meylan, Wave scattering by multiple floating elastic plates with spring or hinged boundary conditions. *Marine Structures* **22** (2009) 712–729.
[261] A.L. Kohout, M.H. Meylan, S. Sakai, K. Hanai, P. Leman & D. Brossard, Linear water wave propagation through multiple floating elastic plates of variable properties. *J. Fluids & Struct.* **23** (2007) 649–663.
[262] D.N. Konispoliatis & S.A. Mavrakos, Hydrodynamic analysis of an array of interacting free-floating oscillating water column (OWC's) devices. *Ocean Eng.* **111** (2016) 179–197.
[263] V.A. Korneev & L.R. Johnson, Scattering of P and S waves by a spherically symmetric inclusion. *Pure & Appl. Geophys.* **147** (1996) 675–718.
[264] A.D. Kotsis & J.A. Roumeliotis, Acoustic scattering by a penetrable spheroid. *Acoustical Physics* **54** (2008) 153–167.
[265] D. Koyama, An a priori error estimate of the Dirichlet-to-Neumann finite element method for multiple scattering problems. *Japan J. Industrial & Appl. Math.* **31** (2014) 165–192.
[266] R. Kress, Minimizing the condition number of boundary integral operators in acoustic and electromagnetic scattering. *Quart. J. Mech. Appl. Math.* **38** (1985) 323–341.

[267] G.A. Kriegsmann, A. Taflove and K.R. Umashankar, A new formulation of electromagnetic wave scattering using an on-surface radiation boundary condition approach. *IEEE Trans. Antennas & Propag.* **AP-35** (1987) 153–161.
[268] G. Kristensson, *Scattering of Electromagnetic Waves by Obstacles*. Edison: SciTech Publishing, 2016.
[269] V.D. Kubenko, I.V. Yanchevskyi & V.O. Liskin, Kinematic excitation of fluid in an infinite cylindrical cavity with two spherical bodies. *Int. Appl. Mech.* **60** (2024) 20–33.
[270] K.A. Kuo, H.E.M. Hunt & M.F.M. Hussein, The effect of a twin tunnel on the propagation of ground-borne vibration from an underground railway. *J. Sound Vib.* **330** (2011) 6203–6222.
[271] V.D. Kupradze, Fundamental problems in the mathematical theory of diffraction (Steady state processes). US National Bureau of Standards Report 2008, 1952, viii+160 pp. Translated from the Russian original, published in 1935.
[272] A. Kyurkchan & E.A. Skorodoumova, Modeling the characteristics of the waves scattering by a group of scatterers. *J. Quant. Spectroscopy & Radiative Transfer* **100** (2006) 207–219.
[273] A.G. Kyurkchan, On solving the problem of wave scattering by several bodies. *Doklady Mathematics* **53** (1996) 453–457.
[274] A.G. Kyurkchan, B.Y. Sternin & V.E. Shatalov, Singularities of continuation of wave fields. *Physics–Uspekhi* **39** (1996) 1221–1242.
[275] J. Lai, M. Kobayashi & L. Greengard, A fast solver for multi-particle scattering in a layered medium. *Optics Express* **22** (2014) 20481–20499.
[276] J. Lai & P. Li, A framework for simulation of multiple elastic scattering in two dimensions. *SIAM J. Sci. Computing* **41** (2019) A3276–A3299.
[277] J.-K. Lee, Y.-B. Han & Y.-J. Ahn, SH wave scattering problems for multiple orthotropic elliptical inclusions. *Advances in Mechanical Engineering* **2013** (2013) 370893 (14 pages).
[278] S.-C. Lee, Light scattering by closely spaced parallel cylinders embedded in a finite dielectric slab. *J. Opt. Soc. Amer.* A **16** (1999) 1350–1361.
[279] S.-C. Lee, Near field scattering by multiple infinite cylinders in an absorbing medium at oblique incidence. *J. Quant. Spectroscopy & Radiative Transfer* **114** (2013) 65–72.
[280] S.-C. Lee, Scattering by multiple perfectly conducting cylinders buried in a lossy half space. *J. Quant. Spectroscopy & Radiative Transfer* **146** (2014) 314–320.
[281] S.-C. Lee, Scattering of evanescent wave by multiple parallel infinite cylinders near a surface. *J. Quant. Spectroscopy & Radiative Transfer* **147** (2014) 252–260.
[282] S.-C. Lee, Scattering at oblique incidence by multiple cylinders in front of a surface. *J. Quant. Spectroscopy & Radiative Transfer* **182** (2016) 119–127.
[283] S.-C. Lee & J.A. Grzesik, Light scattering by closely spaced parallel cylinders embedded in a semi-infinite dielectric medium. *J. Opt. Soc. Amer.* A **15** (1998) 163–173.
[284] V.W. Lee & M.D. Trifunac, Response of tunnels to incident SH-waves. *J. Eng. Mech.* **105** (1979) 643–659.
[285] W.-M. Lee, Acoustic scattering by multiple elliptical cylinders using collocation multipole method. *J. Comp. Phys.* **231** (2012) 4597–4612.
[286] W.-M. Lee, Three-dimensional acoustic scattering by multiple spheres using collocation multipole method. *Int. J. Solids Struct.* **63** (2015) 39–49.
[287] W.M. Lee & J.T. Chen, Scattering of flexural wave in a thin plate with multiple circular inclusions by using the null-field integral equation approach. *J. Sound Vib.* **329** (2010) 1042–1061.
[288] W.-M. Lee & J.-T. Chen, Scattering of flexural wave in a thin plate with multiple circular holes by using the multipole Trefftz method. *Int. J. Solids Struct.* **47** (2010) 1118–1129.
[289] W.M. Lee & J.T. Chen, Scattering of flexural wave in a thin plate with multiple circular inclusions by using the multipole method. *Int. J. Mechanical Sciences* **53** (2011) 617–627.
[290] W.M. Lee & J.T. Chen, The collocation multipole method for solving multiple scattering problems with circular boundaries. *Engineering Analysis with Boundary Elements* **48** (2014) 102–112.
[291] W.M. Lee & J.T. Chen, Computation of scattering of a plane wave from multiple prolate spheroids using the collocation multipole method. *J. Acoust. Soc. Amer.* **140** (2016) 2235–2246.
[292] W.M. Lee & J.T. Chen, Construction of dynamic Green's function for an infinite acoustic field with multiple prolate spheroids. *Acta Mechanica* **233** (2022) 5021–5041.
[293] W.M. Lee, J.T. Chen & W.M. Young, Dynamic Green's functions for multiple circular inclusions

with imperfect interfaces using the collocation multipole method. *Engineering Analysis with Boundary Elements* **94** (2018) 113–121.
[294] J.W. Leonard, M.-C. Huang & R.T. Hudspeth, Hydrodynamic interference between floating cylinders in oblique seas. *Appl. Ocean Res.* **5** (1983) 158–166.
[295] O. Leseur, R. Pierrat, J.J. Sáenz & R. Carminati, Probing two-dimensional Anderson localization without statistics. *Phys. Rev. A* **90** (2014) 053827 (5 pages).
[296] C. Lessig, T. de Witt & E. Fiume, Efficient and accurate rotation of finite spherical harmonics expansions. *J. Comp. Phys.* **231** (2012) 243–250.
[297] E.M. Lewandowski, Multi-vessel seakeeping computations with linear potential theory. *Ocean Eng.* **35** (2008) 1121–1131.
[298] A.-j. Li & Y. Liu, Hydrodynamic performance and energy absorption of multiple spherical absorbers along a straight coast. *Phys. Fluids* **34** (2022) 117102 (17 pages).
[299] A.-j. Li, X.-l. Sun, Y. Liu & H.-j. Li, Analysis of water wave interaction with multiple submerged porous reef balls. *J. Eng. Math.* **127** (2021) article 26 (24 pages).
[300] H. Li & Y. Zhao, The interaction between two close-to-touching convex acoustic subwavelength resonators. *Multiscale Modeling & Simulation* **21** (2023) 804–826.
[301] L.-W. Li, X.-K. Kang & M.-S. Leong, *Spheroidal Wave Functions in Electromagnetic Theory*. New York: Wiley, 2002.
[302] M. Li, H. Guan, H. Zhang & J. Liu, Three-dimensional investigation of wave-pile group interaction using the scaled boundary finite element method. Part II: Application results. *Ocean Eng.* **64** (2013) 185–195.
[303] M. Li, H. Zhang, H. Guan & G. Lin, Three-dimensional investigation of wave-pile group interaction using the scaled boundary finite element method. Part I: Theoretical developments. *Ocean Eng.* **64** (2013) 174–184.
[304] P. Li and A. Wood, A two-dimensional Helmholtz equation solution for the multiple cavity scattering problem. *J. Comp. Phys.* **240** (2013) 100–120.
[305] P. Li and A.W. Wood, Electromagnetic scattering by multiple cavities embedded in the infinite 2D ground plane. *ACES J.* **29** (2014) 584–593.
[306] T.-b. Li & M. Ueda, Sound scattering of a spherical wave incident on a cylinder. *J. Acoust. Soc. Amer.* **87** (1990) 1871–1879.
[307] Z.J. Li, Z.S. Wu, T. Qu, Q.C. Shang & L. Bai, Multiple scattering of zero-order Bessel beam with arbitrary incidence by an aggregate of uniaxial anisotropic spheres. *J. Quant. Spectroscopy & Radiative Transfer* **169** (2016) 1–13.
[308] Z.-J. Li, Z.-S. Wu, Y. Shi, L. Bai & H.-Y. Li, Multiple scattering of electromagnetic waves by an aggregate of uniaxial anisotropic spheres. *J. Opt. Soc. Amer.* A **29** (2012) 22–31.
[309] J. Liao, Foldy–Lax approximation on multiple scattering by many small scatterers, *Applicable Analysis* **92** (2013) 2547–2560.
[310] J. Liao & C. Ji, Extended Foldy–Lax approximation on multiple scattering. *Mathematical Modelling and Analysis* **19** (2014) 85–98.
[311] W.-I Liao, Application of transition matrix to scattering of elastic waves in half-space by a surface scatterer. *Wave Motion* **48** (2011) 325–334.
[312] R. Lim, A more stable transition matrix for acoustic target scattering by elongated objects. *J. Acoust. Soc. Amer.* **138** (2015) 2266–2278.
[313] C.M. Linton & P.A. Martin, Multiple scattering by multiple spheres: a new proof of the Lloyd-Berry formula. *SIAM J. Appl. Math.* **66** (2006) 1649–1668.
[314] H.-T. Liu, Z.-G. Zhou & W.-J. Wu, Dynamic stress intensity factors of two 3D rectangular cracks in a transversely isotropic elastic material under a time-harmonic elastic P-wave. *Wave Motion* **51** (2014) 1309–1324.
[315] J. Liu, A. Guo & H. Li, Analytical solution for the linear wave diffraction by a uniform vertical cylinder with an arbitrary smooth cross-section. *Ocean Eng.* **126** (2016) 163–175.
[316] W. Liu & R.D. Kriz, Multiple wave scattering in fiber-reinforced composites: micromechanical viewpoint. *Wave Motion* **27** (1998) 223–244.
[317] X. Liu, S. Greenhalgh & B. Zhou, Scattering of plane transverse waves by spherical inclusions in a poroelastic medium. *Geophys. J. Int.* **176** (2009) 938–950.

[318] Y. Liu, *Fast Multipole Boundary Element Method: Theory and Applications in Engineering*. Cambridge: Cambridge University Press, 2009.
[319] Y. Liu, R.-S. Wu & C.F. Ying, Scattering of elastic waves by an elastic or viscoelastic cylinder. *Geophys. J. Int.* **142** (2000) 439–460.
[320] Z. Liu & L.-W. Cai, Three-dimensional multiple scattering of elastic waves by spherical inclusions. *J. Vibration & Acoustics* **131** (2009) 061005 (11 pages).
[321] J.A. Lock & M.I. Mishchenko, A persistent feature of multiple scattering of waves in the time-domain: A tutorial. *J. Quant. Spectroscopy & Radiative Transfer* **162** (2015) 221–240.
[322] V.A. Loiko & A.A. Miskevich, Multiple scattering of light in ordered particulate media. In: *Multiple Light Scattering, Radiative Transfer and Remote Sensing* (ed. A. Kokhanovsky) pp. 101–230. Chaim: Springer, 2018.
[323] V. Losenicky, L. Jelinek, M. Capek & M. Gustafsson, Method of moments and T-matrix hybrid. *IEEE Trans. Antennas & Propag.* **70** (2022) 3560–3574.
[324] E. Loukogeorgaki, I. Boufidi & I.K. Chatjigeorgiou, Performance of an array of oblate spheroidal heaving wave energy converters in front of a wall. *Water* **12** (2020) 188 (21 pages).
[325] E. Lucente, A. Monorchio & R. Mittra, An iteration-free MoM approach based on excitation independent characteristic basis functions for solving large multiscale electromagnetic scattering problems. *IEEE Trans. Antennas & Propag.* **56** (2008) 999–1007.
[326] M. Lucido, G. Panariello & F. Schettino, Electromagnetic scattering by multiple perfectly conducting arbitrary polygonal cylinders. *IEEE Trans. Antennas & Propag.* **56** (2008) 425–436.
[327] M. Luna, J. Terradas, R. Oliver & J.L. Ballester, Transverse oscillations of systems of coronal loops. *Astrophysical J.* **692** (2009) 1582–1589.
[328] F. Luppé, J.-M. Conoir & A.N. Norris, Effective wave numbers for thermo-viscoelastic media containing random configurations of spherical scatterers. *J. Acoust. Soc. Amer.* **131** (2012) 1113–1120.
[329] F. Luppé, J.-M. Conoir & T. Valier-Brasier, Longitudinal and transverse coherent waves in media containing randomly distributed spheres. *Wave Motion* **115** (2022) 103082 (20 pages). Corrects [328].
[330] D. Mackowski, The extension of Mie theory to multiple spheres. In [197], pp. 223–256, 2012.
[331] D.W. Mackowski, Discrete dipole moment method for calculation of the T matrix for nonspherical particles. *J. Opt. Soc. Amer.* A **19** (2002) 881–893.
[332] D.W. Mackowski, Exact solution for the scattering and absorption properties of sphere clusters on a plane surface. *J. Quant. Spectroscopy & Radiative Transfer* **109** (2008) 770–788.
[333] W.M. Madigosky & K.P. Scharnhorst, Acoustic wave propagation in materials with inclusions or voids. In: *Sound and Vibration Damping with Polymers* (eds. R.D. Corsaro & L.H. Sperling), ACS Symposium Series **424**, pp. 229–247. Washington, DC: American Chemical Society, 1990.
[334] A.O. Maksimov, Features of Rayleigh scattering by a particle near an interface. *Acoustical Physics* **70** (2024) 1–8.
[335] H.D. Maniar & J.N. Newman, Wave diffraction by a long array of cylinders. *J. Fluid Mech.* **339** (1997) 309–330.
[336] G.D. Manolis & P.S. Dineva, Elastic waves in continuous and discontinuous geological media by boundary integral equation methods: A review. *Soil Dynamics & Earthquake Engineering* **70** (2015) 11–29.
[337] S. Marburg, The Burton and Miller method: unlocking another mystery of its coupling parameter. *J. Computational Acoustics* **24** (2016) 1550016 (20 pages).
[338] S. Marburg & T.-W. Wu, Treating the phenomenon of irregular frequencies. In: *Computational Acoustics of Noise Propagation in Fluids – Finite and Boundary Element Methods* (eds. S. Marburg & B. Nolte) pp. 411–434. Berlin: Springer, 2008.
[339] L. Marin, Electromagnetic scattering from two circular, coaxial disks. *J. Math. Phys.* **15** (1974) 1606–1613.
[340] J. Markkanen, Numerical analysis of the potential formulation of the volume integral equation for electromagnetic scattering. *Radio Sci.* **52** (2017) 1301–1311.
[341] J. Markkanen & A.J. Yuffa, Fast superposition T-matrix solution for clusters with arbitrarily-shaped constituent particles. *J. Quant. Spectroscopy & Radiative Transfer* **189** (2017) 181–188.
[342] P.A. Martin, Two-dimensional acoustic scattering, conformal mapping, and the Rayleigh hypothesis. *J. Acoust. Soc. Amer.* **132** (2012) 2184–2188.
[343] P.A. Martin, Acoustic scattering by one bubble before 1950: Spitzer, Willis, and Division 6. *J. Acoust.*

Soc. Amer. **146** (2019) 920–926.
[344] P.A. Martin & A.J. Hull, Dynamic response of an infinite thin plate loaded with concentrated masses. *Wave Motion* **98** (2020) 102643 (12 pages).
[345] P.A. Martin & A. Maurel, Multiple scattering by random configurations of circular cylinders: weak scattering without closure assumptions. *Wave Motion* **45** (2008) 865–880.
[346] P.A. Martin & A.T. Skvortsov, Scattering by a sphere in a tube, and related problems. *J. Acoust. Soc. Am.* **148** (2020) 191–200.
[347] T. Martin, T-matrix method for closely adjacent obstacles. *J. Quant. Spectroscopy & Radiative Transfer* **234** (2019) 40–46.
[348] M. Martí-Sabaté, B. Djafari-Rouhani & D. Torrent, Bound states in the continuum in circular clusters of scatterers. *Phys. Rev. Res.* **5** (2023) 013131 (8 pages).
[349] V.V. Matus & V.F. Emets, T-matrix method formulation applied to the study of flexural waves scattering from a through obstacle in a plate. *J. Sound Vib.* **329** (2010) 2843–2850.
[350] J.D. Maynard, Wave propagation in arrays of scatterers. Tutorial: part 1. *Acoustics Today* **4**, 4 (2008) 12–21.
[351] J.D. Maynard, Wave propagation in arrays of scatterers. Tutorial: part 2. *Acoustics Today* **6**, 1 (2010) 13–21.
[352] D. Maystre, Electromagnetic scattering by a set of objects: an integral method based on scattering properties. *Progress In Electromagnetics Research* **57** (2006) 55–84.
[353] G. McCauley, H. Wolgamot, J. Orszaghova & S. Draper, Linear hydrodynamic modelling of arrays of submerged oscillating cylinders. *Appl. Ocean Res.* **81** (2018) 1–14.
[354] P. McIver, An extended wide-spacing approximation for two-dimensional water-wave problems in infinite depth. *Quart. J. Mech. Appl. Math.* **67** (2014) 445–468.
[355] N.W. McLachlan, *Theory and Application of Mathieu Functions*. New York: Dover, 1964.
[356] J.C. McNatt, V. Venugopal & D. Forehand, A novel method for deriving the diffraction transfer matrix and its application to multi-body interactions in water waves. *Ocean Eng.* **94** (2015) 173–185.
[357] M. Medvinsky, S. Tsynkov & E. Turkel, Solution of three-dimensional multiple scattering problems by the method of difference potentials. *Wave Motion* **107** (2021) 102822 (18 pages).
[358] S.A. Meguid & X.D. Wang, The dynamic interaction of a microcrack with a main crack under antiplane loading. *Int. J. Solids Struct.* **31** (1994) 1085–1097.
[359] S.A. Meguid & X.D. Wang, On the dynamic interaction between a microdefect and a main crack. *Proc. Roy. Soc.* A **448** (1995) 449–464.
[360] R. Mehra, N. Raghuvanshi, L. Antani, A. Chandak, S. Curtis & D. Manocha, Wave-based sound propagation in large open scenes using an equivalent source formulation. *ACM Transactions on Graphics* **32** (2013) 19:1–19:13.
[361] C. Meiners, J. Psilopoulos & A.F. Jacob, Determining the scattered field of dense particles using recursive algorithms. *Microwave & Optical Technology Letters* **40** (2004) 523–529.
[362] W. Meng & L. Wang, Bounds for truncation errors of Graf's and Neumann's addition theorems. *Numerical Algorithms* **72** (2016) 91–106.
[363] P. Mercadé Ruiz, F. Ferri & J.P. Kofoed, Water-wave diffraction and radiation by multiple three-dimensional bodies over a mild-slope bottom. *Ocean Eng.* **143** (2017) 163–176.
[364] G. Miao, T. Saitoh & H. Ishida, Water wave interaction of twin large scale caissons with a small gap between. *Coastal Engineering Journal* **43** (2001) 39–58.
[365] G.P. Miao, Y.Z. Liu & Z.X. Mi, Computation of hydrodynamic forces on vibrating multiple large vertical cylinders of arbitrary section. *Marine Structures* **6** (1993) 279–294.
[366] A. Mirzaei, A.E. Miroshnichenko, I.V. Shadrivov & Y.S. Kivshar, Optical metacages. *Phys. Rev. Lett.* **115** (2015) 215501 (5 pages).
[367] M.I. Mishchenko, "Independent" and "dependent" scattering by particles in a multi-particle group. *OSA Continuum* **1** (2018) 243–260.
[368] M.I. Mishchenko, J.M. Dlugach, M.A. Yurkin, L. Bi, B. Cairns, L. Liu, R.L. Panetta, L.D. Travis, P. Yang & N.T. Zakharova, First-principles modeling of electromagnetic scattering by discrete and discretely heterogeneous random media. *Physics Reports* **632** (2016) 1–75.
[369] M.I. Mishchenko, L. Liu, D.W. Mackowski, B. Cairns & G. Videen, Multiple scattering by random particulate media: exact 3D results. *Optics Express* **15** (2007) 2822–2836.

[370] M.I. Mishchenko, L. Liu & G. Videen, Conditions of applicability of the single-scattering approximation. *Optics Express* **15** (2007) 7522–7527.
[371] M.I. Mishchenko & M.A. Yurkin, On the concept of random orientation in far-field electromagnetic scattering by nonspherical particles. *Optics Letters* **42** (2017) 494–497.
[372] F.G. Mitri, Pushing, pulling and electromagnetic radiation force cloaking by a pair of conducting cylindrical particles. *J. Quant. Spectroscopy & Radiative Transfer* **206** (2018) 142–150.
[373] F. Montiel, L.G. Bennetts, V.A. Squire, F. Bonnefoy & P. Ferrant, Hydroelastic response of floating elastic discs to regular waves. Part 2. Modal analysis. *J. Fluid Mech.* **723** (2013) 629–652.
[374] I.D. Moore & F. Guan, Three-dimensional dynamic response of lined tunnels due to incident seismic waves. *Earthquake Eng. & Structural Dynamics* **25** (1996) 357–369.
[375] H. Motoshima, Radiation and diffraction of shallow water waves by an arbitrary number of bodies. *J. Soc. Naval Arch. Japan* **147** (1980) 71–78.
[376] A.B. Movchan, R.C. McPhedran & G. Carta, Scattering reduction and resonant trapping of flexural waves: two rings to rule them. *Applied Sciences* **11** (2021) 4462 (14 pages).
[377] A.B. Movchan, R.C. McPhedran, G. Carta & R.V. Craster, Platonic localisation: one ring to bind them. *Arch. Appl. Mech.* **89** (2019) 521–533.
[378] G.J.H. Muijres, G.C. Herman & P.G.J. Bussink, Acoustic wave propagation in two-dimensional media containing small-scale heterogeneities. *Wave Motion* **27** (1998) 137–154.
[379] Y. Murai, J. Kawahara & T. Yamashita, Multiple scattering of *SH* waves in 2-D elastic media with distributed cracks. *Geophys. J. Int.* **122** (1995) 925–937.
[380] V.V. Mykhas'kiv, I.O. Butrak, O.M. Khay, T.I. Kilnytska & Ch. Zhang, A frequency-domain BIEM combining DBIEs and TBIEs for 3-D crack-inclusion interaction analysis. *Comput. Methods Appl. Mech. Engrg.* **200** (2011) 3270–3279.
[381] V.V. Mykhas'kiv & M.V. Khai, Reduction of three-dimensional dynamical elasticity theory problems with arbitrarily located plane slits to integral equations. *J. Appl. Math. & Mech.* **49** (1985) 719–724.
[382] V.V. Mykhas'kiv & M.V. Khai, The interaction of coplanar cracks in an infinite body caused by steady external loads. *J. Soviet Math.* **63** (1993) 345–349.
[383] V.V. Mykhas'kiv & O.M. Khay, Interaction between rigid-disc inclusion and penny-shaped crack under elastic time-harmonic wave incidence. *Int. J. Solids Struct.* **46** (2009) 602–616.
[384] J.-R. Nader, S.-P. Zhu & P. Cooper, Hydrodynamic and energetic properties of a finite array of fixed oscillating water column wave energy converters. *Ocean Eng.* **88** (2014) 131–148.
[385] S. Nag & B.P. Sinha, Electromagnetic plane wave scattering by a system of two uniformly lossy dielectric prolate spheroids in arbitrary orientation. *IEEE Trans. Antennas & Propag.* **43** (1995) 322–327.
[386] M. Nakanishi, N. Ueno, S. Koyama & H. Saruwatari, Two-dimensional sound field recording with multiple circular microphone arrays considering multiple scattering. *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, 2019, pp. 368–372.
[387] D.M. Natarov, V.O. Byelobrov, R. Sauleau, T.M. Benson & A.I. Nosich, Periodicity-induced effects in the scattering and absorption of light by infinite and finite gratings of circular silver nanowires. *Optics Express* **19** (2011) 22176–22190.
[388] T.A. Nieminen, H. Rubinsztein-Dunlop & N.R. Heckenberg, Calculation of the T-matrix: general considerations and application of the point-matching method. *J. Quant. Spectroscopy & Radiative Transfer* **79-80** (2003) 1019–1029.
[389] M. Nigsch, Numerical studies of time-independent and time-dependent scattering by several elliptical cylinders. *J. Computational & Appl. Math.* **204** (2007) 231–241.
[390] A.N. Norris & J.-M. Conoir, Multiple scattering by cylinders immersed in fluid: high order approximations for the effective wavenumbers. *J. Acoust. Soc. Amer.* **129** (2011) 104–113.
[391] A.N. Norris & C. Vemula, Scattering of flexural waves on thin plates. *J. Sound Vib.* 181 (1995) 115–125.
[392] B.A. Noskov, Dynamic properties of heterogeneous surface films: multiple scattering of capillary waves. *J. Chemical Phys.* **108** (1998) 807–815.
[393] J.F. Nye, Numerical solution for diffraction of an electromagnetic wave by slits in a perfectly conducting screen. *Proc. Roy. Soc.* A **458** (2002) 401–427.
[394] C.-J. Ong, D. Miller, L. Tsang, B. Wu & C.-C. Huang, Application of the Foldy–Lax multiple scattering method to the analysis of vias in ball grid arrays and interior layers of printed circuit boards. *Microwave*

& Optical Technology Letters **49** (2007) 225–231.
[395] J. O'Neill, Ö. Selsil, R.C. McPhedran, A.B. Movchan & N.V. Movchan, Active cloaking of inclusions for flexural waves in thin elastic plates. *Quart. J. Mech. Appl. Math.* **68** (2015) 263–288.
[396] Z.Y. Ou & D.W. Lee, Effects of interface energy on multiple scattering of plane compressional waves by two cylindrical fibers. *Int. J. Appl. Mech.* **4** (2012) 1250040 (19 pages).
[397] L. Pajewski, G. Schettini & F. Frezza, Cylindrical-wave approach for the electromagnetic scattering problem by buried two-dimensional objects. *J. Appl. Geophysics* **67** (2009) 318–326.
[398] E. Pan, Green's functions for geophysics: a review. *Reports on Progress in Physics* **82** (2019) 106801 (52 pages).
[399] L. Papillon, R. Costello & J.V. Ringwood, Boundary element and integral methods in potential flow theory: a review with a focus on wave energy applications. *J. Ocean Eng. & Marine Energy* **6** (2020) 303–337.
[400] Y.B. Park & H.J. Eom, Electromagnetic transmission through multiple circular apertures in a thick conducting plane. *IEEE Trans. Antennas & Propag.* **52** (2004) 1049–1055.
[401] W.J. Parnell & P.A. Martin, Multiple scattering of flexural waves by random configurations of inclusions in thin plates. *Wave Motion* **48** (2011) 161–175.
[402] S. Parvanova, P. Dineva & G.D. Manolis, Elastic wave fields in a half-plane with free-surface relief, tunnels and multiple buried inclusions. *Acta Mechanica* **225** (2014) 1843–1865.
[403] S.L. Parvanova, G.D. Manolis & P.S. Dineva, Wave scattering by nanoheterogeneities embedded in an elastic matrix via BEM. *Engineering Analysis with Boundary Elements* **56** (2015) 57–69.
[404] L. Pauling & E.B. Wilson, Jr, *Introduction to Quantum Mechanics*. New York: McGraw-Hill, 1935.
[405] P. Pawliuk & M. Yedlin, Truncating cylindrical wave modes in two-dimensional multiple scattering. *Optics Letters* **35** (2010) 3997–3999.
[406] P. Pawliuk & M. Yedlin, Scattering from cylinders near a dielectric half-space using a general method of images. *IEEE Trans. Antennas & Propag.* **60** (2012) 5296–5304.
[407] P. Pawliuk & M. Yedlin, Multiple scattering between cylinders in two dielectric half-spaces. *IEEE Trans. Antennas & Propag.* **61** (2013) 4220–4228.
[408] M.J. Peake, J. Trevelyan & G. Coates, Extended isogeometric boundary element method (XIBEM) for two-dimensional Helmholtz problems. *Comput. Methods Appl. Mech. Engrg.* **259** (2013) 93–102.
[409] M. Pedneault, C. Turc & Y. Boubendir, Schur complement domain decomposition methods for the solution of multiple scattering problems. *IMA J. Appl. Math.* **82** (2017) 1104–1134.
[410] M. Perton & F.J. Sánchez-Sesma, The indirect boundary element method to simulate elastic wave propagation in a 2-D piecewise homogeneous domain. *Geophys. J. Int.* **202** (2015) 1760–1769.
[411] M.A. Peter & M.H. Meylan, Infinite-depth interaction theory for arbitrary floating bodies applied to wave forcing of ice floes. *J. Fluid Mech.* **500** (2004) 145–167.
[412] P. Piecuch, On the addition theorems for solid spherical harmonics. *Reports on Mathematical Physics* **24** (1986) 187–192.
[413] V.J. Pinfield, Thermo-elastic multiple scattering in random dispersions of spherical scatterers. *J. Acoust. Soc. Amer.* **136** (2014) 3008–3017.
[414] H. Poincaré, Sur la diffraction des ondes hertziennes. *Rendiconti del Circolo Matematico di Palermo* **29** (1910) 169–259.
[415] Y. Pomeau, Henri Poincaré as an applied mathematician. In: *The Scientific Legacy of Poincaré* (eds. É. Charpentier, É. Ghys & A. Lesne) pp. 351–371. Providence: Amer. Math. Soc., 2010.
[416] R. Porter, Surface wave scattering by submerged cylinders of arbitrary cross-section. *Proc. Roy. Soc.* A **458** (2002) 581–606.
[417] R. Porter, Trapping of water waves by pairs of submerged cylinders. *Proc. Roy. Soc.* A **458** (2002) 607–624.
[418] R. Porter & D.V. Evans, Diffraction of flexural waves by finite straight cracks in an elastic sheet over water. *J. Fluids & Struct.* **23** (2007) 309–327.
[419] C.G. Poulton, A.B. Movchan, N.V. Movchan & R.C. McPhedran, Analytic theory of defects in periodically structured elastic plates. *Proc. Roy. Soc.* A **468** (2012) 1196–1216.
[420] X. Pu, A. Palermo & A. Marzani, A multiple scattering formulation for finite-size flexural metasurfaces. *Proc. Roy. Soc.* A **478** (2022) 20210669 (19 pages).
[421] J.M. Putnam & L.N. Medgyesi-Mitschang, Combined field integral equation formulation for inhomo-

geneous two- and three-dimensional bodies: the junction problem. *IEEE Trans. Antennas & Propag.* **39** (1991) 667–672.
[422] M. Rajabi & A. Hajiahmadi, Self-propulsive swimmers: two linked acoustic radiating spheres. *Phys. Rev.* E **98** (2018) 063003 (14 pages).
[423] A.G. Ramm, Electromagnetic scattering by many small particles. *Physics Letters A* **360** (2007) 735–741.
[424] A.G. Ramm, Many-body wave scattering by small bodies. *J. Math. Phys.* **48** (2007) 023512 (6 pages).
[425] A.G. Ramm, Electromagnetic wave scattering by small perfectly conducting particles and applications. *J. Math. Phys.* **55** (2014) 083505 (10 pages).
[426] A.G. Ramm, Scattering of electromagnetic waves by many small perfectly conducting or impedance bodies. *J. Math. Phys.* **56** (2015) 091901 (21 pages).
[427] M.-L. Rapún, On the solution of direct and inverse multiple scattering problems for mixed sound-soft, sound-hard and penetrable objects. *Inverse Problems* **36** (2020) 095014 (30 pages).
[428] E. Renzi, A. Abdolali, G. Bellotti & F. Dias, Wave-power absorption from a finite array of oscillating wave surge converters. *Renewable Energy* **63** (2014) 55–68.
[429] E. Renzi & F. Dias, Hydrodynamics of the oscillating wave surge converter in the open ocean. *European Journal of Mechanics B/Fluids* **41** (2013) 1–10.
[430] C.J. Rodger, J.R. Wait & R.L. Dowden, Electromagnetic scattering from a group of thin conducting cylinders. *Radio Sci.* **32** (1997) 907–912.
[431] A. Rodríguez-Castellanos, F.J. Sánchez-Sesma, F. Luzón & R. Martin, Multiple scattering of elastic waves by subsurface fractures and cavities. *Bull. Seismological Soc. Amer.* **96** (2006) 1359–1374.
[432] A. Rohfritsch, J.-M. Conoir, R. Marchiano & T. Valier-Brasier, Numerical simulation of two-dimensional multiple scattering of sound by a large number of circular cylinders. *J. Acoust. Soc. Amer.* **145** (2019) 3320–3329.
[433] M. Roseau, *Asymptotic Wave Theory*. Amsterdam: North-Holland, 1976.
[434] T. Rother, *Sound Scattering on Spherical Objects*. Cham: Springer, 2020.
[435] J.A. Roumeliotis & A.D. Kotsis, Acoustic scattering from two spheres, one with a small radius. *Acoustical Physics* **53** (2007) 33–43.
[436] J.A. Roumeliotis, A.D. Kotsis & G. Kolezas, Acoustic scattering by an impenetrable spheroid. *Acoustical Physics* **53** (2007) 436–447.
[437] J.A. Roumeliotis & V. Douvalis, Electromagnetic scattering from two external spheres one of which has a small radius. *J. Electromagnetic Waves & Applications* **18** (2004) 591–614.
[438] R. Roy, S. De & B.N. Mandal, Water wave scattering by multiple thin vertical barriers. *Appl. Math. & Computation* **355** (2019) 458–481.
[439] W.M. Rucker, E. Schlemmer & K.R. Richter, The solution of 3D multiple scattering problems using the boundary element method. *IEEE Transactions on Magnetics* **30** (1994) 3132–3135.
[440] T. Saitoh, S. Hirose & T. Fukui, Application of fast multipole boundary element method to multiple scattering analysis of acoustic and elastic waves. In: *Review of Quantitative Nondestructive Evaluation* vol. 26 (eds. D.O. Thompson & D.E. Chimenti) pp. 79–86. New York: American Institute of Physics, 2007.
[441] S. Sancar & Y.-H. Pao, Spectral analysis of elastic pulses backscattered from two cylindrical cavities in a solid. Part I. *J. Acoust. Soc. Amer.* **69** (1981) 1591–1596.
[442] S. Sancar & W. Sachse, Spectral analysis of elastic pulses backscattered from two cylindrical cavities in a solid. Part II. *J. Acoust. Soc. Amer.* **69** (1981) 1597–1609.
[443] K. Sankarbabu, S.A. Sannasiraj & V. Sundar, Interaction of regular waves with a group of dual porous circular cylinders. *Appl. Ocean Res.* **29** (2007) 180–190.
[444] S.A. Sannasiraj, R. Sundaravadivelu & V. Sundar, Diffraction-radiation of multiple floating structures in directional waves. *Ocean Eng.* **28** (2000) 201–234.
[445] K. Sarabandi & P.F. Polatin, Electromagnetic scattering from two adjacent objects. *IEEE Antennas & Propag.* **42** (1994) 510–517.
[446] D. Sarkar, E. Contal, N. Vayatis & F. Dias, Prediction and optimization of wave energy converter arrays using a machine learning approach. *Renewable Energy* **97** (2016) 504–517.
[447] D. Sarkar, E. Renzi & F. Dias, Wave farm modelling of oscillating wave surge converters. *Proc. Roy. Soc.* A **470** (2014) 20140118 (22 pages).

[448] K. Sarkar, Rayleigh limit extended: scattering from a fluid sphere. *J. Acoust. Soc. Amer.* **152** (2022) R7–R8.
[449] F. Schäfer, F. Eckert & T. Wellens, Cooperative scattering of scalar waves by optimized configurations of point scatterers. *J. Phys. B: Atomic, Molecular & Optical Physics* **50** (2017) 235502 (10 pages).
[450] J. Schäfer, S.-C. Lee & A. Kienle, Calculation of the near fields for the scattering of electromagnetic waves by multiple infinite cylinders at perpendicular incidence. *J. Quant. Spectroscopy & Radiative Transfer* **113** (2012) 2113–2123.
[451] D. Schebarchov, E.C. Le Ru, J. Grand & B. Auguié, Mind the gap: testing the Rayleigh hypothesis in T-matrix calculations with adjacent spheroids. *Optics Express* **27** (2019) 35750–35760.
[452] D. Sébilleau, On the computation of the integrated products of three spherical harmonics. *J. Phys. A: Math. & Gen.* **31** (1998) 7157–7168.
[453] E. Sever, F. Dikmen, Y.A. Tuchkin & C. Sabah, A numerically stable algorithm for scattering from several circular cylinders including metamaterials with different boundary conditions. *Optik* **168** (2018) 667–676.
[454] A.V. Shanin, Diffraction of a plane wave by two ideal strips. *Quart. J. Mech. Appl. Math.* **56** (2003) 187–215.
[455] R. Sheikhhassani & M. Dravinski, Scattering of a plane harmonic SH wave by multiple layered inclusions. *Wave Motion* **51** (2014) 517–532.
[456] L. Shen & Y.J. Liu, An adaptive fast multipole boundary element method for three-dimensional acoustic wave problems based on the Burton–Miller formulation. *Computational Mech.* **40** (2007) 461–472.
[457] X. Shen, Q. Qin, Y. Hong & G. Zhang, A matrix inversion approach of computing T-matrix for axially symmetrical particles of extreme shape and dielectrically large dimension. *Radio Sci.* **47** (2012) RS5005 (14 pages).
[458] Y.M. Shen, Y.H. Zheng & C.-O. Ng, Interaction of oblique waves with an array of long horizontal circular cylinders. *Science in China Series E: Technological Sciences* **50** (2007) 490–509.
[459] P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena*, second edition. Berlin: Springer, 2006. The first edition is [M1100].
[460] X. Sheng, C.J.C. Jones & D.J. Thompson, Modelling ground vibration from railways using wavenumber finite- and boundary-element methods. *Proc. Roy. Soc.* A **461** (2005) 2043–2070.
[461] D.M. Shin & Y. Cho, Diffraction of waves past two vertical thin plates on the free surface: A comparison of theory and experiment. *Ocean Eng.* **124** (2016) 274–286.
[462] R.A. Shore, Scattering of an electromagnetic linearly polarized plane wave by a multilayered sphere: obtaining a computational form of Mie coefficients for the scattered field. *IEEE Antennas & Propagation Magazine* **57**, 6 (2015) 69–116.
[463] J.F. Shriner & W.J. Thompson, Angular-momentum coupling coefficients: new and improved algorithms. *Computers in Physics* **7** (1993) 144–148.
[464] P. Siddorn & R. Eatock Taylor, Diffraction and independent radiation by an array of floating cylinders. *Ocean Eng.* **35** (2008) 1289–1303.
[465] B. Sieger, Die Beugung einer ebenen elektrischen Welle an einem Schirm von elliptischem Querschnitt. *Annalen der Physik*, 4 Folge, **27** (1908) 626–664.
[466] A. Silbiger, Scattering of sound by an elastic prolate spheroid. *J. Acoust. Soc. Amer.* **35** (1963) 564–570.
[467] R. Silva, G. Govaere & P. Salles, Wave interaction with cylindrical porous piles. *Ocean Eng.* **30** (2003) 1719–1740.
[468] B.E. Simon & M.F. Hamilton, Analytical solution for acoustic radiation force on a sphere near a planar boundary. *J. Acoust. Soc. Amer.* **153** (2023) 627–642.
[469] J. Singh & A. Babarit, A fast approach coupling boundary element method and plane wave approximation for wave interaction analysis in sparse arrays of wave energy converters. *Ocean Eng.* **85** (2014) 12–20.
[470] H. Sircar, On sound-waves due to prescribed vibrations on a spherical surface in the presence of a rigid and fixed spheroidal obstacle. *Tohoku Mathematical Journal*, series 1, **31** (1929) 251–265.
[471] M.J.A. Smith, M.H. Meylan & R.C. McPhedran, Scattering by cavities of arbitrary shape in an infinite plate and associated vibration problems. *J. Sound Vib.* **330** (2011) 4029–4046.
[472] W.R.C. Somerville, B. Auguié & E.C. Le Ru, Severe loss of precision in calculation of T-matrix integrals. *J. Quant. Spectroscopy & Radiative Transfer* **113** (2012) 524–535.

[473] W.R.C. Somerville, B. Auguié & E.C. Le Ru, A new numerically stable implementation of the T-matrix method for electromagnetic scattering by spheroidal particles. *J. Quant. Spectroscopy & Radiative Transfer* **123** (2013) 153–168.

[474] H. Song, L. Tao & S. Chakrabarti, Modelling of water wave interaction with multiple cylinders of arbitrary shape. *J. Comp. Phys.* **229** (2010) 1498–1513.

[475] N. St. Clair, D. Davenport, A.D. Kim & D. Kleckner, Dynamics of acoustically bound particles. *Phys. Rev. Res.* **5** (2023) 013051 (13 pages).

[476] E.O. Steinborn & K. Ruedenberg, Rotation and translation of regular and irregular solid spherical harmonics. *Advances in Quantum Chemistry* **7** (1973) 1–81.

[477] D.R. Steward, Waves in collections of circular shoals and bathymetric depressions. *J. Waterway, Port, Coastal, & Ocean Engineering* **146** (2020) 04020018 (13 pages).

[478] M. Stojek, M. Markiewicz & O. Mahrenholtz, Diffraction loads on multiple vertical cylinders with rectangular cross section by Trefftz-type finite elements. *Computers & Structures* **75** (2000) 335–345.

[479] S. Ström & W. Zheng, The null field approach to electromagnetic scattering from composite objects. *IEEE Trans. Antennas & Propag.* **36** (1988) 376–383.

[480] J.W. Strutt, Investigation of the disturbance produced by a spherical obstacle on the waves of sound. *Proceedings of the London Mathematical Society*, Series 1, **4** (1872) 253–283.

[481] T. Sumiya, S. Biwa & G. Haïat, Computational multiple scattering analysis of elastic waves in unidirectional composites. *Wave Motion* **50** (2013) 253–270.

[482] B. Sun, L. Bi, P. Yang, M. Kahnert & G. Kattawar, *Invariant Imbedding T-Matrix Method for Light Scattering by Nonspherical and Inhomogeneous Particles*. Amsterdam: Elsevier, 2020.

[483] L. Sun, R. Eatock Taylor & P.H. Taylor, First- and second-order analysis of resonant waves between adjacent barges. *J. Fluids & Struct.* **26** (2010) 954–978.

[484] V. Tabatadze, K. Karaçuha & E.I. Veliyev, The solution of the plane wave diffraction problem by two strips with different fractional boundary conditions. *J. Electromag. Waves & Appl.* **34** (2020) 881–893.

[485] T. Takaishi, M. Miyazawa & C. Kato, A computational method of evaluating noncompact sound based on vortex sound theory. *J. Acoust. Soc. Amer.* **121** (2007) 1353–1361.

[486] K. Tharmmapornphilas & S. Hirose, Multiple scattering approximation of anti-plane elastic waves in infinite and half-space domains with distributed inclusions. *Electronic Journal of Boundary Elements* **4** (2006) 1–30.

[487] D. Theobald, A. Egel, G. Gomard & U. Lemmer, Plane-wave coupling formalism for T-matrix simulations of light scattering by nonspherical particles. *Phys. Rev. A* **96** (2017) 033822 (8 pages).

[488] B. Thierry, A remark on the single scattering preconditioner applied to boundary integral equations. *J. Math. Anal. & Appl.* **413** (2014) 212–228.

[489] B. Thierry & X. Antoine, Spectral and condition number estimates of the acoustic single-layer operator for low-frequency multiple scattering in dense media. *J. Computational & Appl. Math.* **239** (2013) 380–395.

[490] B. Thierry, X. Antoine, C. Chniti & H. Alzubaidi, μ-diff: an open-source Matlab toolbox for computing multiple scattering problems by disks. *Computer Physics Comm.* **192** (2015) 348–362.

[491] D.P. Thomas, Electromagnetic diffraction by two coaxial discs. *Proc. Camb. Phil. Soc.* **60** (1964) 621–634.

[492] G. Tokić & D.K.P. Yue, Hydrodynamics of large wave energy converter arrays with random configuration variations. *J. Fluid Mech.* **923** (2021) R1 (12 pages).

[493] L.A. Tolokonnikov, The scattering of a plane sound wave by an elastic sphere with an inhomogeneous coating. *J. Appl. Math. & Mech.* **78** (2014) 367–373.

[494] D. Torrent, Introduction to multiple scattering theory for scalar waves. In: *Acoustic Waves in Periodic Structures, Metamaterials, and Porous Media: From Fundamentals to Industrial Applications* (eds N. Jiménez, O. Umnova & J.-P. Groby) pp. 43–64. Cham: Springer, 2021. Topics in Appl. Phys. **143**.

[495] D. Torrent, A. Håkansson, F. Cervera & J. Sánchez-Dehesa, Homogenization of two-dimensional clusters of rigid rods in air. *Phys. Rev. Lett.* **96** (2006) 204302 (4 pages).

[496] D. Torrent & J. Sánchez-Dehesa, Effective parameters of clusters of cylinders embedded in a nonviscous fluid or gas. *Phys. Rev.* B **74** (2006) 224305 (10 pages).

[497] R.J.A. Tough, The transformation properties of vector multipole fields under a translation of coordinate origin. *J. Phys. A: Math. Gen.* **10** (1977) 1079–1087.

[498] T. Triantafyllidis & B. Prange, Dynamic subsoil-coupling between rigid, circular foundations on the halfspace. *Soil Dynamics & Earthquake Engineering* **8** (1989) 9–21.
[499] C.-C. Tsai, H. Behera & T.-W. Hsu, Analysis of water wave interaction with multiple submerged semi-circular porous structures. *Arch. Appl. Mech.* **93** (2023) 2693–2709.
[500] L. Tsang, Van de Hulst essay: Multiple scattering of waves by discrete scatterers and rough surfaces. *J. Quant. Spectroscopy & Radiative Transfer* **224** (2019) 566–587.
[501] L. Tsang, H. Chen, C.-C. Huang & V. Jandhyala, Modeling of multiple scattering among vias in planar waveguides using Foldy–Lax equations. *Microwave & Optical Technology Letters* **31** (2001) 201–208.
[502] L. Tsang, K.-H. Ding, S. Huang & X. Xu, Electromagnetic computation in scattering of electromagnetic waves by random rough surface and dense media in microwave remote sensing of land surfaces. *Proc. IEEE* **101** (2013) 255–279.
[503] L. Tsang, K.H. Ding, S.E. Shih & J.A. Kong, Scattering of electromagnetic waves from dense distributions of spheroidal particles based on Monte Carlo simulations. *J. Opt. Soc. Amer.* A **15** (1998) 2660–2669.
[504] V. Twersky, Regular polygonal arrays of resonant scatterers. *J. Acoust. Soc. Amer.* **89** (1991) 991–998.
[505] O. Umnova, K. Attenborough & C.M. Linton, Effects of porous covering on sound attenuation by periodic arrays of cylinders. *J. Acoust. Soc. Amer.* **119** (2006) 278–284.
[506] A. Unsöld, Beiträge zur Quantenmechanik der Atome. *Annalen der Physik* **82** (1927) 355–393.
[507] J.Y. Vaishnav, J.D. Walls, M. Apratim & E.J. Heller, Matter-wave scattering and guiding by atomic arrays. *Phys. Rev.* A **76** (2007) 013620 (13 pages).
[508] T. Valier-Brasier & J.-M. Conoir, Resonant acoustic scattering by two spherical bubbles. *J. Acoust. Soc. Amer.* **145** (2019) 301–311.
[509] T. Valier-Brasier & D. Royer, *Elastic Waves in Solids 2. Radiation, Scattering, Generation.* Hoboken: Wiley-ISTE, 2022.
[510] B. van der Aa & J. Forssén, Scattering by an array of perforated cylinders with a porous core. *J. Acoust. Soc. Amer.* **136** (2014) 2370–2380.
[511] B. van der Aa & J. Forssén, The 2.5D MST for sound propagation through an array of acoustically rigid cylinders perpendicular to an impedance surface. *J. Phys. D: Appl. Phys.* **48** (2015) 295501 (14 pages).
[512] B. Van Genechten, B. Bergen, D. Vandepitte & W. Desmet, A Trefftz-based numerical modelling framework for Helmholtz problems with complex multiple-scatterer configurations. *J. Comp. Phys.* **229** (2010) 6623–6643.
[513] E. van 't Wout, S.R. Haqshenas, P. Gélat, T. Betcke & N. Saffari, Benchmarking preconditioned boundary integral formulations for acoustics. *Int. J. Numer. Meth. Eng.* **122** (2021) 5873–5897.
[514] E. Veliev, M. Ivakhnychenko & T. Ahmedov, Fractional boundary conditions in plane waves diffraction on a strip. *Progress In Electromagnetics Research* **79** (2008) 443–462.
[515] J.V. Venås & T. Jenserud, Exact 3D scattering solutions for spherical symmetric scatterers. *J. Sound Vib.* **440** (2019) 439–479. Corrigendum: **474** (2020) 115270.
[516] J.V. Venås & T. Jenserud, Generalization and stabilization of exact scattering solutions for spherical symmetric scatterers. *J. Sound Vib.* **539** (2022) 117263 (19 pages).
[517] F. Vico, L. Greengard & Z. Gimbutas, Boundary integral equation analysis on the sphere. *Numer. Math.* **128** (2014) 463–487.
[518] V. Villamizar, J.C. Badger & S. Acosta, High order local farfield expansions absorbing boundary conditions for multiple scattering. *J. Comp. Phys.* **460** (2022) 111187 (32 pages).
[519] A. Vion, R.V. Sabariego & C. Geuzaine, A model reduction algorithm for solving multiple scattering problems using iterative methods. *IEEE Trans. Magnetics* **47** (2011) 1470–1473.
[520] M.S. Vogelius & D. Volkov, Asymptotic formula for perturbations in the electromagnetic fields due to the presence of inhomogeneities of small diameter. *Mathematical Modelling and Numerical Analysis* **34** (2000) 723–748.
[521] J.L. Volakis, Alternative field representations and integral equations for modeling inhomogeneous dielectrics. *IEEE Trans. Microwave Theory & Techniques* **40** (1992) 604–608.
[522] J.L. Volakis & K. Sertel, *Integral Equation Methods for Electromagnetics.* Raleigh: SciTech Publishing, 2012.
[523] T. von Petersdorff, Boundary integral equations for mixed Dirichlet, Neumann and transmission problems. *Math. Meth. Appl. Sci.* **11** (1989) 185–213.

[524] J.R. Wait, On the scattering of spherical waves by a cylindrical object. *Applied Scientific Research*, Series B **4** (1955) 464–468.
[525] J.R. Wait, *Electromagnetic Radiation from Cylindrical Structures*. New York: Pergamon, 1959.
[526] D.A.G. Walker & R. Eatock Taylor, Wave diffraction from linear arrays of cylinders. *Ocean Eng.* **32** (2005) 2053–2078.
[527] A. Walther & A.H.E. Müller, Janus particles: synthesis, self-assembly, physical properties, and applications. *Chemical Reviews* **113** (2013) 5194–5261.
[528] G. Wang & D. Liu, Scattering of SH-wave by multiple circular cavities in half space. *Earthquake Engineering & Engineering Vibration* **1** (2002) 36–44.
[529] G.F. Wang, Multiple diffraction of plane compressional waves by two circular cylindrical holes with surface effects. *J. Appl. Phys.* **105** (2009) 013507 (6 pages).
[530] G.L. Wang, W.C. Chew & M.J. White Coupled integral equations for sound propagation above a hard ground surface with trench cuttings. *J. Acoust. Soc. Amer.* **120** (2006) 1209–1216.
[531] H. Wang & J. Liu, On decomposition method for acoustic wave scattering by multiple obstacles. *Acta Mathematica Scientia* **33B** (2013) 1–22.
[532] J.H. Wang, X.L. Zhou & J.F. Lu, Dynamic stress concentration around elliptic cavities in saturated poroelastic soil under harmonic plane waves. *Int. J. Solids Struct.* **42** (2005) 4295–4310.
[533] X. Wang & L.J. Sudak, Scattering of elastic waves by multiple elastic circular cylinders with imperfect interface. *Waves in Random & Complex Media* **17** (2007) 159–187.
[534] Z. Wang & S. Biwa, Multiple scattering and stop band characteristics of flexural waves on a thin plate with circular holes. *J. Sound Vib.* **416** (2018) 80–93.
[535] P.C. Waterman, T-matrix methods in acoustic scattering. *J. Acoust. Soc. Amer.* **125** (2009) 42–51.
[536] D.E. Weston, Acoustic interaction effects in arrays of small spheres. *J. Acoust. Soc. Amer.* **39** (1966) 316–322.
[537] H. Weyl, Ausbreitung elektromagnetischer Wellen über einem ebenen Leiter. *Annalen der Physik* **60** (1919) 481–500. doi:10.1002/andp.19193652104
[538] B. Wilks, F. Montiel & S. Wakes, Rainbow reflection and broadband energy absorption of water waves by graded arrays of vertical barriers. *J. Fluid Mech.* **941** (2022) A26 (25 pages).
[539] D.R. Wilton & R. Mittra, A new numerical approach to the calculation of electromagnetic scattering properties of two-dimensional bodies of arbitrary cross section. *IEEE Trans. Antennas & Propag.* **AP-20** (1972) 310–317.
[540] A.A. Wolf, Rotation operators. *Amer. J. Phys.* **37** (1969) 531–536.
[541] H.A. Wolgamot, R. Eatock Taylor & P.H. Taylor, Radiation, trapping and near-trapping in arrays of floating truncated cylinders. *J. Eng. Math.* **91** (2015) 17–35.
[542] M.B. Woodworth & A.D. Yaghjian, Multiwavelength three-dimensional scattering with dual-surface integral equations. *J. Opt. Soc. Amer.* A **11** (1994) 1399–1413.
[543] H. Wu, Y. Liu & W. Jiang, A fast multipole boundary element method for 3D multi-domain acoustic scattering problems based on the Burton–Miller formulation. *Engineering Analysis with Boundary Elements* **36** (2012) 779–788.
[544] J.H. Wu, A.Q. Liu, H.L. Chen & T.N. Chen, Multiple scattering of a spherical acoustic wave from fluid spheres. *J. Sound Vib.* **290** (2006) 17–33.
[545] R. Wu, B.-Y. Wu, P.-X. He, K.-Y. Guo & X.-Q. Sheng, A fast plane wave expansion algorithm for rigorous scattering analysis from swarm targets. *IEEE Trans. Antennas & Propag.* **71** (2023) 7426–7437.
[546] X. Wu & W. Zheng, An adaptive perfectly matched layer method for multiple cavity scattering problems. *Comm. Comput. Phys.* **19** (2016) 534–558.
[547] Z. Xie, R. Zhang, B. Wang & L.-L. Wang, An efficient iterative method for solving multiple scattering in locally inhomogeneous media. *Comput. Methods Appl. Mech. Engrg.* **358** (2020) 112642 (26 pages).
[548] G. Xu, Improved recursive computation of [C]lebsch–Gordan coefficients. *J. Quant. Spectroscopy & Radiative Transfer* **254** (2020) 107210 (6 pages).
[549] Y. Yang, Q. Gui, Y. Zhang, Y. Chai & W. Li, Acoustic scattering of a pair of rigid spheroids based on the T-matrix method. *Frontiers in Physics* **11** (2023) 1170811 (15 pages).
[550] H.-X. Ye, J.-W. Dai & Y.-Q. Jin, Analytical iterative algorithm for fast computation of scattering from multiple conductive cylinders and the image reconstruction. *European Physical J. Applied Phys.*

48 (2009) 20903 (7 pages).
[551] C. Yeh, Scattering of acoustic waves by a penetrable prolate spheroid. I. Liquid prolate spheroid. *J. Acoust. Soc. Amer.* **42** (1967) 518–521.
[552] R.W. Yeung & R.K.M. Seah, On Helmholtz and higher-order resonance of twin floating bodies. *J. Eng. Math.* **58** (2007) 251–265.
[553] M.P. Yu, Y.P. Han, Z.W. Cui & A.T. Chen, Electromagnetic scattering by multiple dielectric particles under the illumination of unpolarized high-order Bessel vortex beam. *J. Quant. Spectroscopy & Radiative Transfer* **195** (2017) 107–113.
[554] Y.-K. Yu, Electrostatics of charged dielectric spheres with application to biological systems. II. A formalism bypassing Wigner rotation matrices. *Phys. Rev. E* **100** (2019) 012401 (13 pages).
[555] Z. Yuan, A. Boström & Y. Cai, An analytical model for calculating vibrations from twin tunnels in a saturated poroelastic half-space. *Soil Dynamics & Earthquake Engineering* **120** (2019) 23–27.
[556] Z. Yuan, A. Boström, Y. Cai & Z. Cao, Analytical solution for calculating vibrations from twin circular tunnels. *Soil Dynamics & Earthquake Engineering* **117** (2019) 312–327.
[557] Z. Yuan, A. Boström, Y. Cai & Z. Cao, Analytical wave function method for modelling a twin tunnel embedded in a saturated poroelastic full-space. *Computers & Geotechnics* **114** (2019) 103114 (15 pages).
[558] Z. Yuan, A. Boström, Y. Cai, X. Pan, Z. Cao & L. Shi, The wave function method for calculation of vibrations from a twin tunnel in a multi-layered half-space. *Soil Dynamics & Earthquake Engineering* **125** (2019) 105716 (17 pages).
[559] X. Zeng & A.S. Cakmak, Scattering of plane SH waves by multiple cavities. *Proc. 1985 Pressure Vessels and Piping Conference*, PVP–Volume 98-4, pp. 155–163. New York: ASME, 1985.
[560] X. Zeng, M. Shi & S. Huang, Hydrodynamic interactions of water waves with a group of independently oscillating truncated circular cylinders. *Acta Mechanica Sinica* **32** (2016) 773–791.
[561] T.R. Zhan, S.T. Chui & Z.F. Lin, Multiple scattering of metallic wire structures. *J. Appl. Phys.* **118** (2015) 163104 (6 pages).
[562] Ch. Zhang & J.D. Achenbach, Scattering by multiple crack configurations. *J. Appl. Mech.* **55** (1988) 104–110.
[563] H. Zhang & Y. Han, Addition theorem for the spherical vector wave functions and its application to the beam shape coefficients. *J. Opt. Soc. Amer.* B **25** (2008) 255–260.
[564] J. Zhang, A.A. Taflanidis & J.T. Scruggs, Surrogate modeling of hydrodynamic forces between multiple floating bodies through a hierarchical interaction decomposition. *J. Comp. Phys.* **408** (2020) 109298 (25 pages).
[565] N. Zhang, J. Pan, Y. Gao, X. Chen, D. Dai & Y. Zhang, Surface motion of an arbitrary number of semi-circular viscoelastic alluvial valleys for incident plane *SH* waves. *Geophys. J. Int.* **228** (2022) 1607–1620.
[566] N. Zhang, J. Pan, Y. Gao, Y. Zhang, D. Dai & X. Chen, Analytical approach to scattering of SH waves by an arbitrary number of semicircular canyons in an elastic half space. *Soil Dynamics & Earthquake Engineering* **146** (2021) 106762 (14 pages).
[567] Y.J. Zhang & E.P. Li, Fast multipole accelerated scattering matrix method for multiple scattering of a large number of cylinders. *Progress In Electromagnetics Research* **72** (2007) 105–126.
[568] Z. Zhang, G. He, W. Wang, S. Liu & Z. Wang, Broadband cloaking of multiple truncated cylinders in water waves using the arrangement defects. *Phys. Fluids* **32** (2020) 067111 (13 pages).
[569] K. Zhao, V. Rawat & J.-F. Lee, A domain decomposition method for electromagnetic radiation and scattering analysis of multi-target problems. *IEEE Trans. Antennas & Propag.* **56** (2008) 2211–2221.
[570] S. Zheng, A. Antonini, Y. Zhang, D. Greaves, J. Miles & G. Iglesias, Wave power extraction from multiple oscillating water columns along a straight coast. *J. Fluid Mech.* **878** (2019) 445–480.
[571] S. Zheng, M.H. Meylan, G. Zhu, D. Greaves & G. Iglesias, Hydroelastic interaction between water waves and an array of circular floating porous elastic plates. *J. Fluid Mech.* **900** (2020) A20 (25 pages).
[572] S. Zheng, Y. Zhang, J. Liu & G. Iglesias, Wave diffraction from multiple truncated cylinders of arbitrary cross sections. *Applied Mathematical Modelling* **77** (2020) 1425–1445.
[573] S. Zheng, Y. Zhang, Y. Liu & G. Iglesias, Wave radiation from multiple cylinders of arbitrary cross sections. *Ocean Eng.* **184** (2019) 11–22.

[574] W. Zheng, The null field approach to electromagnetic scattering from composite objects: the case with three or more constituents. *IEEE Trans. Antennas & Propag.* **36** (1988) 1396–1400.
[575] W. Zheng & S. Ström, The null field approach to electromagnetic scattering from composite objects: the case of concavo-convex constituents. *IEEE Trans. Antennas & Propag.* **37** (1989) 373–383.
[576] Y.H. Zheng, Y.M. Shen, Y.G. You, B.J. Wu & L. Rong, Hydrodynamic properties of two vertical truncated cylinders in waves. *Ocean Eng.* **32** (2005) 241–271.
[577] J. Zhou, J. Ren & W. Bai, Survey on hydrodynamic analysis of ship-ship interaction during the past decade. *Ocean Eng.* **278** (2023) 114361 (20 pages).
[578] X.-L. Zhou, J.-H. Wang & L.-F. Jiang, Dynamic response of a pair of elliptic tunnels embedded in a poroelastic medium. *J. Sound Vib.* **325** (2009) 816–834.
[579] R.-c. Zhu, G.-p. Miao & H.-r. Zhu, The radiation problem for multiple structures with small gaps in between. *J. Hydrodynamics*, Series B **18** (2006) 520–526.