

# Symmetry Anyone?

## Part 1

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**Thursday, February 23, 2017, 4:00p.m.**

# Acknowledgements

Marcus du Sautoy: Symmetry, Reality's Riddle

Ian Stewart: Why Beauty is Truth – The History of Symmetry

Mario Livio: The Equation That Couldn't Be Solved

Martin Gardner: The New Ambidextrous Universe – Symmetry and Asymmetry

Keshlan Govinder: A Symmetric View of Life

Stephen Strogatz: Sync: How Order Emerges from Chaos in the Universe, Nature, and Daily Life



# Outline

- A couple of puzzles
- Nature & architecture
- Art & design
- Music & literature
- Symmetry in science
- Symmetries in mathematics
- Rubik's cube
- My research:
  - Lie-point symmetries
  - Conservation laws
  - Lax pairs
- Conclusions

# What is Symmetry?

Galileo Galilei (1564–1642)

The universe cannot be read  
until we have learnt the  
language and become  
familiar with the characters  
in which it is written.

It is written in  
mathematical language,  
and the letters are triangles,  
circles and  
other geometrical figures,  
without which means it is  
humanly impossible  
to comprehend a single word.



# What is Symmetry?

- Dictionary: **Exact correspondence** of form and constituent configuration on opposite sides of a dividing line or plane or about a center or an axis.
- **Invariance** under motions (flips, turns, translations) or mathematical operations (symmetries are transformations).
- Symmetry preserves distances, angles, sizes, shapes, etc.
- Symmetry creates **stability** and **order**.
- Paraphrasing Wigner: “ “... the unreasonable effectiveness of using **symmetries** in mathematics... .”

## Father – Daughter Puzzle

Today, the ages of a father and his daughter add up to 40 years. Five years from now, the father will be 4 times older than his daughter. How old is each today?

Solution:  $F$  : age of the father (today).  
 $D$  : age of the daughter (today).

$$F + D = 40$$

$$F + 5 = 4(D + 5)$$

Eliminate  $F = 40 - D$  and solve for  $D$  (smaller number):

$$45 - D = 4(D + 5) \quad \text{or} \quad 5D = 25$$

Hence,  $D = 5$  and  $F = 40 - D = 35$ .

## Symmetry Reduces Complexity

Solution:             $20 + x$  : age of the father (today).  
                          $20 - x$  : age of the daughter (today).

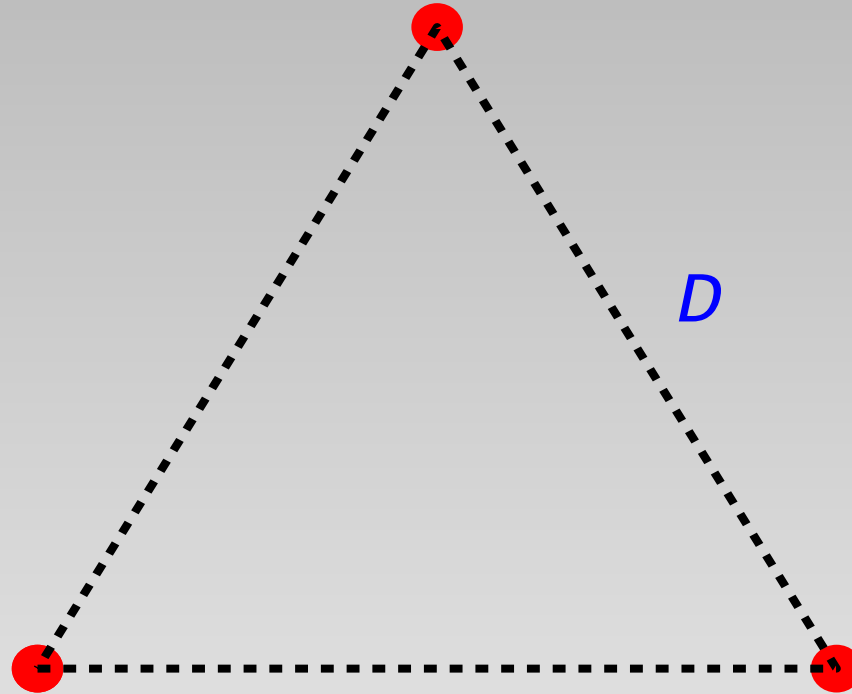
$$25 + x = 4(25 - x) \quad \text{or} \quad 5x = 75$$

Hence,  $x = 15$ .

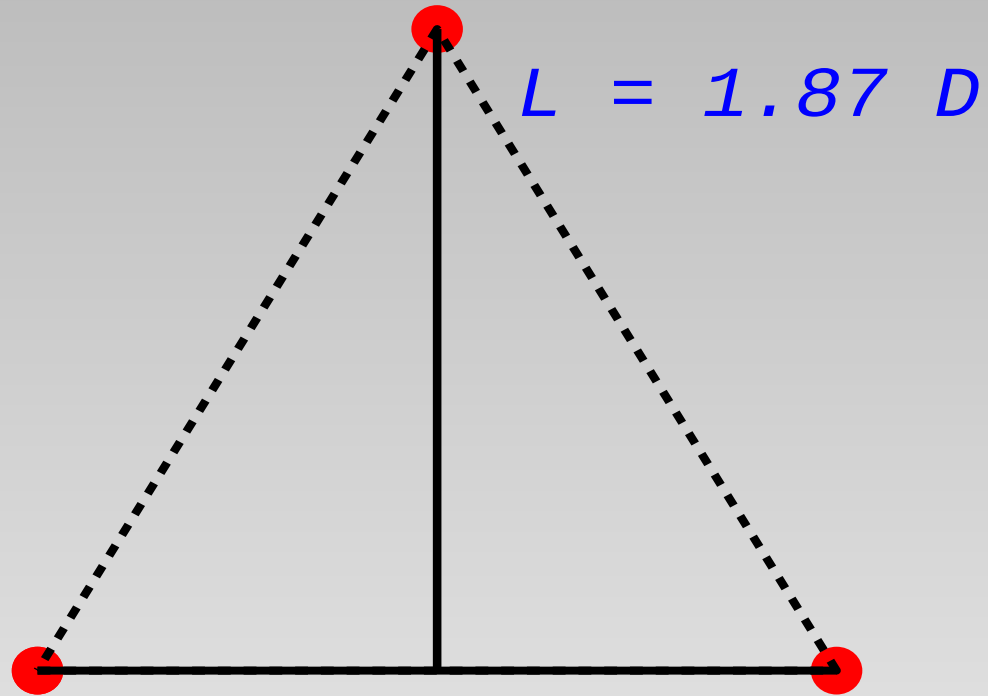
Father is  $20 + x = 35$ .      Daughter is  $20 - x = 5$ .

# Shortest path puzzle

Connecting Cities with Shortest Road System

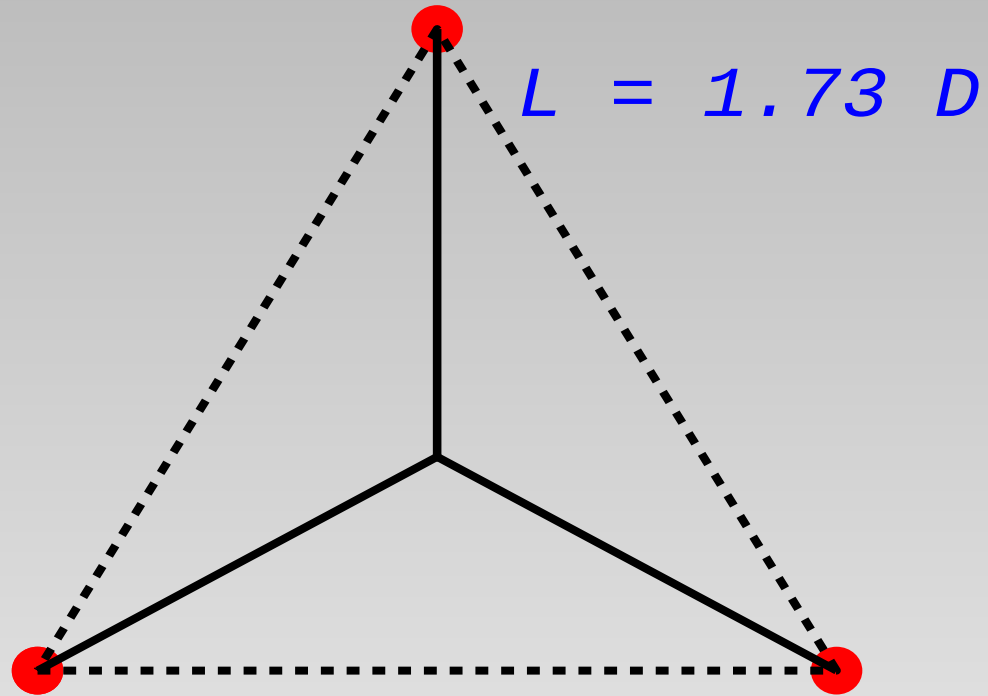


Three cities in equilateral triangle

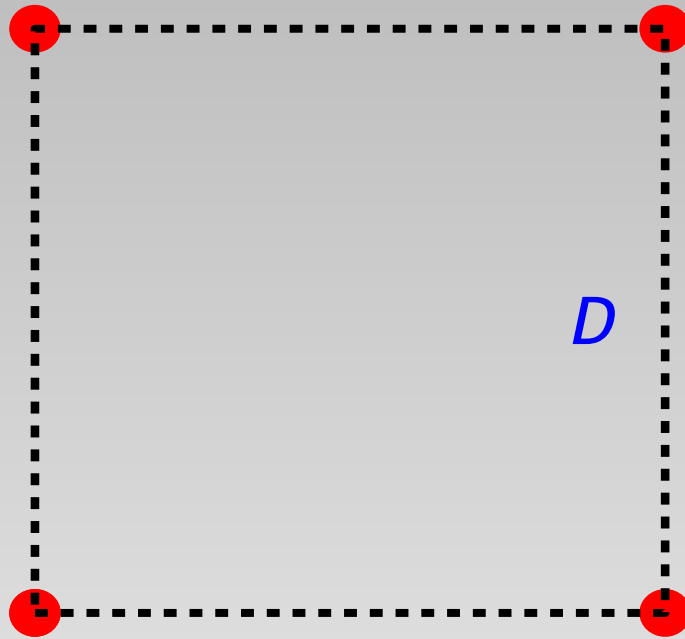


One choice of a road system



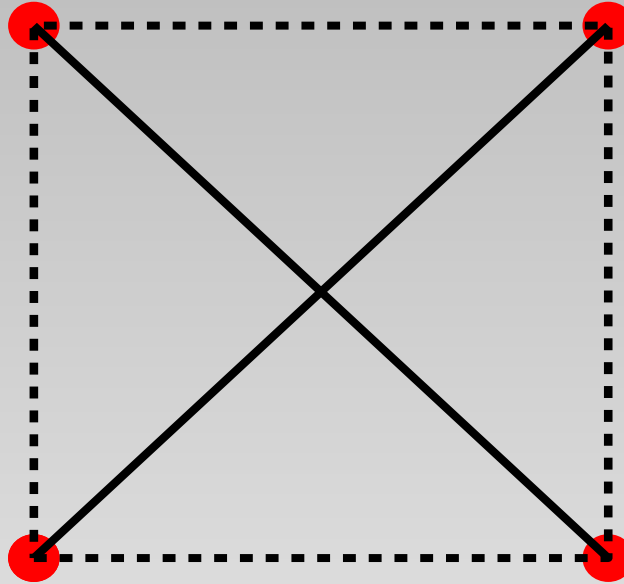


Shortest road system connecting three cities



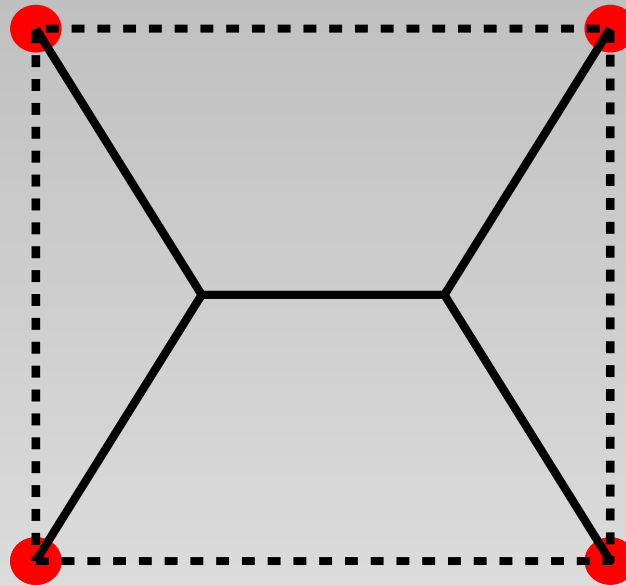
Four cities in a square

$$L = 2.83 D$$

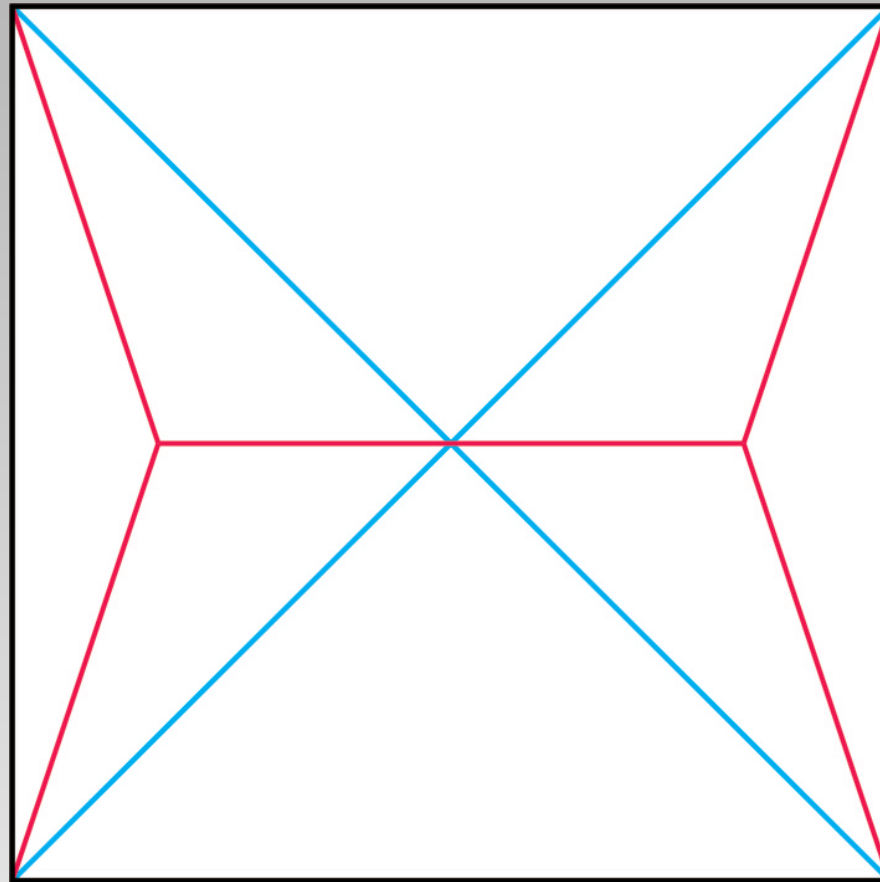


One choice of a road system

$$L = 2.73 D$$



**Shortest road system connecting 4 cities**



**Add the triangles on the square!**

# Nature





# Nature



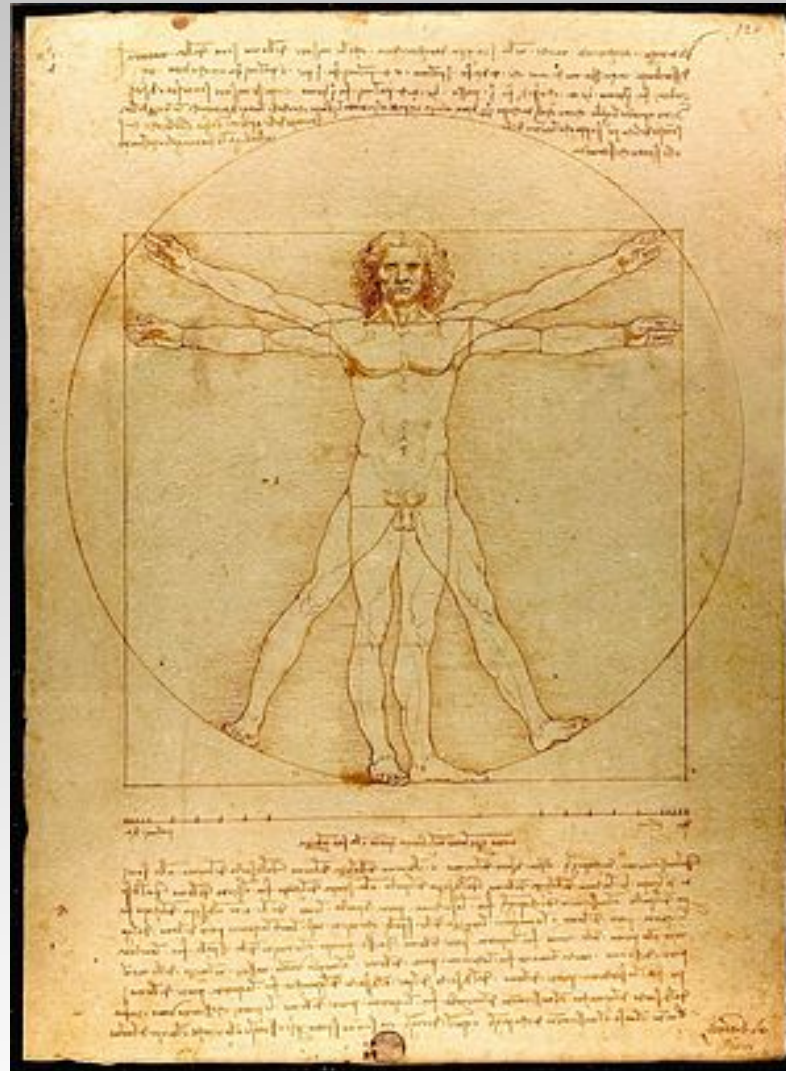
# Human Face





# Human Body

## Vitruvian Man – Leonardo da Vinci (c. 1490)



# Architecture





# Architecture: From Symmetry to Loss of Symmetry





# Symmetric or Not?



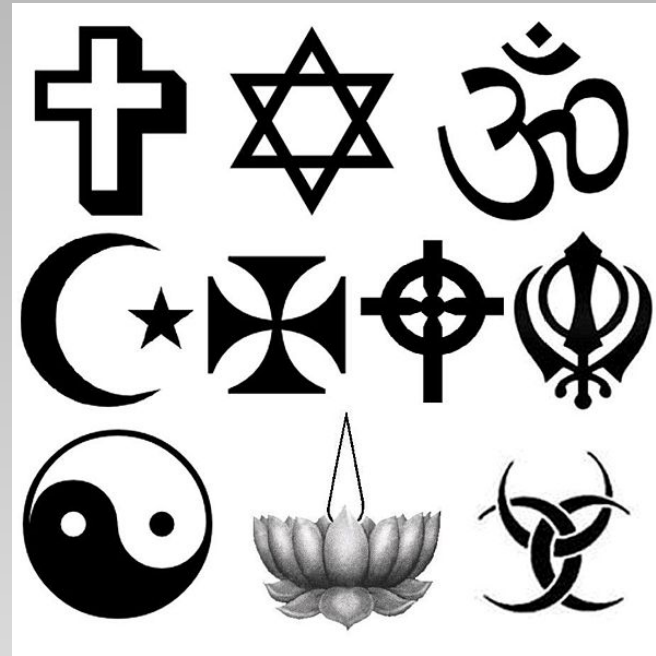


# Art and Design





# Symmetric or Not?



# Music

## Fugues – Bach

FIGURE 2. Intervals in the well-tempered system.

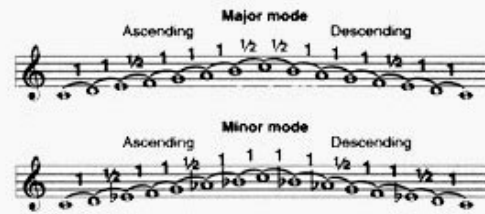
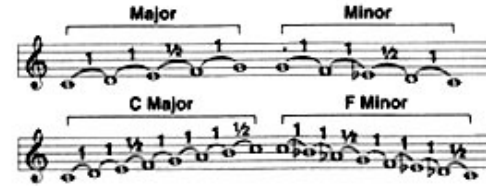
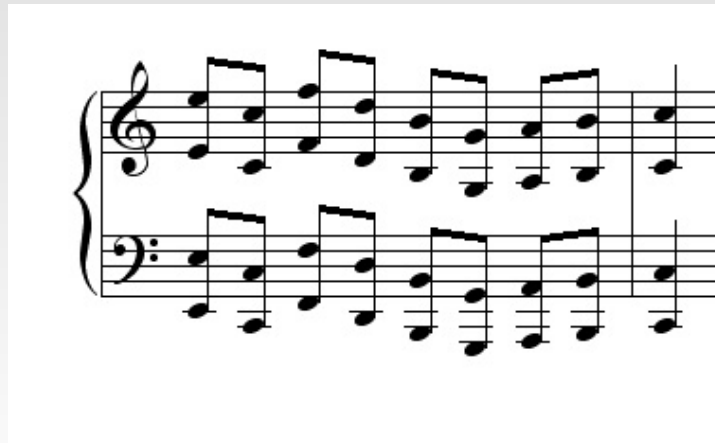


FIGURE 4. Inversion of the major mode generates a minor mode of a different key.



Arch form – **A****B****C****B****A** – Béla Bartók.

Scales/chords – Mozart



# Literature

## Palindromes: Words and Sentences

dud, civic, madam

“Madam, I’m Adam”

A man, a plan, a canal: Panama

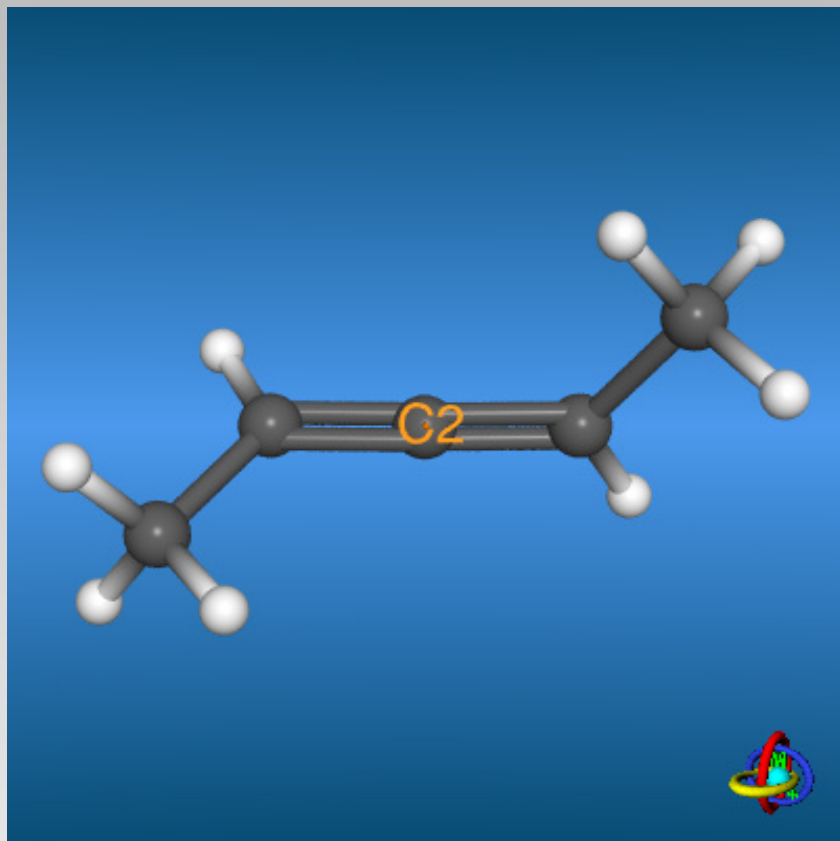
Sit on a potato pan, Otis

Palindrome Story – 2002 words long; published on  
Web on February 20th, 2002 or 20-02-2002.

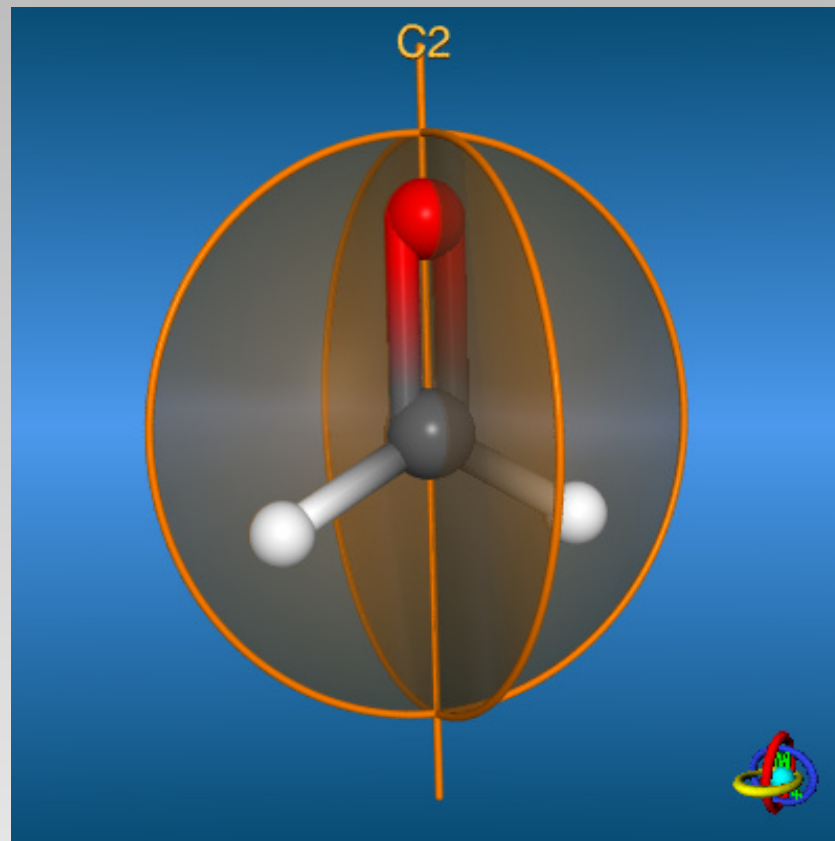
Longest sentence: 17,826 words!



# Symmetry in Chemistry



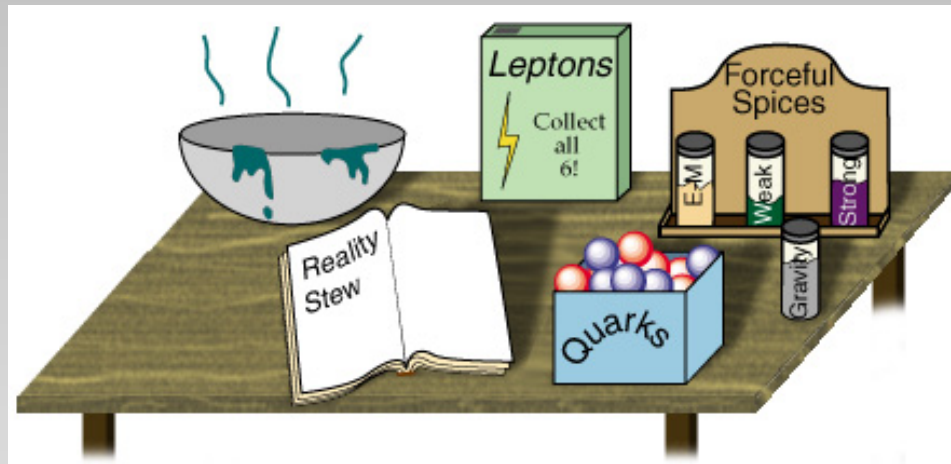
Allene



Methanal

# Symmetry in Physics

## Universality of the Laws of Physics



Laws of motion are independent of location (space invariant).

The Lorentz group (of Lorentz transformations) expresses the fundamental symmetry of space and time of all known fundamental laws of nature.

For example, the following laws, equations, and theories respect Lorentz symmetry:

- The kinematical laws of special relativity,
- Maxwell's field equations in the theory of electromagnetism,
- The Dirac equation in the theory of the electron,
- The Standard Model of Particle Physics.