Time-Domain Scattering

P. A. Martin

Cambridge University Press, 2021, xiii+250 pp., ISBN 978-1-108-83559-6

CORRECTIONS AND ADDITIONS

June 13, 2024 `tdbook-add.tex`

Corrections

p. 127. Half way down, change "compressional flow" to "compressible flow".

p. 163. Missing) in (10.11): should be $(\mathrm{S}(\partial u/\partial n))(P,t)$.

p. 176. Last line: "§5.14], For" should be "§5.14]. For"

p. 184. Two lines below (10.81): $h = \frac{1}{3}T$ should be $h = \frac{1}{3}T/N$. Error found by M. Ganesh.

Opening quotation for Chapter 1:

> *The velocities of pulses propagated in an elastic fluid, are in a ratio compounded of the subduplicate ratio of the elastic force directly, and the subduplicate ratio of the density inversely; supposing the elastic force of the fluid to be proportional to its condensation.*
>
> Proposition XLVII in Book 2 of Newton's *Principia*, as translated from the Latin by A. Motte in 1729. In more modern terms, speed of sound = $\sqrt{\text{bulk modulus/density}}$.

1.1.6. Non-zero ambient velocity and homogeneous fluid

[End of §1.1.6, p. 6]: For time-harmonic motions, with $p(\boldsymbol{r},t) = \mathrm{Re}\,\{P(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\omega t}\}$, the convected wave equation (1.26) becomes

$$\nabla^2 P + k^2 P + 2\mathrm{i}kM\frac{\partial P}{\partial x} - M^2\frac{\partial^2 P}{\partial x^2} = 0, \tag{1}$$

where $k = \omega/c$, $M = |\boldsymbol{U}|/c$ is the Mach number and we have taken the uniform ambient flow $\boldsymbol{U}$ to be in the x-direction. Numerical solutions of (1) have been obtained [66, 35, 36].

1.3.4. A simple initial-boundary value problem

[Bottom of p. 16]: "where (1.62) ... can be found." See also [16, Appendix] and [23, eqn (25)].

1.3.7. Damping, dissipation, absorption and losses

[Bottom of p. 19]: "see Holm's book [M417]". See also [11].

2.3.5. Multipole representation

[Top of p. 40]: "See ... [M147, eqn (4)]." Add [9, §2.2].

2.4.1. Integral representations

[p. 46, bottom]: Poritsky [46] has given a Weyl-type plane-wave integral representation for $r^{-1}f(r-ct)$; see [49, §3.10].

2.6. Moving sources

[Bottom of p. 57]: For much more on electromagnetic fields generated by moving sources, see [67, Chapter 23].

4.6. Further problems and boundary conditions

[p. 80, just before §4.6.1]: Yet another variant of (4.16) has been studied in [7],

$$\frac{\partial u}{\partial n}(P,t) + \int_0^t g(t-\tau)\,\frac{\partial u}{\partial \tau}(P,\tau)\,\mathrm{d}\tau = f(P,t), \quad P \in S, \quad t > 0,$$

where g is given (it could be a generalised function) and the integral term is recognised as a Laplace convolution, (1.76). The special case with $g(t) = t^{-1/2}$ is [41, eqn (1.13)]. For electromagnetic analogues, see [42].

4.8. Existence and uniqueness

[p. 89, just before §4.8.1] ... book by Sayas [M742]; see also [8].

7.2.1. Dirichlet boundary condition [Scattering by a sphere]

[p .114, end of 2nd paragraph]: For numerical computation of θ_n and $\beta_{n,m}$, see [21].

7.2.4. Literature

[p. 120, end of section, "For elastodynamic scattering... [M623, §III.C]."] Bahari *et al.* [4] consider scattering by a thick fluid-filled spherical shell, and give a good review of the relevant literature.

7.4.1. Dirichlet boundary condition [Scattering by a sphere]

[Bottom of p. 123, add]: For other integral equations with kernels involving P_n, see [M259, eqn (49)] and [20, eqn (5-2)].

7.5. Moving spheres

[p. 125, middle of first paragraph]: 'problems involving spheres [M713, M772, M325]' add [62, 28, 65].

8.2. Scattering frequencies

[p. 133, 3rd paragraph]: "Numerical methods ...": add [47, 3]

8.3. Boundary integral equations

[p. 135, 2nd paragraph]: "Boundary integral equations ...": add [37]

8.6. Singularity expansion method (SEM)

[p. 137, above quotation]: "... collaborators... M70]": add [61]

[Top of p. 138]: "Later papers include ... M73]": add [63].

[Bottom of p. 138]: Conventional SEM has been used for hydrodynamic problems, with frequency-domain software employed to compute natural frequencies [56, eqn (28)].

9.1.1. Kirchhoff's formula for bounded domains

[p. 146, just before §9.1.2, after $\delta/\delta n$.] For an early discussion, see [19, §III.6], especially [19, p. 179, eqn (35)].

9.1.7. Literature [on Kirchhoff's formula]

[Top of p. 150, with insertions:] ... has been translated [M485], [32] and analysed [M138], [29].

9.2. Kirchhoff's formula: a space-time derivation

Last line, p. 151, "... IBVPs [M693, M694]": add [43].

9.3.2. Moving surface $S(t)$

[p. 154, 2nd paragraph, about Ffowcs Williams–Hawkings, after [M345, Chapter 5]]: For associated inverse problems, see [10].

[p. 156, end of §9.3.2]: Additional references: [27]

10.2. Integral equations: direct method

[p. 163, below (10.13)]:

Transmission problems (Section 4.6.2) can be reduced to coupled pairs of time-domain boundary integral equations (TDBIEs) [M549, eqn (2)], [13, eqns (5) and (6)], much as they are in the frequency domain [M599, §6.2]. See also [24, §8.2]. It turns out to be advantageous to use a time derivative of standard TDBIEs, just as (10.13) is related to (10.12):

> Ideally, one could remove the time derivative and still solve the resulting equation. However, as explained in [M377, M378, M721, M722], for the solution of TDBIEs to be stable, basis and testing functions should be selected from appropriate Sobolev spaces. ... Following the analysis in [M721, M722], the testing function for TDBIEs without the time derivative should be selected as the time-derivative of the Dirac delta function. This means that a marching method which relies on point-testing in time (i.e., testing with Dirac deltas) does not yield a stable solution when it is used to solve TDBIEs without the time derivative. [13, p. 1066]

For acoustic scattering by inhomogeneous obstacles, see [24, §8.1]. For piecewise homogeneous obstacles, see [48].

10.3.1. Basic time-stepping method

[p. 166, just before §10.3.2, add:] For the damped wave equation (1.69), see [58].

10.3.2. Instabilities and remedies

[p. 166, 2nd paragraph]: "Frequency-domain integral equations ... M318]": add [13, 60, 59].

[p. 167, line 1]: "B-splines [M717, M223]": add [60, 59].

[p. 167, 3rd paragraph]: Pölz & Schanz [M702]... variable; see also [45].

[p. 167, end of 5th paragraph]: For another strategy, see [26].

10.4.2. Application [of CQM] to time-domain BIEs

[p. 170, 4 lines below (10.39)]: "For overviews ... M390]": add [33]. Then, after "For some criticism, see [M19, §2.2.1]": For many details and applications, see the book by Banjai & Sayas [8].

[Top of p. 171, after "CQMs with BDF 2 ... [M706]."] For mixed boundary conditions, see [52]. For the damped wave equation (1.69), see [6].

[Next paragraph]: "For some applications of Runge–Kutta ... M391]", [8, Chapter 5] and [48, 25].

10.5. Electromagnetics

[p. 176, above the quotation]: after "... scattering [M90, M399]." Add "See also the 1987 review by Miller [40]." Then "Subsequent work ... M219]", add [39, 50, 51].

[p. 176, below the quotation]: Nevertheless, explicit marching-on-in-time methods continue to be developed [M845, M171], [12].

"Time-domain versions ... M169]." Add [64]. [Further down]: "For fast methods ..." Add [57].

[Last paragraph]: After "M158]", add [17]. Then, change "For a review, see [M550]." to "For reviews, see [M550], [18] and [8, Chapter 6]."

10.6. Elastodynamics

[p. 177, line 3]: "For three-dimensional ... M679]": add [5, 15, 30, 55, 1].

[p. 178, penultimate paragraph]: "Another option ... [M281]." Add [68]

[p. 178, end of last paragraph]: See also [68]. The space-time energetic Galerkin method of Aimi et al. [M9, M10] has been extended to elastodynamics [2].

10.7.1. Clément's equation

For more on Clément's approach, see [14].

10.8. Method of fundamental solutions

[p. 182]: The representation (10.75) has been used [9, §3] in a Foldy-type scheme (see [M599, §8.3] for a description in the frequency domain) for multiple scattering by N small soft spheres. The problem of scattering by two soft spheres has been solved exactly, using time-domain addition theorems, by Azizoglu et al. [M32].

10.9. Use of Fourier transforms

[Bottom of p. 183]: "can be effective [M487]": add [34].

10.10.3. Comments and literature [on cracks and screens]

[p. 192, end of 2nd paragraph]: For more on scattering by thin flat screens, see [24, §9.2].

[Bottom of p. 192]: Boundary integral equations in the Laplace-transform domain have been used for plane-strain problems [22, 44] and for curved cracks in three dimensions [54].

[New section] 10.11. Small scatterers

Acoustic scattering by small obstacles in the frequency domain has an extensive literature, going back to Lord Rayleigh in the 19th century [M599, §8.2]. However, analogous results in the time domain are rare. A typical problem concerns the interaction of a sound pulse with a small obstacle. The beginning of a theory for such problems is outlined in [38], using time domain boundary integral equations. A key question is: what does it mean for an obstacle to be "small"? Earlier work includes [9, 53].

For scattering by a cluster of small scatterers, see [9, 53, 31]. In particular, Kachanovska [31] shows that merely using a Fourier transform of Foldy's frequency-domain method [M599, §8.3] can lead to instabilities.

References

p. 199. Reference [M138] is reprinted in [29], pp. 63–123.

References

[1] A. Aimi, G. Di Credico, H. Gimperlein & E.P. Stephan, Higher-order time domain boundary elements for elastodynamics: graded meshes and *hp* versions. *Numer. Math.* **154** (2023) 35–101. (doi:10.1007/s00211-023-01355-x)

[2] A. Aimi, S. Dallospedale, L. Desiderio & C. Guardasoni, A space-time energetic BIE method for 3D elastodynamics: the Dirichlet case. *Comput. Mech.* **72** (2023) 885–905. (doi:10.1007/s00466-023-02312-z)

[3] C.J.S. Alves & P.R.S. Antunes, Wave scattering problems in exterior domains with the method of fundamental solutions. *Numer. Math.* **156** (2024) 375–394. (doi:10.1007/s00211-024-01395-x)

[4] A. Bahari, G. Lefeuve-Mesgouez, A. Mesgouez & N. Popplewell, A comprehensive time-domain elasto-acoustics study of a fluid-filled spherical shell embedded in an elastic medium. *Soil Dynamics & Earthquake Engineering* **132** (2020) 106002 (27 pages). (doi:10.1016/j.soildyn.2019.106002)

[5] P.K. Banerjee, S. Ahmad & G.D. Manolis, Transient elastodynamic analysis of three-dimensional problems by boundary element method. *Earthquake Engineering & Structural Dynamics* **14** (1986) 933–949. (doi:10.1002/eqe.4290140609)

[6] L. Banjai & V. Gruhne, Efficient long-time computations of time-domain boundary integrals for 2D and dissipative wave equation. *J. Comp. Appl. Math.* **235** (2011) 4207–4220. (doi:10.1016/j.cam.2011.03.015)

[7] L. Banjai, C. Lubich & J. Nick, Time-dependent acoustic scattering from generalized impedance boundary conditions via boundary elements and convolution quadrature. *IMA J. Numer. Anal.* **42** (2022) 1–26. (doi:10.1093/imanum/draa091)

[8] L. Banjai & F.-J. Sayas, *Integral Equation Methods for Evolutionary PDE: A Convolution Quadrature Approach*. Cham: Springer, 2022.

[9] H. Barucq, J. Diaz, V. Mattesi & S. Tordeux, Asymptotic behavior of acoustic waves scattered by very small obstacles. *ESAIM: Mathematical Modelling & Numerical Analysis* **55** (2021) S705–S731. (doi:10.1051/m2an/2020047)

[10] C.-X. Bi, Y. Xu, Y.-B. Zhang & X.-Z. Zhang, A time-domain inverse method for the localization and quantification of unsteady rotating loading sources. *J. Sound & Vib.* **512** (2021) 116405 (15 pages). (doi:10.1016/j.jsv.2021.116405)

[11] W. Cai, W. Chen, J. Fang & S. Holm, A survey on fractional derivative modeling of power-law frequency-dependent viscous dissipative and scattering attenuation in acoustic wave propagation. *Appl. Mech. Rev.* **70** (2018) 030802 (12 pages). (doi:10.1115/1.4040402)

[12] R. Chen, S.B. Sayed, H.A. Ulku, & H. Bagci, Explicit marching-on-in-time solvers for second-kind time domain integral equations. In: *Advances in Time-Domain Computational Electromagnetic Methods* (ed. Q. Ren, S. Yan & A.Z. Elsherbeni) pp. 275–320. Hoboken, NJ: Wiley, 2022. (doi:10.1002/9781119808404.ch7)

[13] R. Chen, Y. Shi, S.B. Sayed & H. Bagci, On the spurious resonance modes of time domain integral equations for analyzing acoustic scattering from penetrable objects. *J. Acoust. Soc. Amer.* **151** (2022) 1064–1076. (doi:10.1121/10.0009401)

[14] Y.-M. Choi, B. Bouscasse, A.H. Clement, L. Gentaz, P. Ferrant, Y.J. Kim & S. Malenica, An alternative expression of the time domain free surface Green function for deep water. *Ocean Eng.* **267** (2023) 113267 (6 pages). (doi:10.1016/j.oceaneng.2022.113267)

[15] H.B. Coda & W.S. Venturini, Further improvements on three dimensional transient BEM elastodynamic analysis. *Engineering Analysis with Boundary Elements* **17** (1996) 231–243. (doi:10.1016/S0955-7997(96)00019-7)

[16] B. Deconinck, Q. Guo, E. Shlizerman & V. Vasan, Fokas's Unified Transform Method for linear systems. *Quart. Appl. Math.* **76** (2018) 463–488. (doi:10.1090/qam/1484)

[17] A. Dély, F.P. Andriulli & K. Cools, Large time step and DC stable TD-EFIE discretized with implicit Runge–Kutta methods. *IEEE Trans. Antennas & Propagation* **68** (2020) 976–985. (doi:10.1109/TAP.2019.2943443)

[18] A. Dély, A. Merlini, K. Cools & F.P. Andriulli, Convolution quadrature time domain integral equation methods for electromagnetic scattering. In: *Advances in Time-Domain Computational Electromagnetic Methods* (ed. Q. Ren, S. Yan & A.Z. Elsherbeni) pp. 321–359. Hoboken, NJ: Wiley, 2022. (doi:https://doi.org/10.1002/9781119808404.ch8)

[19] P. Drude, *The Theory of Optics*. New York: Longmans, Green & Co., 1902.

[20] D.B. Duncan, Positivity of a weakly singular operator and approximation of wave scattering from the sphere. *J. Integral Equations & Applications* **34** (2022) 317–333. (doi:10.1216/jie.2022.34.317)

[21] T.M. Dunster, A. Gil, D. Ruiz-Antolín & J. Segura, Computation of the reverse generalized Bessel polynomials and their zeros. *Computational & Mathematical Methods* **3** (2021) e1198 (12 pages). (doi:10.1002/cmm4.1198)

[22] S. Ebrahimi & A.-V. Phan, Dynamic analysis of cracks using the SGBEM for elastodynamics in the Laplace-space frequency domain. *Engineering Analysis with Boundary Elements* **37** (2013) 1378–1391. (doi:10.1016/j.enganabound.2013.07.004)

[23] A.S. Fokas & K. Kalimeris, Extensions of the d'Alembert formulae to the half line and the finite interval obtained via the unified transform. *IMA J. Appl. Math.* **87** (2022) 1010–1042. (doi:10.1093/imamat/hxac030)

[24] J.T. Fokkema & P.M. van den Berg, *Seismic Applications of Acoustic Reciprocity*. Amsterdam: Elsevier, 1993.

[25] M. Ganesh & F. Le Louër, A high-order algorithm for time-domain scattering in three dimensions. *Advances in Comput. Math.* **49** (2023), article 46 (43 pages). (doi:10.1007/s10444-023-10033-3)

[26] Z. Gao, Z. Li & Y. Liu, A time-domain boundary element method using a kernel-function library for 3D acoustic problems. *Engineering Analysis with Boundary Elements* **161** (2024) 103–112. (doi:10.1016/j.enganabound.2024.01.001)

[27] G. Ghorbaniasl, L. Siozos-Rousoulis & C. Lacor, A time-domain Kirchhoff formula for the convective acoustic wave equation. *Proc. R. Soc. A* **472** (2016) 20150689 (17 pages). (doi:10.1098/rspa.2015.0689)

[28] C.C. Handapangoda, M. Premaratne & P.N. Pathirana, Plane wave scattering by a spherical dielectric particle in motion: a relativistic extension of the Mie theory. *Progress In Electromagnetics Research* **112** (2011) 349–379. (doi:10.2528/PIER10102901)

[29] K. Hentschel & N.Y. Zhu (eds.), *Gustav Robert Kirchhoff's Treatise "On the Theory of Light Rays"*. Hackensack: World Scientific, 2017.

[30] F. Janod & O. Coutant, Seismic response of three-dimensional topographies using a time-domain boundary element method. *Geophys. J. Int.* **142** (2000) 603–614. (doi:10.1046/j.1365-246x.2000.00183.x)

[31] M. Kachanovska, A new class of uniformly stable time-domain Foldy–Lax models for scattering by small particles. Acoustic sound-soft scattering by circles. *Multiscale Modeling & Simulation* **22** (2024) 1–38. (doi:10.1137/22M1495512)

[32] G. Kirchhoff, On the theory of light rays. Translation of [M482]. In: [29], pp. 31–61.

[33] S. Langer & M. Schanz, Time domain boundary element method. In: *Computational Acoustics of Noise Propagation in Fluids – Finite and Boundary Element Methods* (eds. S. Marburg & B. Nolte) pp. 495–516. Berlin: Springer, 2008.

[34] J. Lin, An efficient and high-order frequency-domain approach for transient acoustic-structure interactions in three dimensions. *Int. J. Numer. Meth. Eng.* **108** (2016) 790–816. (doi:10.1002/nme.5243)

[35] X. Liu, H. Wu, W. Jiang & R. Sun, A fast multipole boundary element method for three-dimensional acoustic problems in a subsonic uniform flow. *Int. J. Numer. Meth. Fluids* **93** (2021) 1669–1689. (doi:10.1002/fld.4947)

[36] X. Liu & J. Xu, A low-frequency fast multipole boundary element method for acoustic problems in a subsonic uniform flow. *Engineering Analysis with Boundary Elements* **162** (2024) 102–116. (doi:10.1016/j.enganabound.2024.01.026)

[37] Y. Ma & J. Sun, Computation of scattering poles using boundary integrals. *Appl. Math. Lett.* **146** (2023) 108792 (7 pages). (doi:10.1016/j.aml.2023.108792)

[38] P.A. Martin, Acoustic scattering by a small obstacle in the time domain. *J. Acoust. Soc. Amer.* **151** (2022) 3066–3069. (doi:10.1121/10.0010449)

[39] H. Mieras & C.L. Bennett, Space-time integral equation approach to dielectric targets. *IEEE Trans. Antennas & Propagation* **30** (1982) 2–9. (doi:10.1109/TAP.1982.1142753)

[40] E.K. Miller, An overview of time-domain integral-equation models in electromagnetics. *J. Electromagnetic Waves & Applications* **1** (1987) 269–293. (doi:10.1163/156939387X00054)

[41] H.-M. Nguyen & L.V. Nguyen, Generalized impedance boundary conditions for strongly absorbing obstacle: The full wave equation. *Math. Models & Methods in Applied Sciences* **25** (2015) 1927–1960. (doi:10.1142/S0218202515500499)

[42] J. Nick, B. Kovács & C. Lubich, Time-dependent electromagnetic scattering from thin layers. *Numer. Math.* **150** (2022) 1123–1164. (doi:10.1007/s00211-022-01277-0)

[43] S. Petropavlovsky, S. Tsynkov & E. Turkel, 3D time-dependent scattering about complex shapes using high order difference potentials. *J. Comp. Phys.* **471** (2022) 111632 (16 pages). (doi:10.1016/j.jcp.2022.111632)

[44] D.-H. Phan, T.-T. Phan, T.-K. Nguyen & A.-V. Phan, Dynamic stress intensity factors for multiple parallel cracks in an infinite domain under the passage of a normal incident impact or blast P-wave. *Engineering Analysis with Boundary Elements* **106** (2019) 75–85. (doi:10.1016/j.enganabound.2019.04.030)

[45] D. Pölz & M. Schanz, On the space-time discretization of variational retarded potential boundary integral equations. *Computers & Math. with Applications* **99** (2021) 195–210. (doi:10.1016/j.camwa.2021.08.004)

[46] H. Poritsky, Extension of Weyl's integral for harmonic spherical waves to arbitrary wave shapes. *Comm. Pure Appl. Math.* **4** (1951) 33–42. (doi:10.1002/cpa.3160040106)

[47] A.G. Ramm, Calculating resonances (natural frequencies) and extracting them from transient fields. *J. Math. Phys.* **26** (1985) 1012–1020. (doi:10.1063/1.526585). Erratum: **27** (1986) 419.

[48] A. Rieder, F.-J. Sayas & J.M. Melenk, Time domain boundary integral equations and convolution quadrature for scattering by composite media. *Math. Comp.* **91** (2022) 2165–2195. (doi:10.1090/mcom/3730)

[49] M. Roseau, *Aymptotic Wave Theory.* Amsterdam: North-Holland, 1976.

[50] B.P. Rynne, Time domain scattering from arbitrary surfaces using the Electric Field Integral Equation. *J. Electromagnetic Waves & Applications* **5** (1991) 93–112. (doi:10.1163/156939391X00491)

[51] B.P. Rynne, Time domain scattering from dielectric bodies. *Electromagnetics* **14** (1994) 181–193. (doi:10.1080/02726349408908378)

[52] D. Seibel, Boundary element methods for the wave equation based on hierarchical matrices and adaptive cross approximation. *Numer. Math.* **150** (2022) 629–670. (doi:10.1007/s00211-021-01259-8)

[53] M. Sini, H. Wang & Q. Yao, Analysis of the acoustic waves reflected by a cluster of small holes in the time-domain and the equivalent mass density. *Multiscale Modeling & Simulation* **19** (2021) 1083–1114. (doi:10.1137/20M1319693)

[54] V. Sládek & J. Sládek, Transient elastodynamic three-dimensional problems in cracked bodies. *Applied Mathematical Modelling* **8** (1984) 2–10. (doi:10.1016/0307-904X(84)90169-0)

[55] A. Sohrabi-Bidar, M. Kamalian & M.K. Jafari, Time-domain BEM for three-dimensional site response analysis of topographic structures. *Int. J. Numer. Meth. Eng.* **79** (2009) 1467–1492.

(doi:10.1002/nme.2619)
[56] J. Sun, S.-L.J. Hu & H. Li, Computing free motion of floating structures by pole-residue operations. *Appl. Ocean Res.* **109** (2021) 102558 (11 pages). (doi:10.1016/j.apor.2021.102558)
[57] T. Takahashi, A fast time-domain boundary element method for three-dimensional electromagnetic scattering problems. *J. Comp. Phys.* **482** (2023) 112053 (26 pages). (doi:10.1016/j.jcp.2023.112053)
[58] T. Takahashi, A time-domain boundary element method for the 3D dissipative wave equation: Case of Neumann problems. *Int. J. Numer. Meth. Eng.* **124** (2023) 5263–5292. (doi:10.1002/nme.7343)
[59] T. Takahashi, N. Miyazawa & M. Tanigawa, A three-dimensional shape optimization for transient acoustic scattering problems using the time-domain boundary element method. *Int. J. Numer. Meth. Eng.* **124** (2023) 482–512. (doi:10.1002/nme.7130)
[60] T. Takahashi, M. Tanigawa & N. Miyazawa, An enhancement of the fast time-domain boundary element method for the three-dimensional wave equation. *Computer Phys. Comm.* **271** (2022) 108229 (25 pages). (doi:10.1016/j.cpc.2021.108229)
[61] F.M. Tesche, On the analysis of scattering and antenna problems using the singularity expansion technique. *IEEE Trans. Antennas & Propagation* **21** (1973) 53–62. (doi:10.1109/TAP.1973.1140398)
[62] V. Twersky, Relativistic scattering of electromagnetic waves by moving obstacles. *J. Math. Phys.* **12** (1971) 2328–2341. (doi:10.1063/1.1665539)
[63] P. Vincent, Singularity expansions for cylinders of finite conductivity. *Appl. Phys.* **17** (1978) 239–248. (doi:10.1007/BF00886952)
[64] X. Wang, Y. Shi, M. Lu, B. Shanker, E. Michielssen & H. Bağcı, Stable and accurate marching-on-in-time solvers of time domain EFIE, MFIE, and CFIE based on quasi-exact integration technique. *IEEE Trans. Antennas & Propag.* **69** (2021) 2218–2229. (doi:10.1109/TAP.2020.3026867)
[65] M.R. Whittam, A.G. Lamprianidis, Y. Augenstein & C. Rockstuhl, Identifying regions of minimal backscattering by a relativistically moving sphere. *Phys. Rev. A* **108** (2023) 043510 (14 pages). (doi:10.1103/PhysRevA.108.043510)
[66] T.W. Wu & L. Lee, A direct boundary integral formulation for acoustic radiation in a subsonic uniform flow. *J. Sound Vib.* **175** (1994) 51–63. (doi:10.1006/jsvi.1994.1310)
[67] A. Zangwill, *Modern Electrodynamics*. Cambridge: Cambridge University Press, 2013.
[68] L. Zhao, H. Dong & F. Ma, Inverse obstacle scattering for elastic waves in the time domain. *Inverse Problems* **38** (2022) 045005 (30 pages). (doi:10.1088/1361-6420/ac531c)